Today, we will:

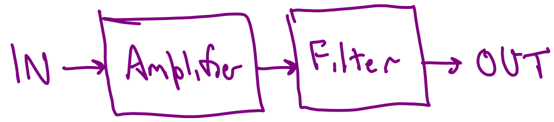
- Do some example problems – Filters
- Review the pdf module: **Digital Filters**

Example: Low-pass filter circuit

Given: The following components are available:

- two resistors, 10.0 kΩ and 87.0 kΩ
- a capacitance decade box

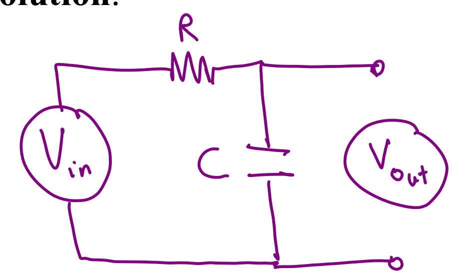
Input has signal and noise →



The input voltage contains a signal frequency of 50 Hz, and some unwanted noise at 1000 Hz. We want to amplify the signal, with $G_{\text{amplifier}} \approx 8.5$, and we decide to use a first-order low-pass filter with a cutoff frequency of 200 Hz to attenuate the noise.

(*a*) **To do**: Draw the filter circuit and calculate the required capacitance.

Solution:



LOW-PASS FILTER

$$f_{\text{cutoff}} = \frac{1}{2\pi RC} \rightarrow \text{Solve for } C:$$

$$\boxed{C = \frac{1}{2\pi R f_{\text{cutoff}}}}$$

Let's pick the 87.0 kΩ resistor

Coulomb, ampere, Farad

$$C = \frac{1}{2\pi(87{,}000\ \cancel{\Omega})(200\ \frac{1}{\cancel{s}})} \underbrace{\left(\frac{\cancel{C}}{\cancel{s}\cdot\cancel{A}}\right)\left(\frac{\cancel{\Omega}\cdot\cancel{A}}{\cancel{V}}\right)\left(\frac{F\cdot\cancel{V}}{\cancel{C}}\right)}_{\text{Unity conversion factors}}$$

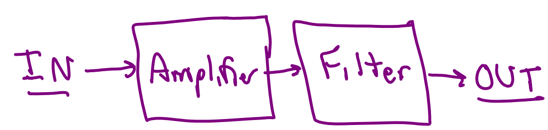
- So, $C = 9.1468 \times 10^{-9}$ F → $\boxed{C = 0.00915\ \mu F}$

[In real life, a capacitor of 0.01 μF is close enough & this is a common capacitance]

(*b*) **To do**: Calculate the overall gain of both the signal and the noise, and calculate how well we have improved the signal-to-noise ratio (SNR). *Note*: ***SNR is the ratio of signal power to noise power.*** Since electrical power is proportional to voltage^2, we define SNR as

$$\text{SNR} = \frac{\text{Power}_{\text{signal}}}{\text{Power}_{\text{noise}}} = \left(\frac{|V_{\text{signal}}|}{|V_{\text{noise}}|}\right)^2$$, or in terms of decibels, $$\text{SNR}_{\text{dB}} = 20\log_{10}\left(\frac{|V_{\text{signal}}|}{|V_{\text{noise}}|}\right).$$

Solution:



- Signal: $f = 50$ Hz → $G_{\text{overall}} = G_{\text{amp}} \cdot G_{\text{filter}}$

$$= (8.50)\frac{1}{\sqrt{1+\left(\frac{50}{200}\right)^2}}$$

(given)

recall, for a 1st-order low-pass filter,

$$G = \frac{1}{\sqrt{1+\left(f/f_{\text{cutoff}}\right)^2}}$$

$$G_{\text{overall}} = 8.2462$$ ← This is the overall gain of the signal (the 50 Hz component)

• Repeat for the noise @ $f = 1000$ Hz

$$G_{overall} = G_{amp} G_{filter} = (8.50)\frac{1}{\sqrt{1+\left(\frac{1000}{200}\right)^2}} = \underline{\underline{1.6670}}$$

This is the overall gain of the noise

Notice → $G_{overall}$ of signal is > $G_{overall}$ of noise, which is what we desire → the filter will improve the signal-to-noise ratio

• Compare the original SNR to the signal-conditioned (i.e., low-pass filtered) SNR.

$$(SNR)_{original} = \left(\frac{|V|_{signal}}{|V|_{noise}}\right)^2$$

$$(SNR)_{filtered} = \left(\frac{8.2462\,|V|_{signal}}{1.6670\,|V|_{noise}}\right)^2$$

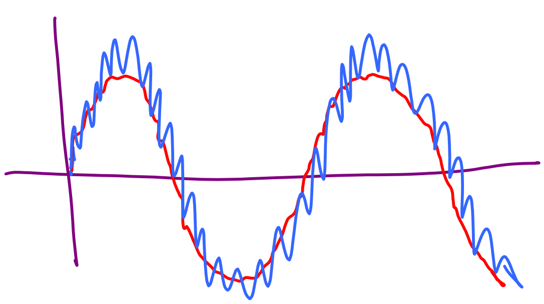
The ratio of these is the improvement.

Note: We are not given the values of $|V|_{signal}$ & $|V|_{noise}$, but they cancel out when we calculate the ratio.

$$\frac{(SNR)_{filtered}}{(SNR)_{original}} = \left(\frac{8.2462}{1.6670}\right)^2 = 24.47 \approx 24.5$$

$\boxed{\text{So, we have improved the SNR by a factor of 24.5 by putting a low-pass filter in line between the amplifier \& the DAQ}}$

Bottom line: When there is unwanted high frequency noise, we can improve the signal-to-noise ratio by using a low-pass filter, and the filter also reduces the potential of aliasing



—— unfiltered (has high-frequency noise)

—— filtered (we have reduced or attenuated the high-frequency noise [but not completely]

Example: Digital data acquisition and Filters

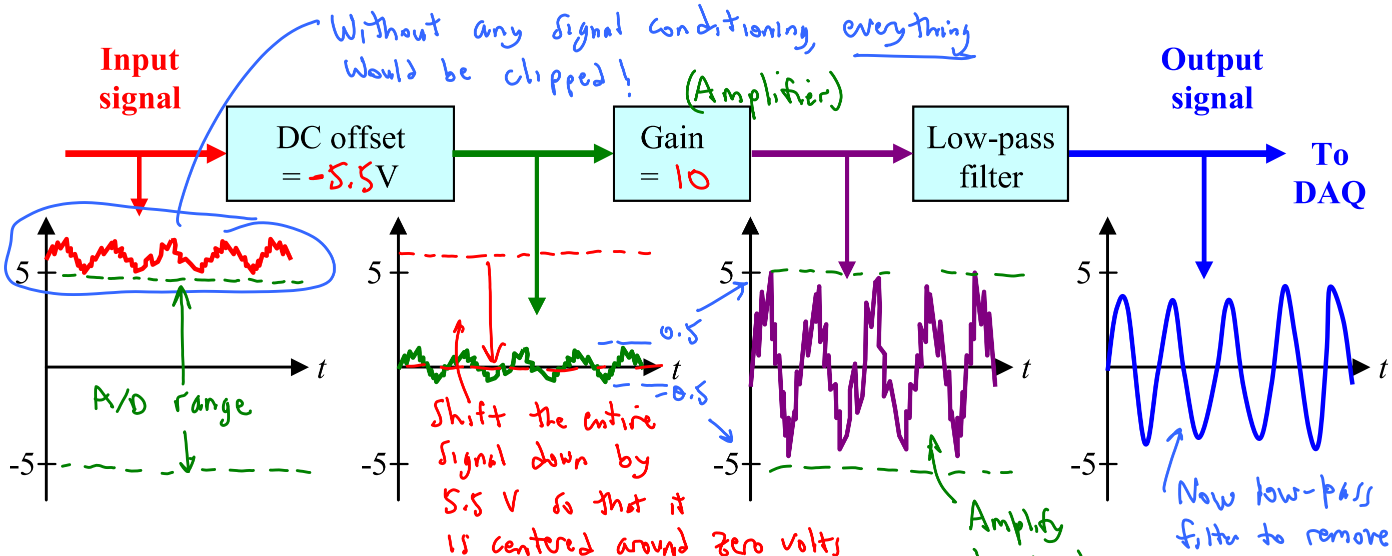
Given: A voltage signal has a frequency around 100 Hz, and ranges from 5.0 to 6.0 volts. There is also some unwanted AC noise at a frequency around 5000 Hz, with an amplitude of ±0.005 V. The digital data acquisition system is 12-bit, and has a range from -5 to 5 volts.

(***a***) **To do**: Calculate an appropriate DC offset and gain in order to utilize the full range of the A/D system.

[We will learn later how to build these circuits (DC offset & amplifier)]

Solution: $\boxed{\text{DC offset} = -5.5\text{ V}}$, $\boxed{\text{Gain} = 10}$.

Schematic diagram of the circuitry:



(***b***) **To do**: Design a first-order filter with these requirements:

- Use resistors and capacitors only
- Don't attenuate the 100 Hz signal by more than 25% (after the gain).
- Attenuate the 5000 Hz noise to lower than the quantizing error of the A/D system (in other words, remove it completely, since its amplitude will be reduced to the noise level of the A/D converter).

Solution: (a) See answers and discussions above.

✱ There are two problems with our original setup:

(1) Signal is clipped (all our voltages are > 5 V, so they would be clipped by the -5 to 5 V A/D converter)

(2) Even after we shift the signal down to avoid clipping, we are using a small portion of the A/D range

(-0.5 to 0.5 V = 1 V range out of -5 to 5 V = 10 V A/D)

[We are using only 1/10th of the available range of the A/D!]

(b) Low-pass filter:

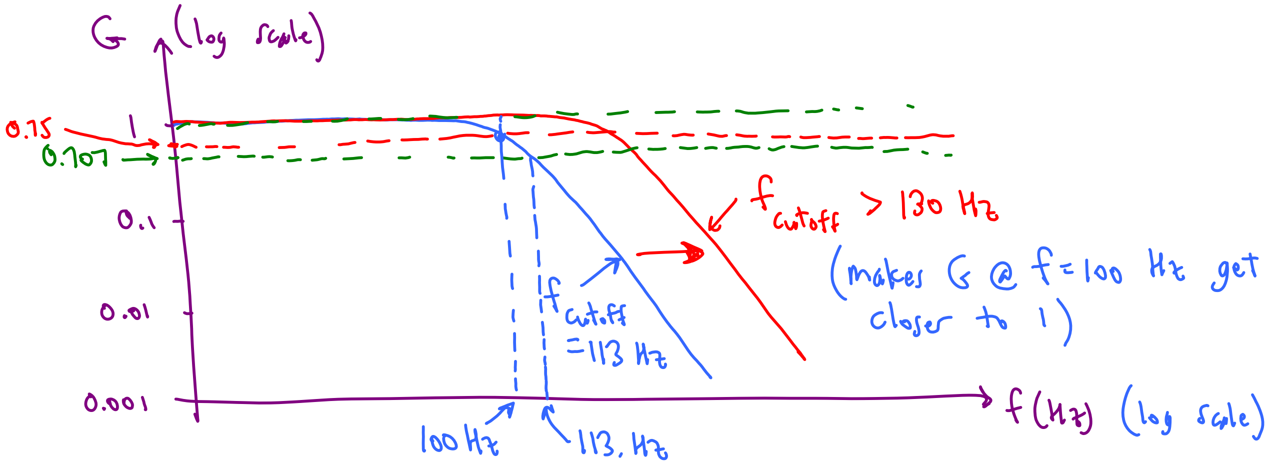
- First criterion — use R & C only (passive 1^{st}-order low-pass filter)
- Now add a low-pass filter to remove the high frequency noise.
- Calculate the minimum cutoff frequency such that the second criterion is met → the signal @ 100 Hz is attenuated by not more than 25% → $\boxed{\text{Set } G \geq 0.75 \text{ @ } 100 \text{ Hz}}$

- For a first-order filter, n=2, $G = \dfrac{1}{\sqrt{1+\left(\dfrac{f}{f_{cutoff}}\right)^2}}$
- Solve for f_{cutoff} → $\boxed{f_{cutoff} = \dfrac{f}{\sqrt{\dfrac{1}{G^2}-1}}}$

  (leaving out the algebra)

- Plug in $f = 100$ Hz & $G = 0.75$ → $f_{cutoff} = \dfrac{100 \text{ Hz}}{\sqrt{\dfrac{1}{(0.75)^2}-1}} = 113.$ Hz

This value of $f_{cutoff} = 113.$ Hz is the lower limit, because any $f_{cutoff} > 113$ Hz will shift the Bode plot to the right & will make $G > 0.75$, which would be good.



• Third Criterion → Remove the high frequency noise completely

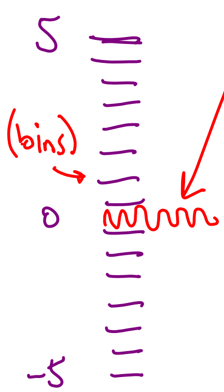
• Recall, Quantizing error of the A/D converter $= \pm \dfrac{V_{max} - V_{min}}{2^{N+1}}$

Here, A/D is 12-bit ($N=12$) & range is -5 (V_{min}) to 5 (V_{max}) V

So, quantizing error $= \pm \dfrac{5-(-5)\,V}{2^{12+1}} = \pm 0.00122\ V$

• Our goal is to attenuate the 5000 Hz noise amplitude so much that it gets "lost" in the quantizing error of the A/D, as illustrated below

A/D bins



☆ This is the key

If the amplitude is reduced such that the noise fluctuations are smaller than one bin, we will not be able to measure it!

• So, now we calculate the upper limit of f_{cutoff} that will kill the noise as shown to the left

• Caution → The noise amplitude is $\pm 0.005\ V$, but we amplify by 10.

So, the amplitude of the noise going into the A/D is not $\pm 0.005\ V$, but is $\pm 0.05\ V$

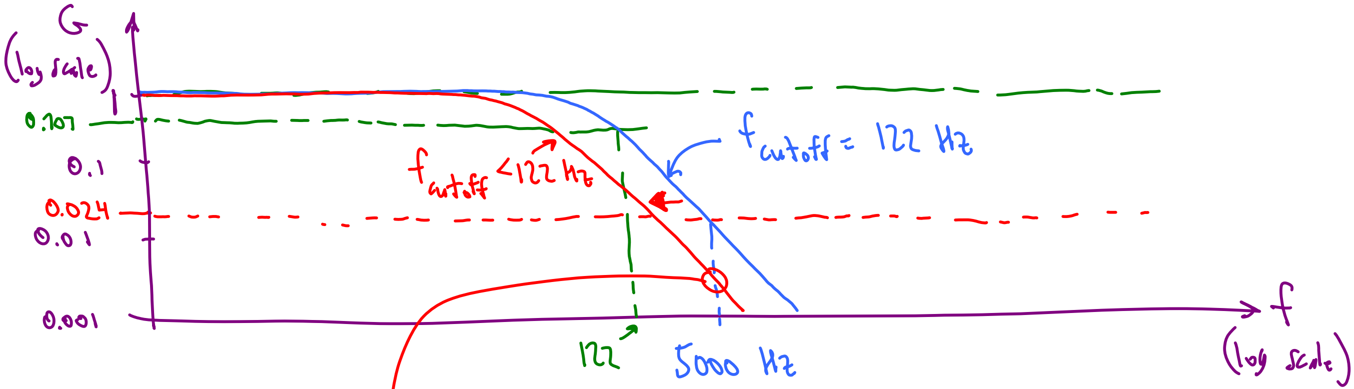
• Set $G = \left|\dfrac{V_{out}}{V_{in}}\right| = \left|\dfrac{\pm 0.00122\ V}{\pm 0.05\ V}\right| = 0.0244$ = minimum G to kill all the noise

Quantizing error

amplitude of the noise

• Calculate $f_{cutoff} = \dfrac{f}{\sqrt{\frac{1}{G^2} - 1}} = \dfrac{5000\ Hz}{\sqrt{\frac{1}{0.0244^2} - 1}} = 122.\ Hz$

☆ This value of f_{cutoff} is the upper limit, since any $f_{cutoff} < 122\ Hz$ would attenuate the noise even more!

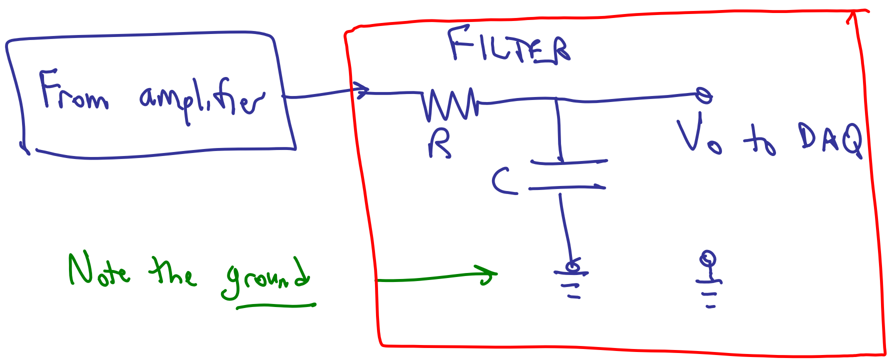


Any f_{cutoff} < 122 Hz shifts the Bode plot to the left & G at f = 5000 Hz would be even smaller than the requirement

- Bottom line → need $113 < f_{cutoff} < 122$ Hz to meet all the criteria

- We pick f in the middle, i.e. pick $\boxed{f_{cutoff} = 117 \text{ Hz}}$

- Circuit: Low-pass filter with $f_{cutoff} = 117$ Hz:



- Pick a capacitor → C = 0.100 μF is a popular one. Thus,

$$R = \frac{1}{2\pi f_{cutoff} C} = \frac{1}{2\pi \left(117 \frac{1}{\cancel{s}}\right)\left(0.100 \times 10^{-6} \cancel{F}\right)} \underbrace{\left(\frac{\cancel{F} \cdot \cancel{V}}{\cancel{C}}\right)\left(\frac{\cancel{C}}{\cancel{s} \cdot \cancel{A}}\right)\left(\frac{\Omega \cdot \cancel{A}}{\cancel{V}}\right)}_{\text{Unity conversion factors}}$$

$R = 13{,}603\ \Omega$, or $\boxed{R = 13.6\ k\Omega}$

- Final answer $\left.\begin{array}{l} R = 13.6\ k\Omega \\ C = 0.100\ \mu F \end{array}\right\}$ These values will do the job for us!