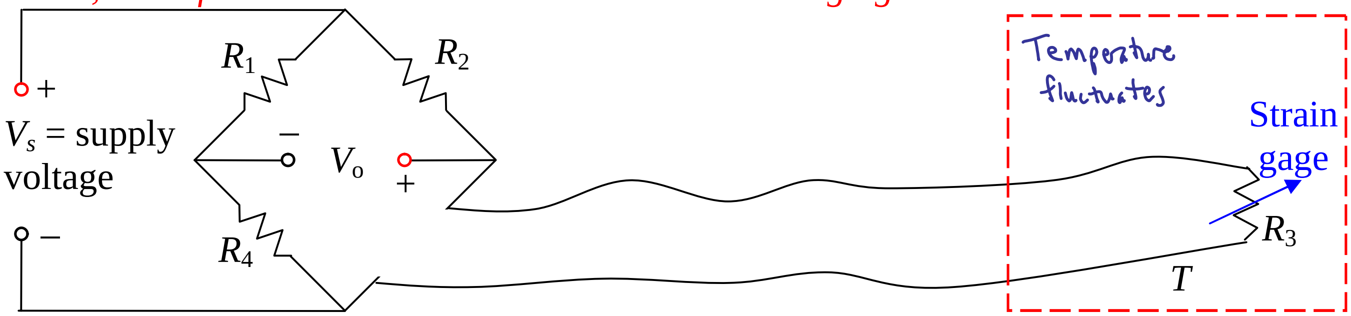
Today, we will:

- Discuss additional items about strain gages that are not in the pdf notes: **temperature compensation, transverse strain, strain on arbitrary surfaces**
- Discuss the learning module **strain gage rosettes**
- Do some review example problems – stress, strain, and strain gages

Example: Strain gages and temperature compensation

Given: A quarter-bridge strain gage circuit is constructed using R_3 as the strain gage, as sketched. The beam being measured is located a short distance from the bridge circuit. Unfortunately, the temperature at the location of the experiment fluctuates a lot, and the resistance of the strain gage is also quite sensitive to temperature. **We get a *false reading* when the temperature changes –** *a change in temperature appears (falsely) like a change in strain, even if the actual strain on the beam is not changing.*



To do: Figure out a way to eliminate the temperature effect.

Solution: Use a "dummy" gage, of opposite sign, as a temperature compensating gage

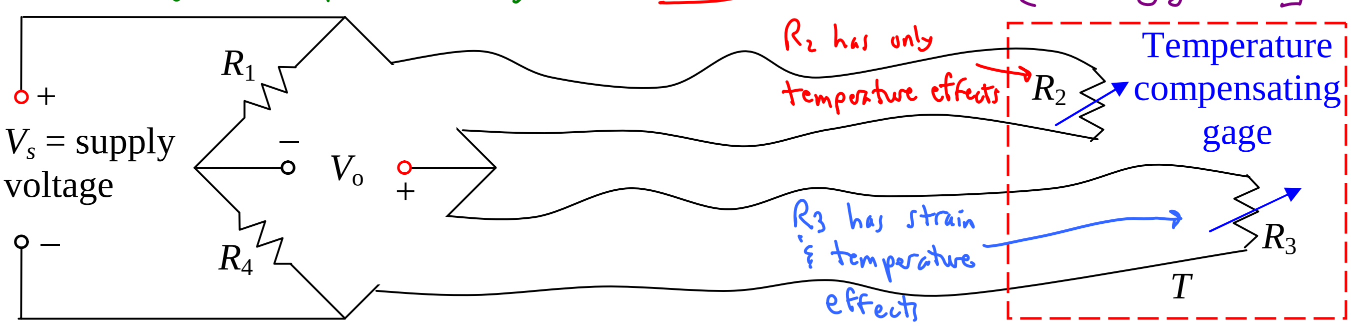
Recall the approximate equation for V_o when all four resistors of the Wheatstone bridge have small changes in resistance:

$$\frac{V_o}{V_s} \approx \frac{R_{2,\text{initial}} R_{3,\text{initial}}}{\left(R_{2,\text{initial}} + R_{3,\text{initial}}\right)^2} \left(\frac{\delta R_1}{R_{1,\text{initial}}} - \frac{\delta R_2}{R_{2,\text{initial}}} + \frac{\delta R_3}{R_{3,\text{initial}}} - \frac{\delta R_4}{R_{4,\text{initial}}} \right)$$

our strain gage, exposed to strain and to T

Pick R_2 (or R_4) for temperature compensation, since δR_2 & δR_4 have $\ominus$ve signs

Mount strain gage for R_2 on a part of the beam or structure that is exposed to the same temperature as R_3, but is not exposed to strain → [It is a "dummy" strain gage]



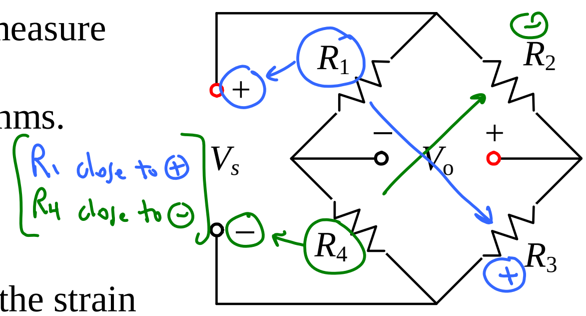
Since δR_2 & δR_3 have opposite signs, changes in T cancel out! Clever!!

Example: Strain gages

Given: A Wheatstone bridge circuit is constructed to measure strain in a component of a truss beam on a bridge.

- All resistors and strain gages are nominally 120 ohms.
- The strain gage factor is 2.05.
- The supply voltage to the bridge is 6.00 V.
- With no load, the bridge is balanced (V_o = 0).
- An axial strain of 350 μstrain is applied such that the strain gage is in tension.

How to rember the signs of each resistor:



(*a*) To do: Calculate the output voltage in mV when resistor 2 is the strain gage.

(*b*) To do: Calculate the output voltage in mV when resistors 1 and 2 are the strain gages, and *both* strain gages are in tension with ε_a = 350 μstrain.

(*c*) To do: Calculate the output voltage in mV when resistors 1 and 3 are the strain gages, and *both* strain gages are in tension with ε_a = 350 μstrain..

Solution:

(a) • Recall, R_1 & R_3 are the "positive" resistors
R_2 & R_4 are the "negative" resistors. Here, R_2 is the strain gage

• So, $\varepsilon_a \approx -\frac{V_o}{V_s}\frac{4}{n}\frac{1}{S}$ n=1 for a quarter bridge

$$\boxed{V_o = -\frac{V_s \varepsilon_a S}{4}}$$

• Numbers: $V_o = \frac{-(6.00\text{V})(350\times10^{-6})(2.05)}{4}\left(\frac{1000\text{ mV}}{\text{V}}\right) = -1.07625\text{ mV}$

$$\boxed{V_o \approx -1.08\text{ mV}}$$

(b) R_1 (+) & R_2 (−) are of opposite signs → ∴ They cancel each other out!

$$\boxed{V_o = 0}$$

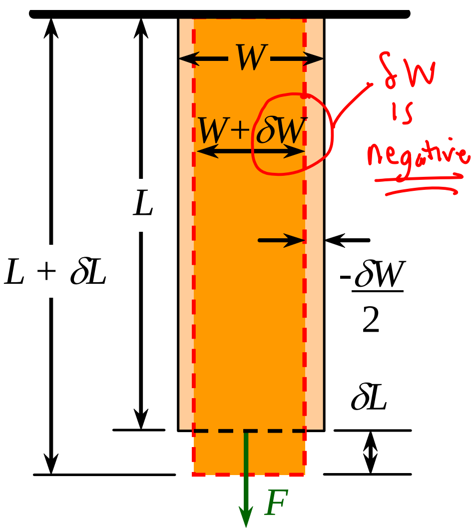
(c) • R_1 (+) & R_3 (+) are of the same sign, so the voltage adds up
(If identical strain gages both exposed to the same strain, V_o doubles)

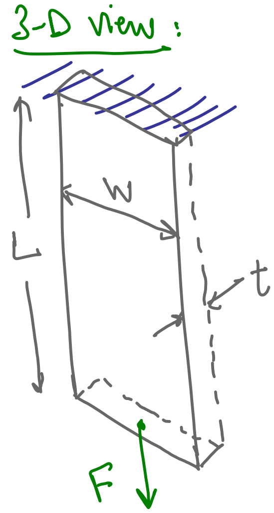
• Also, R_1 & R_3 are positive, so $\varepsilon_a = +\frac{V_o}{V_s}\frac{4}{n}\frac{1}{S} \rightarrow \boxed{V_o = \frac{V_s \varepsilon_a S}{2}}$

n=2 for half bridge

• We have doubled the sensitivity → ∴ $\boxed{V_o = 2.15\text{ mV}}$ [Sensitivity = $\frac{\Delta\text{ output}}{\Delta\text{ input}}$]

Tranverse Strain: Consider a hanging beam, of dimensions L and W, with thickness t.





- Axial strain ε_a is in one direction.
- However, strain in one direction leads to strain of the opposite sign in other directions
- We call this transverse strain

Equations: • $\boxed{\varepsilon_a = \frac{\delta L}{L} = \text{axial strain}}$

But, as we stretch vertically, the beam contracts horizontally, as stretched, where δW is negative

- Define transverse strain = 90° to axial strain → $\boxed{\varepsilon_t = \text{transverse strain} = \frac{\delta W}{W} \quad (<0)}$

- Here, $\boxed{\varepsilon_a > 0, \therefore \varepsilon_t < 0}$

[Similar contraction occurs in the third direction → $\varepsilon_t = \frac{\delta t}{t}$, also <0 since $\delta t < 0$]

- Define $\boxed{\text{Poisson's Ratio} = \nu = -\frac{\varepsilon_t}{\varepsilon_a}}$ → Poisson's ratio is a property of the material

[We define it with a ⊖ sign so that ν is ⊕ve]

- For most metals, $\nu \approx \frac{1}{4}$ to $\frac{1}{3}$ & is dimensionless

[$\nu \approx \frac{1}{4}$ for most isotropic materials (no preferred direction)
[E.g., wood has grain and is not isotropic]]

Example: Strain gages and transverse strain

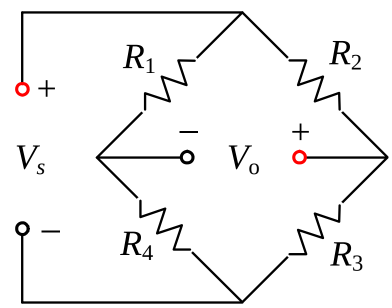
Given: A 2.8 cm × 5.0 cm rectangular rod is stretched from its initial length of 0.4000 m to a length of 0.4005 m.

- The modulus of elasticity of the rod material is 95.0 GPa (gigapascals).
- Poisson's ratio of the rod material is 0.333.

(*a*) To do: Calculate the axial stress in units of MPa.

(*b*) To do: Calculate the transverse strain in units of microstrain.

(*c*) To do: A strain gage with a strain gage factor of 2.10 is glued to the rod before it is stretched, aligned with the direction of stretching. A quarter bridge Wheatstone bridge circuit is constructed, with the strain gage as resistor R_1. The strain gage is balanced before the rod is stretched. The bridge supply voltage is 7.50 V. Calculate the output voltage (in mV) of the bridge after the rod is stretched.



Solution:

(a) · By definition, ε_a = axial strain $= \dfrac{\delta L}{L} = \dfrac{(0.4005 - 0.4000)\text{ m}}{0.4000\text{ m}} = \boxed{0.00125 = \varepsilon_a}$ [1250 μ strain

· Axial stress $\boxed{\sigma_a = E\,\varepsilon_a} = (95.0\times10^9\ \text{Pa})(0.00125)\left(\dfrac{1\ \text{MPa}}{10^6\ \text{Pa}}\right) = 118.75$ MPa

(Hooke's law)

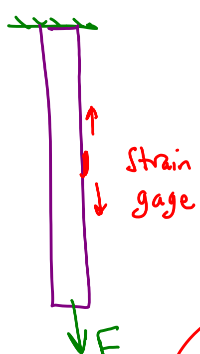
Answer: $\boxed{\sigma_a = 119\ \text{MPa}}$

(b) ε_t = transverse strain $= -\nu\,\varepsilon_a = -0.333\,(0.00125) = \boxed{-0.00041625 = \varepsilon_t}$

or, $\times 10^6 \rightarrow \boxed{\varepsilon_t = -416\ \mu\text{strain}}$

[Note: $\sigma_t = 0$ since we are not applying any stress in the transverse direction]

(c)



· The strain gage is in tension (stretched) when F is applied as sketched

· R_1 is one of the (+)ve resistors in the bridge circuit

[recall, R_1 & R_3 are (+)ve, R_2 & R_4 are (−)ve]

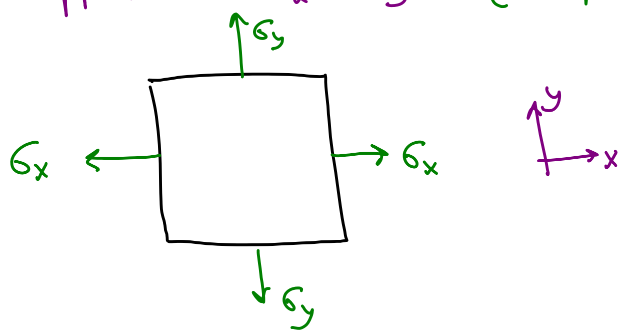
· $V_o = \dfrac{n}{4}\,\varepsilon_a\,S\,V_s = \dfrac{1}{4}(0.00125)(2.10)(7.50\text{V})\left(\dfrac{1000\ \text{mV}}{\text{V}}\right) = 4.9219$ mV

[n = 1 for a quarter bridge]

Answer: $\boxed{V_o = 4.92\ \text{mV}}$

STRESS AND STRAIN ON A SURFACE

- Consider an element on the surface of a material that has two normal stresses applied $\longrightarrow$ σ_x & σ_y [Principal stresses]



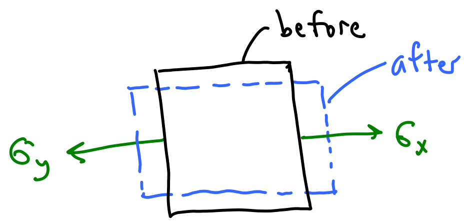
- How do we calculate the <u>strains</u>, ε_x & ε_y?

- At first glance, we might say $\cancel{\varepsilon_x = \frac{\sigma_x}{E}}$ & $\cancel{\varepsilon_y = \frac{\sigma_y}{E}}$

  No! → There are also transverse strains, so the solution is more involved than this

- <u>ANALYSIS</u>: Let's pretend that stress σ_x occurs first, by itself

<u>STAGE 1</u>:



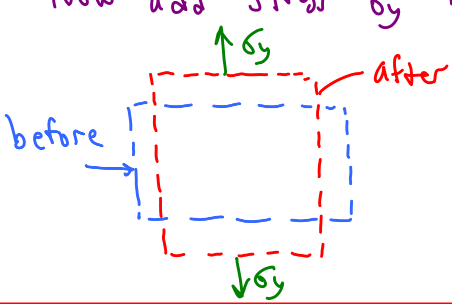
We analyze for stage 1 (subscript 1)

- $\boxed{\varepsilon_{x_1} = \frac{\sigma_x}{E}}$ (<u>positive</u> in x direction for tension, as shown)

- But, there will also be <u>transverse strain</u> → $\boxed{\varepsilon_{y_1} = -\nu \frac{\sigma_x}{E}}$ (<u>negative</u> because it shrinks in y direction)

recall, E = Young's modulus, ν = Poisson's ratio } Assume these are <u>known</u> material properties

<u>STAGE 2</u>: Now add stress σ_y in the y-direction



Here, $\boxed{\varepsilon_{y_2} = \frac{\sigma_y}{E}}$ (positive in y-direction)

& transverse: $\boxed{\varepsilon_{x_2} = -\nu \frac{\sigma_y}{E}}$ (negative in x-direction)

· Combining these (since strain adds linearly),

Total: $\varepsilon_x = \varepsilon_{x_1} + \varepsilon_{x_2} = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} \rightarrow \boxed{\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)}$ (1)

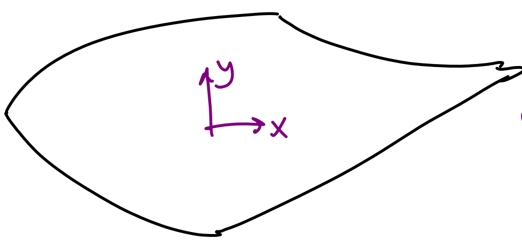
$\varepsilon_y = \varepsilon_{y_1} + \varepsilon_{y_2} = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} \rightarrow \boxed{\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)}$ (2)

· Bottom line:
- ε_x is smaller than $\frac{\sigma_x}{E}$ because of the effect of σ_y.
- ε_y is smaller than $\frac{\sigma_y}{E}$ because of the effect of σ_x.

(assuming both σ_x & σ_y are ⊕ve, as sketched in our analysis)

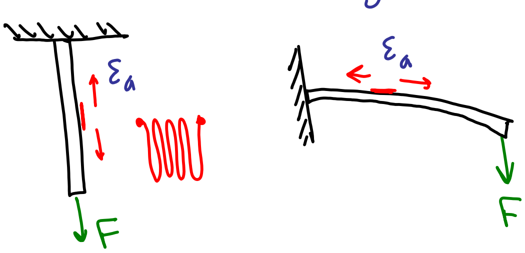
[I will leave it as an exercise to solve the other way, i.e., σ_x & σ_y as functions of ε_x, ε_y, E, & ν.]

What about an arbitrary surface in which we do not know beforehand which are the directions of principal stresses or principal strains?



[Here x & y are chosen for convenience, and may have nothing to do with the directions of principal stress or strain]

Note: For simple axial strain problems, we need only one strain gage, and we align it in the direction of the axial (principal) strain

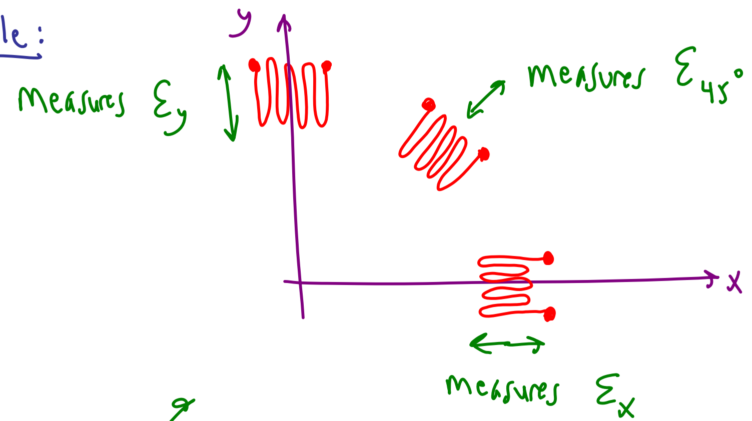


→ Align the strain gage in the direction of the applied stress

This is automatically in the direction of principal strain, & we call it ε_a

- But, for the general case, we don't know the directions of principal strain
- It turns out that we need to measure a minimum of 3 strains in 3 directions along the surface in order to determine the strain field.
- We use a $\boxed{\text{strain gage rosette}}$ ☆

Example:



Three strain gages, mounted close together:
- In x direction
- In y direction
- @ 45° angle

[You can buy strain gage rosettes already made, with all 3 on one backing]

- To analyze this, use Mohr's circle & equations you learned in E. Mech. class → can determine principal strains & their directions

[See learning module on website for pictures of strain gage rosettes]