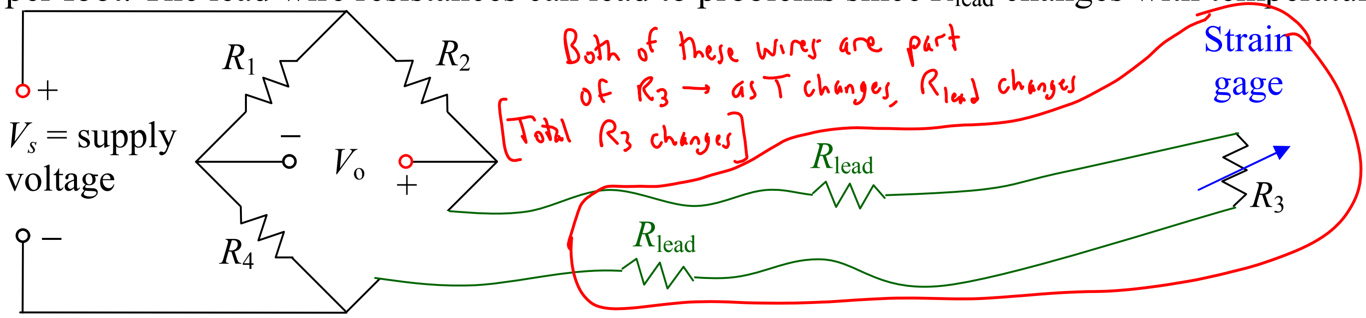
Today, we will:
- Discuss how to compensate for strain gages with long lead wires
- Do a **review** example problem – A/D converters, op-amps, filters, and strain gages

Example: Strain gage with long lead wires – how to compensate

Given: A quarter-bridge strain gage circuit is constructed with 120-Ω resistors and a 120-Ω strain gage as usual. The main problem here is that the experiment is very far away – the lead wires going to the strain gage are 40 ft long, and the lead wires have a resistance of 0.026 Ω per foot. The lead wire resistances can lead to problems since R_{lead} changes with temperature.



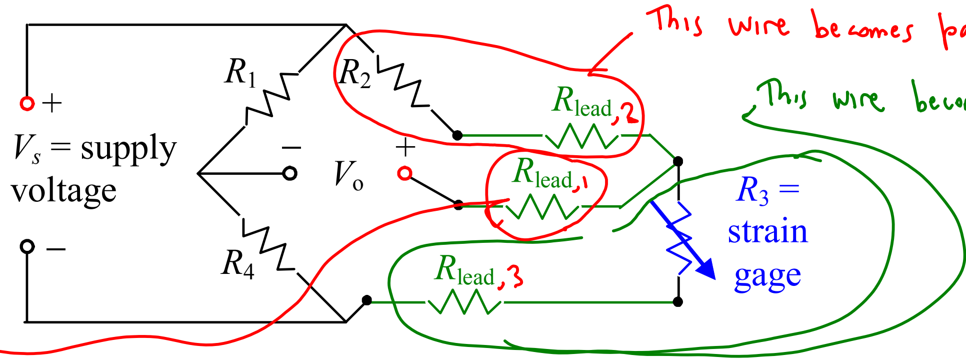
To do: Devise a modified circuit that will cancel out the effect of the lead wires.

Solution: A clever "trick", using 3 wires connected to the strain gage rather than just 2:

Label the lead wires as shown $R_{\text{lead},1}$, 2, 3 →

This wire becomes part of R_2

This wire becomes part of R_3



Recall the approximate equation for V_o when all four resistors of the Wheatstone bridge have small changes in resistance:

$$\frac{V_o}{V_s} \approx \frac{R_{2,\text{initial}} R_{3,\text{initial}}}{\left(R_{2,\text{initial}} + R_{3,\text{initial}}\right)^2} \left(\frac{\delta R_1}{R_{1,\text{initial}}} - \frac{\delta R_2}{R_{2,\text{initial}}} + \frac{\delta R_3}{R_{3,\text{initial}}} - \frac{\delta R_4}{R_{4,\text{initial}}} \right)$$

⊖ ⊕

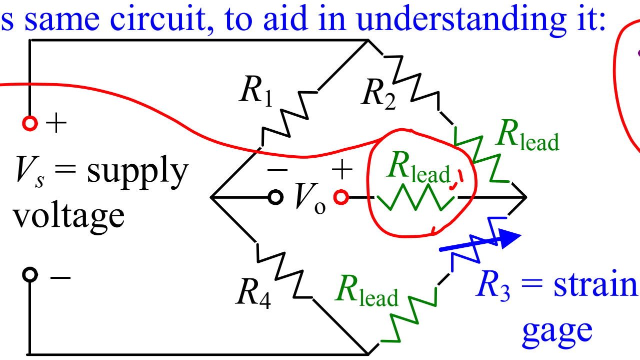
$\delta R_{\text{lead},2} = \delta R_{\text{lead},3}$

Since the wires are exposed to the same temperature variations

So – the lead wire resistances cancel each other out!

Another way to draw this same circuit, to aid in understanding it:

What about this remaining lead wire?



Clever!

V_o is unaffected by changes in $R_{\text{lead},1}$

Answer:

V_o goes to a voltmeter or op-amp or A/D or something with a very high input impedance → So, negligible current flows through $R_{\text{lead},1}$ ∴ negligible ΔV

Example: Strain gages and digital data acquisition [a comprehensive review problem]

Given: A quarter-bridge Wheatstone bridge circuit is used with a strain gage to measure strains up to ±1000 μstrain for a beam vibrating at a maximum frequency of 50 Hz.

- the supply voltage to the Wheatstone bridge is V_s = 5.00 V DC
- all Wheatstone bridge resistors and the strain gage itself are 120 Ω
- the circuit is initially balanced at zero strain
- the strain gage factor for the strain gage is S = 2.04
- there is some unwanted noise in output V_o: f_{noise} = 3600 Hz and amplitude = ±0.50 mV
- the output voltage V_o is sent into a 12-bit A/D converter with a range of ± 5 V
- op-amps, resistors, and capacitors are available in the lab
- all op-amps have a GBP (noninverting) of 1.2 MHz

To do: Design a circuit with the highest possible resolution (i.e., a circuit that utilizes the full range of the A/D converter), and minimizes the noise.

Max strain = 1000 μstrain = 1000×10^{-6}

Solution:

n=1 here (quarter bridge)

First, we calculate $V_{o,max}$ = max voltage output from the strain gage circuit:

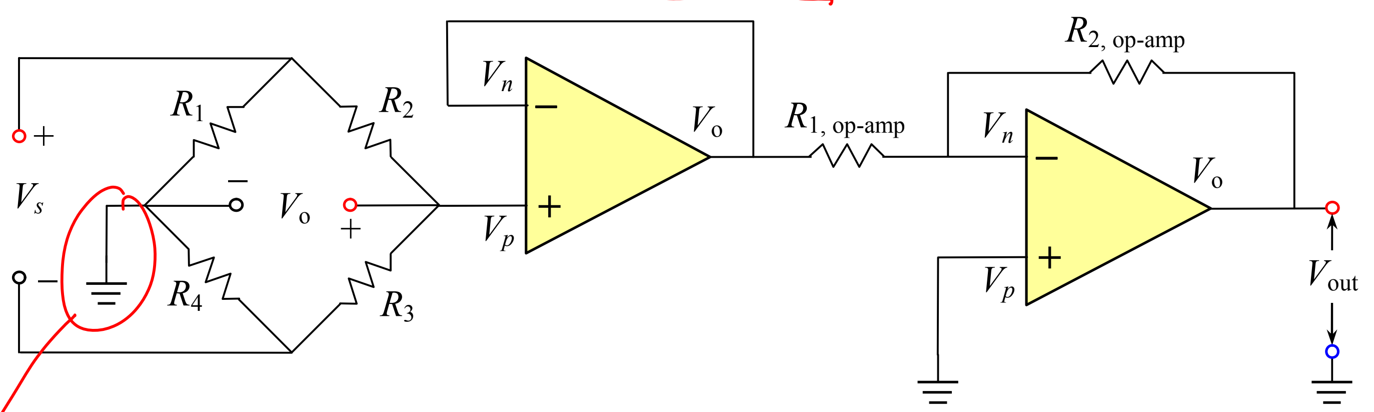
$$V_{o,\,max} = V_s \varepsilon_a \frac{n}{4} S = (5.00\text{ V})(1000 \times 10^{-6})\frac{1}{4}(2.04) = 2.55 \times 10^{-3}\text{ V} \quad = \text{maximum}$$

Thus, V_o varies between -2.55 mV and +2.55 mV. Now, to utilize the full range of the A/D converter, we need to amplify by a factor of

Too small!

$$G = \frac{\pm 5.00\text{ V}}{\pm 2.55 \times 10^{-3}\text{ V}} = \pm 1960.78 \text{ [sign depends on type of amplifier]}$$

We suggest this circuit (single stage inverting amplifier) as a starting point:



Wheatstone bridge   Buffer   Inverting amplifier

To achieve the required gain, we would use, say, $\boldsymbol{R_{1,\,op\text{-}amp}}$ **= 1 kΩ**, and $\boldsymbol{R_{2,\,op\text{-}amp}}$ **= 1.96 MΩ**.

Questions:

1. Why do we ground the Wheatstone bridge at V_0^- instead of at the bottom of the bridge?
   - Have to ground somewhere
   - Forcing $V_o^- = 0$ guarantees that $\boxed{V_o^+ \text{ will range from } -2.55 \text{ to } +2.55 \text{ mV relative to ground}}$
2. Why did we put a buffer between the Wheatstone bridge and the amplifier?
   - The bridge circuit is sensitive to downstream circuits → V_o can change if current is drawn
   - So → Add a buffer to provide a very high input impedance [Nothing downstream of the buffer will affect V_o]

3. Why are we using an *inverting* (rather than a noninverting) amplifier?

Recall – Inverting amps are better at reducing noise & we care about noise here. (CMRR noise)

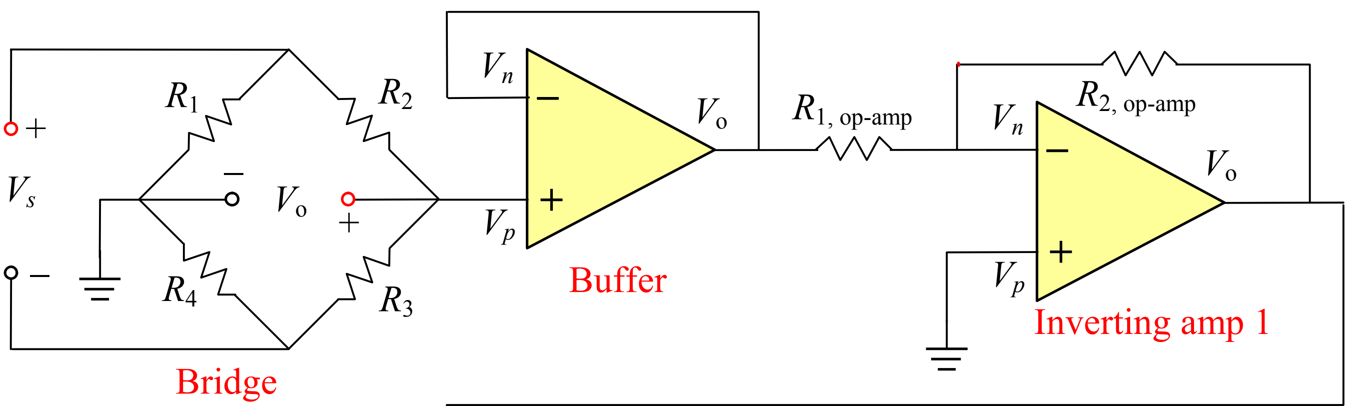
4. To get positive V_{out} when the strain is positive, which resistor should be the strain gage?

Since we are inverting, use one of the ⊖ve resistors → $\boxed{R_2 \text{ or } R_4}$

5. Why is this circuit not so great, and what should we do to improve it?

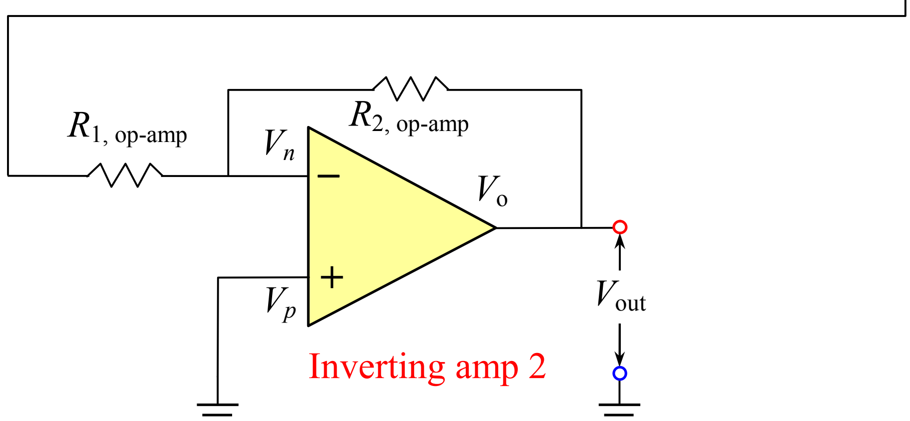
Answers:

- $R_{1,\text{ op-amp}} = 1\ \text{k}\Omega$ is at the lower recommended limit (not so great input impedance)
- $R_{2,\text{ op-amp}}$ exceeds the recommended 1 MΩ limit (leads to stray capacitance effects)
- G is huge, and can lead to GBP effects as discussed previously
- To improve this circuit, let's instead construct a ***two-stage amplifier***:



Set $G_{\text{stage 1}} = G_{\text{stage 2}} = -\sqrt{G_{\text{amp}}} = -\sqrt{1960.78}$

$\underline{-44.281}$



In this case, we can choose better values of the op-amp resistors. Namely, the gain of each amplifier is the square root of the total gain, i.e.,

$$G_{\text{stage}} = -\sqrt{1960.78} = -44.281$$

so we choose, for example, $R_{1,\text{ op-amp}} = 10.0\ \text{k}\Omega$, and $R_{2,\text{ op-amp}} = 442.8\ \text{k}\Omega$.

much better since $R_1 > 1\ \text{k}\Omega$ & $R_2 < 1\ \text{M}\Omega$

Questions:

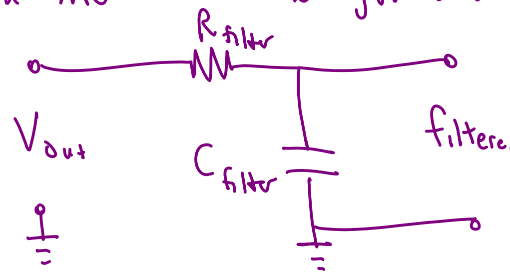
1. To get positive V_{out} when the strain is positive, which resistor should be the strain gage?

Now we are inverting twice [⊖⊖=⊕] → So use R_1 or R_3 as the strain gage ☆

2. What should we add to get rid of the high frequency (3600 Hz) noise?

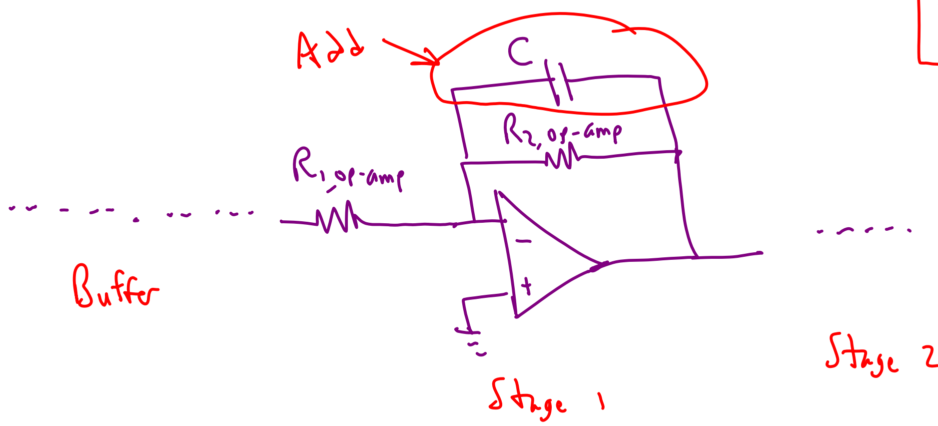
Recall: $f_{\text{signal}} \approx 50$ Hz, $f_{\text{noise}} \approx 3600$ Hz } Use a low-pass filter to attenuate the high frequency noise

- How to add a filter → could just add it at the end:



(a passive low-pass filter)

- OR – better, since we already have an inverting amplifier, add a capacitor to convert the inverting amplifier into an $\boxed{\text{active low-pass filter: Inverting amplifier}}$



What C to use? ⟶ Since $f_{signal} = 50\ Hz$, let's pick $\boxed{f_{cutoff} = 150\ Hz}$ so that we do not attenuate the desired frequency content.

- Recall, $f_{cutoff} = \dfrac{1}{2\pi R_{2,op\text{-}amp} C} \rightarrow C = \dfrac{1}{2\pi R_{2,op\text{-}amp} f_{cutoff}}$

[Recall, R_2 is the one that forms f_{cutoff} for an active low-pass filter]

$$C = \frac{1}{2\pi (442800\ \Omega)(150\ \tfrac{1}{s})} \left(\frac{\Omega \cdot A}{V}\right)\left(\frac{C}{s \cdot A}\right)\left(\frac{F \cdot V}{C}\right)$$

$$C = 2.396 \times 10^{-9}\ F \approx 2.40\ pF$$

(picoFarad)

or $\boxed{C \approx 0.00240\ \mu F}$

- For practice, calculate the GBP effects for our 2-stage amplifier ← Try this on your own

Summary: $GBP_{inverting} = 1.173\ MHz$; $f_c = 26500\ Hz$

@ $f = 50\ Hz$, $G_{GBP} = 0.99998$ (negligible)

@ $f = 3600\ Hz$, $G_{GBP} = 0.99089$ (also almost negligible effect)

QUESTION: Does this filter "completely" remove the noise for our A/D? I.e., have we reduced the noise amplitude below the resolution of the A/D?

SOLUTION:

- Our A/D is 12-bit ($N=12$) with a range from -5 to 5 V (V_{min} = -5, V_{max} = 5)

- Quantization error $= \pm \dfrac{V_{max}-V_{min}}{2^{N+1}} = \dfrac{5-(-5)\text{ V}}{2^{13}} = \pm 0.00122 \text{ V}$

$$= \pm 1.22 \text{ mV}$$

- Compare to the noise amplitude:

[Careful! We amplify twice, filter once, & have GBP effects too]

[We don't care about the ⊕ or ⊖ signs here]

$$|V_{noise}|_{in} = \pm 0.50 \text{ mV} \quad \text{(given)}$$

$$|V_{noise}|_{out} = |V_{noise}|_{in} \cdot |G_{amp\,1}| \cdot |G_{GBP,\,amp\,1}| \cdot |G_{filter}| \cdot |G_{amp\,2}| \cdot |G_{GBP,\,amp\,2}|$$

$$= (0.50 \text{ mV})(44.281)(0.99089)\left(\frac{1}{\sqrt{1+\left(\frac{3600}{150}\right)^2}}\right)(44.281)(0.99089)$$

$$= \underline{40.07 \text{ mV}}$$

→ Since this is greater than the quantizing error, we have not removed the noise ☆

- What to do? — Add another capacitor to the stage 2 amp to turn it also into an active low-pass filter / inverting amplifier ☆

- Re-calculate (multiply the above by $\dfrac{1}{\sqrt{1+\left(\frac{3600}{150}\right)^2}}$ again)

- Get $|V_{noise}|_{out} = \underline{1.67 \text{ mV}}$ → Still > 1.22 mV = quantizing error but not by much

[We almost remove the noise with two filters]

• EXTRA → Calculate the Signal-to-noise ratio (SNR) for the original input & for the output of our circuit (signal conditioned)

• Recall, $\boxed{SNR = \left(\frac{\text{Ampl. signal}}{\text{Ampl. noise}}\right)^2}$ by definition & $\boxed{SNR_{dB} = 20\log_{10}\left(\frac{\text{Ampl. signal}}{\text{Ampl. noise}}\right)}$

Here, Ampl. noise $= |V_{noise}|$; Ampl. signal $= |V_{signal}|$, etc.

• INPUT (original signal) : $|V_{signal}| = 2.55$ mV, $|V_{noise}| = 0.50$ mV

$$SNR_{IN} = \left(\frac{2.55}{0.50}\right)^2 = 26.01$$

$$SNR_{dB,\,in} = 20\log_{10}\left(\frac{2.55}{0.50}\right)$$

Not very good → $\boxed{SNR_{dB\,in} = 14.2 \text{ dB}}$

• OUTPUT (after signal conditioning)

$$|V_{signal}|_{out} = |V_{signal}|_{in} \cdot G_{amp}^2 \cdot G_{filter}^2 \cdot G_{GBP}^2$$

$$= (2.55 \text{ mV}) \cdot (44.281)^2 \left(\frac{1}{\sqrt{1+\left(\frac{50}{150}\right)^2}}\right)^2 (0.999998)^2 = \underline{4500 \text{ mV}}$$

$$|V_{noise}|_{out} = |V_{noise}|_{in} \cdot G_{amp}^2 \cdot G_{filter}^2 \cdot G_{GBP}^2$$

$$= (0.50 \text{ mV})(44.281)^2 \left(\frac{1}{\sqrt{1+\left(3600/150\right)^2}}\right)^2 (0.99089)^2 = \underline{1.668 \text{ mV}}$$

• Thus, $SNR_{out} = \left(\frac{|V_{signal}|_{out}}{|V_{noise}|_{out}}\right)^2 = \left(\frac{4500 \text{ mV}}{1.668 \text{ mV}}\right)^2 = 7.28 \times 10^6$

or $SNR_{dB\,out} = 20\log\left(\frac{4500}{1.668}\right) = \boxed{68.6 \text{ dB} = SNR_{dB,\,out}}$

☆ much better SNR !