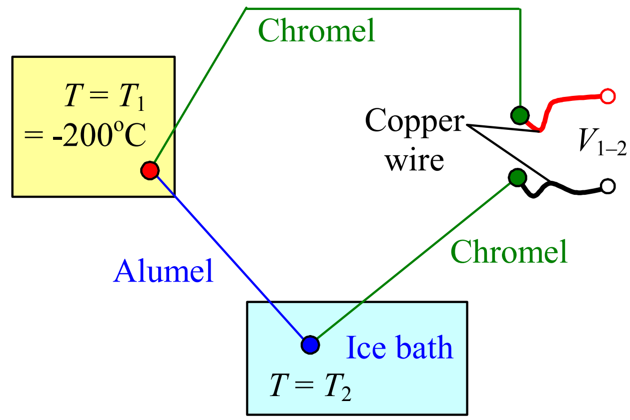
Today, we will:
- Continue reviewing the pdf module: **Temperature Measurement**
- Do some example problems – thermocouples, bulb thermometers, RTDs, thermistors

Example: Thermocouples

Given: Fred uses a type K thermocouple to measure the temperature in a freezer. The actual (*true*) temperature in the freezer is T_1 = -200.0°C. The thermocouple voltages are measured properly with an ice bath reference. Unfortunately, the ice bath has too much water, and its temperature is actually 1.00°C instead of 0.00°C. Fred is not aware of this – he assumes that the ice bath is at 0.00°C.



To do: Calculate the temperature that Fred measures, and the percentage error. Use the "brief" thermocouple tables for consistency.

Solution: Use our "workhorse" equation for thermocouples: $\boxed{V_{1-2} = V_{1-R} - V_{2-R}}$.

Note: This eq. comes from the Law of Intermediate Temperatures:

$V_{2-R} = V_{2-1} + V_{1-R}$

$V_{2-R} = -V_{1-2} + V_{1-R} \Rightarrow V_{1-2} = V_{1-R} - V_{2-R}$ ✓

- Table for Type K @ T_1 = -200°C, V_{1-R} = -5.891 mV
  - " " @ T_2 = 1.00°C, V_{2-R} = 0.0399 mV (interpolating)
- $V_{1-2} = V_{1-R} - V_{2-R}$ = -5.891 − 0.0399 = -5.931 mV = V_{1-2}

This is the voltage that Fred measures

- Since Fred is unaware that his ice bath has problems, he thinks that

V_{1-R} = -5.931 mV (which is incorrect)

↓

Fred interpolates on the table → $\boxed{\text{Fred thinks } T_1 = -203.9\,°C}$

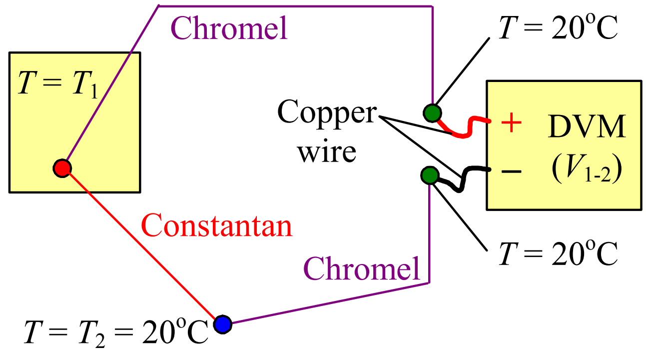
- Percent error $= \dfrac{-203.9 - (-200)}{200}$ $\boxed{\cong -2.0\% \text{ error}}$ [Small error in ice bath leads to large error in T]

WRONG WAY → Add temperatures: Fred measures -200.0 −1.00 = -201.0°C ✗ Wrong!

☆ [You cannot add temperatures – add voltages only.] ☆

Example: Thermocouples

Given: James uses a type E thermocouple to measure the temperature of an oven. He has no ice bath, but uses a reference bath exposed to the ambient air, which he measures with a thermometer to be 20°C. The voltage James reads is 8.714 mV.



To do: Calculate the temperature in the oven. Use the "brief" thermocouple tables for consistency, a portion of which is shown here for convenience:

Temperature (°C)	T	E	J	K	R	S
0	0.000	0.000	0.000	0.000	0.000	0.000
20	0.789	1.192	1.019	0.798	0.111	0.113
40	1.611	2.419	2.058	1.611	0.232	0.235
60	2.467	3.683	3.115	2.436	0.363	0.365
80	3.357	4.983	4.186	3.266	0.501	0.502
100	4.277	6.317	5.268	4.095	0.647	0.645
120	5.227	7.683	6.359	4.919	0.800	0.795
140	6.204	9.078	7.457	5.733	0.959	0.950
160	7.207	10.501	8.560	6.539	1.124	1.109
180	8.235	11.949	9.667	7.338	1.294	1.273

Solution: Use our "workhorse" equation for thermocouples: $\boxed{V_{1-2} = V_{1-R} - V_{2-R}}$.

• $V_{1-R} = V_{1-2} + V_{2-R}$

Table, Type E @ T_1

measured $= 8.714$ mV

table @ $T_2 = 20.0^{\circ}$C

$V_{2-R} = 1.192$ mV

$V_{1-R} = 8.714 + 1.192 = \boxed{9.906 \text{ mV} = V_{1-R}}$

• Now interpolate using $V_{1-R} = 9.906$ & the Type E table → $\boxed{T_1 = 151.6^{\circ}\text{C}}$

• Wrong way → • At $V_{1-2} = 8.714$ mV (voltage reading), interpolate on the table, and get $T_{1-2} = 134.8^{\circ}$C

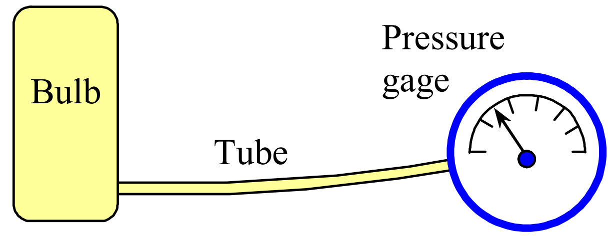
• Add 20°C to compensate for the "ice bath" being @ 20°C

→ $T_1 = 134.8 + 20 = 154.8^{\circ}$C ✗ WRONG!

☆ Again → never add temperatures; add only voltages ☆

Bottom line: Don't really need an ice bath if can measure the reference T & do algebra properly

Example: Bulb thermometer



Given: A ***pressure thermometer*** (also called a ***gas thermometer*** or ***bulb thermometer***) relies on the ideal gas law $\boxed{PV = mRT}$ to calculate temperature T at a given measured pressure P (the gage is actually a pressure gage).

- V = volume of the gas in the bulb = constant
- R = ideal gas constant for the particular gas in the bulb = constant
- m = mass of the gas in the bulb = constant

(*a*) To do: Develop an equation for the ***sensitivity*** of the device. Is it constant? *Note*: Constant sensitivity implies that the output is a *linear* function of the input.

(*b*) To do: Determine whether a change in the *initial* filling pressure P_i and/or the *initial* filling temperature T_i have any effect on the sensitivity of the device.

Solution:

(a) $$\text{Sensitivity} \equiv \frac{\text{change in output}}{\text{change in input}} = \frac{d(\text{output})}{d(\text{input})} \approx \frac{\Delta(\text{output})}{\Delta(\text{input})}$$

• Here, since we have an equation, we can use the derivative form:

$$\text{Sensitivity} = \frac{dP}{dT} \quad \begin{matrix} \leftarrow \text{output} = P \\ \leftarrow \text{input} = T \end{matrix}$$

• But we know that $P = \underbrace{\frac{mR}{\forall}}_{\text{constant}} T \longrightarrow \boxed{\frac{dP}{dT} = \frac{mR}{\forall} = \text{sensitivity} = \text{constant}}$

(b) Initially, $P = P_i$ & $T = T_i$. So $P_i = \frac{mR}{\forall} T_i \Rightarrow \frac{mR}{\forall} = \frac{P_i}{T_i}$

& $$\boxed{\text{Sensitivity} = \frac{mR}{\forall} = \frac{P_i}{T_i}}$$

• As you pump up the bulb to increase P_i, m goes up & thus the sensitivity also goes up.

Answer: Yes – the initial P_i & T_i do affect the sensitivity

Bottom line: We can set the sensitivity to any value we want by setting P_i & T_i. From then on, however (once everything is sealed), the sensitivity remains constant

Example: RTDs

Given: A standard 100-Ω platinum RTD is used to measure the temperature of warm air in a tank. The resistance is measured to be 125.9 Ω.

To do: Calculate the temperature of the air in the tank.

Solution: Need to interpolate. Here is a screen shot from the **Platinum 100-Ω RTD Table:**

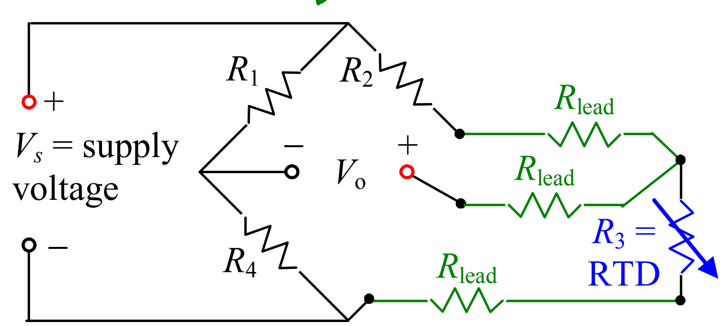
°C	Ohms
65	125.16
70	127.07

at 125.9, interpolate: $\boxed{T = 66.9\,^\circ C}$

For simple applications we measure R for an RTD. But if lead wires are an issue, and/or we want better sensitivity, we use a Wheatstone bridge.

Example: RTDs

Given: A standard 100-Ω platinum RTD is used to measure temperature. A Wheatstone bridge is set up in the three-wire configuration, using resistor R_3 as the RTD, as sketched.



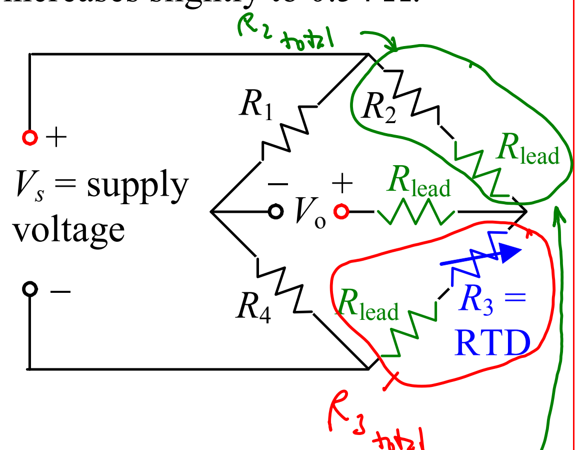
- R_{lead} = 0.51 Ω at 20°C
- V_s = 10.00 V
- We want the bridge to be exactly balanced when T = 20°C

(*a*) To do: What resistors (R_1, R_2, and R_4,) should we use to balance the bridge at 20°C?

(*b*) To do: At T = 40°C, predict V_o. *Note*: At 40°C, R_{lead} increases slightly to 0.54 Ω.

Solution:

For clarity, we re-draw the circuit as shown to the right. Thus, $R_{2,total} = R_2 + R_{lead}$ and $R_{3,total} = R_3 + R_{lead}$.



$R_{2\,total} = R_2 + R_{lead}$

(a) RTD table → @ 20°C, $R_3 = 107.79\ \Omega$

- $R_{3\,total} = R_3 + R_{lead}$ (see diagram)

$= 107.79 + 0.51 = 108.30\ \Omega$

(at 20°C)

- There are many ways to balance the bridge. The easiest way is to have symmetry → Set all 4 legs of bridge to 108.30 Ω ($R_{3\,total}$)

Answers → $\boxed{R_1 = R_4 = 108.30\ \Omega}$

$$R_{2\,total} = R_2 + R_{lead} = 108.30\ \Omega \qquad R_2 = 108.30 - 0.51\ \Omega$$

$(0.51\ \Omega)$

$$\boxed{R_2 = 107.79\ \Omega}$$

✱ These values of R_1, R_2, R_4 will exactly balance the bridge @ 20°C

(b) At T = 40°C, predict V_o

Sol'n: Exact Wheatstone bridge equation:

$$\boxed{V_o = V_s \frac{R_3 R_1 - R_4 R_2}{(R_2+R_3)(R_1+R_4)}}$$

Resistor values to plug in:

Careful: These are the total R values! ✱

- $R_1 = \underline{108.30\ \Omega}$ (fixed)
- $R_2 = 107.79\ \Omega$, but $R_{2\,total} = 107.79 + 0.54 = \underline{108.33\ \Omega} = R_{2\,total}$

  R_{lead} @ T = 40°C (given)
- $R_3 = R_{RTD}$ @ T = 40°C → From table, $R_{RTD} = 115.54\ \Omega$

  But $R_{3\,total} = R_{RTD} + R_{lead} = 115.54 + 0.54 = \underline{116.08\ \Omega} = R_{3\,total}$
- $R_4 = \underline{108.30\ \Omega}$ (fixed)

· Finally,
$$V_o = (10.00\ V)\frac{(116.08)(108.30) - (108.30)(108.33)}{(108.33 + 116.08)(108.30 + 108.30)} = 0.1733\ V$$

Answer: $\boxed{V_o \text{ @ } 40°C = 0.173\ V}$

Comments: • We had to use R_{total} for each leg of the bridge

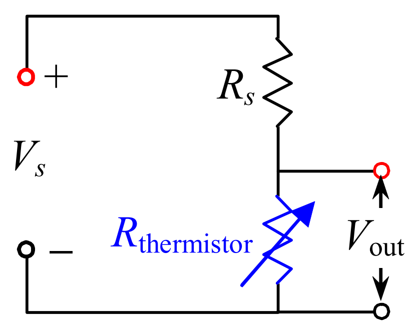
• This is exact since Eq. (1) is exact & we even accounted for the change in lead resistance with temperature

Example: Thermistors

Given: A thermistor has a resistance of 1633<u>0</u> Ω at 0.0°C, and it drops to 624<u>7</u> Ω at 20.0°C. This thermistor is used in a simple voltage divider circuit, as sketched.

(this is a type 5000 thermistor)

- V_s = supply voltage = 5.00 V DC.
- R_s = supply resistance = 10.00 kΩ.



(*a*) To do: Calculate the output voltage at T = 0.0°C.

(*b*) To do: Calculate the output voltage at T = 20.0°C.

(*c*) To do: Suppose the output voltage is 2.353 V. Estimate the temperature as best as you can, given the limited amount of information provided in the problem statement.

Solution:

(a) At $T = 0°C$, $\underline{R_t = 16330\ \Omega}$ (given)

Voltage divider → $V_{out} = V_s \dfrac{R_t}{R_t + R_s} = (5.00\text{V}) \dfrac{16330\ \Omega}{16330 + 10000\ \Omega}$

$V_{out} = 3.101$ V $\quad \boxed{V_{out} \cong 3.01\ \text{V}}$

(b) At $T = 20°C$, $\underline{R_t = 6247\ \Omega}$ (given)

$V_{out} = (5.00\text{V}) \dfrac{6247\ \Omega}{6247 + 10000\ \Omega} = 1.923$ V $\quad \boxed{V_{out} \cong 1.92\ \text{V}}$

(c) @ $V_{out} = 2.353$ V, interpolate (assuming <u>linear</u> V_{out} with T, for lack of any further information)

T (°C)	V_{out} (V)
0	3.101
$\boxed{T}$	$\boxed{2.353}$
20	1.923

→ interpolate: $T = 12.70°C$ → $\boxed{T = 12.7\ °C}$

Comment: This is <u>not exact</u> since V_{out} vs T is not exactly linear

<u>EXTRA</u>: Use the thermistor table → @ $V_{out} = 2.353$ V, solve for R_t & interpolate. <u>TRY IT ON YOUR OWN</u> → $\boxed{T = 12.4\ °C}$ = <u>Exact</u>