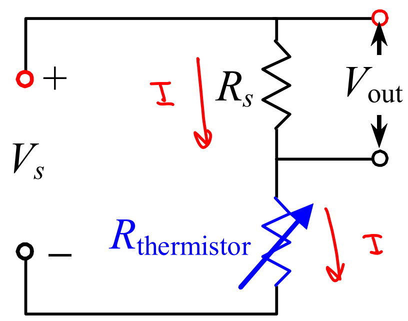
**Today, we will**:
- Finish reviewing the pdf module: **Temperature Measurement** and do an example
- Begin to review the pdf module: **Measurement of Mechanical Quantities**
- Do some example problems – mechanical quantities

## Example: Thermistors




**Given**: A thermistor with $R_{\text{thermistor}} = 1633\underline{0}\ \Omega$ at 0.0°C and $624\underline{7}\ \Omega$ at 20.0°C is used in a simple voltage divider circuit, as sketched. (*Note*: Here, $V_{\text{out}}$ is across $R_s$ instead of $R_{\text{thermistor}}$.)
- $V_s$ = supply voltage = 5.00 V DC.
- $R_s$ = supply resistance = 10.00 kΩ.

**(*a*) To do**: Calculate the output voltage at $T$ = 0.0°C.

**(*b*) To do**: Calculate the output voltage at $T$ = 20.0°C.

**(*c*) To do**: Suppose the output voltage is 2.353 V. Estimate the temperature as best as you can, given the limited amount of information provided in the problem statement.

**Solution**:

(a) • Negligible current flows into the DMM or DAQ or whatever is reading $V_{out}$ since we assume the device has a huge input impedance.

• So, Ohm's law: $I = \dfrac{V_s}{R_s + R_{therm}}$ & $V_{out} = I \cdot R_s = \dfrac{V_s}{R_s + R_{therm}} \cdot R_s$

i.e., $\boxed{V_{out} = V_s \dfrac{R_s}{R_s + R_{therm}}}$

[This is a voltage divider & it does not matter if the resistor is on the top or on the bottom]

@ $T = 0°C$, $R_{therm} = 16330\,\Omega \rightarrow V_{out} = (5.00\text{ V}) \dfrac{10{,}000\ \Omega}{(10{,}000 + 16{,}330)\ \Omega}$

$V_{out} = 1.89897\text{ V} \approx \boxed{1.90\text{ V @ } T = 0°C}$

(b) @ $T = 20°C$, $R_{therm} = 6247\,\Omega \rightarrow V_{out} = (5.00)\dfrac{6247}{10000 + 6247} = 3.07749\text{ V}$

$V_{out} \approx \boxed{3.08\text{ V @ } T = 20°C}$

NOTE: THIS CIRCUIT IS "BETTER" THAN THE PREVIOUS ONE BECAUSE AS $T \uparrow$, $V_{out} \uparrow$

(In the previous circuit, $V_{out} \downarrow$ as $T \uparrow$)

(c) Assuming linear behavior, interpolate:

| $T$ (°C) | $V_{out}$ (V) |
|---|---|
| 0 | 1.89897 |
| $\square$ | 2.353 → $T = 7.7051$°C |
| 20 | 3.07749 |

[This is approximate since it is not exactly linear] → $\boxed{T \approx 7.71\text{°C}}$

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ADDITIONAL → THIS TURNS OUT TO BE A TYPE 5000 THERMISTOR.
(we have tables of $R_{therm}$ vs. $T$ for this thermistor)

- Let's compare our approximate answer to Part (c) to the "exact" answer from the thermistor table:

- At $V_{out} = 2.353$ V, work "backwards" with the voltage divider equation to solve for $R_{therm}$

$$V_{out} = V_s \frac{R_s}{R_s + R_{therm}} \rightarrow \boxed{R_{therm} = \frac{R_s V_s}{V_{out}} - R_s}$$

- Numbers → $R_{therm} = \dfrac{(10000\,\Omega)(5.00\text{ V})}{2.353\text{ V}} - 10000\,\Omega = \underline{\underline{11249.47\,\Omega}}$

- Finally, interpolate on the thermistor table to get $T$:

| $T$ (°C) | $R$ (Ω) |
|---|---|
| 7 | 11510 |
| $\square$ | 11249.47 → $\underline{T = 7.4737\text{°C}}$ |
| 8 | 10960 |

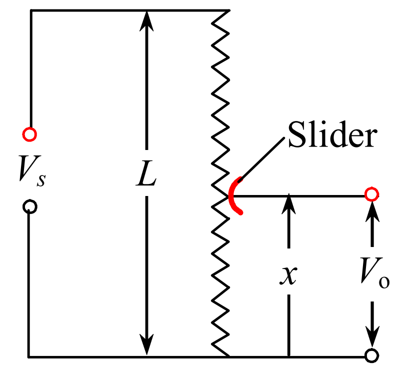
$\boxed{\text{The "exact" } T \text{ is } 7.47\text{°C}}$

(Still linearly interpolating, but between 7 & 8 °C instead of between 0 & 20 °C — more accurate.)

(The error of the previous estimate is ≈ 3%)

**Example: Displacement measurement**

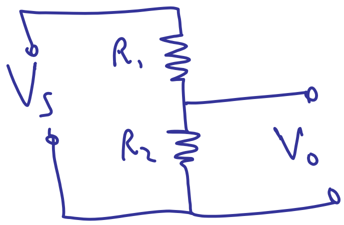
**Given**: A linear potentiometer is constructed using a resistor with resistance $R$ = 10.0 kΩ and length $L$ = 10 cm. The supply voltage is 5.00 V DC.




**To do**: When $V_o$ = 2.25 V, calculate distance $x$.

**Solution**:

- This is just a voltage divider




$$\boxed{V_o = V_s \frac{R_2}{R_1+R_2}}$$

- Here, $\frac{R_2}{R_1+R_2} = \frac{x}{L}$ since resistance is linearly proportional to the length of the resistor

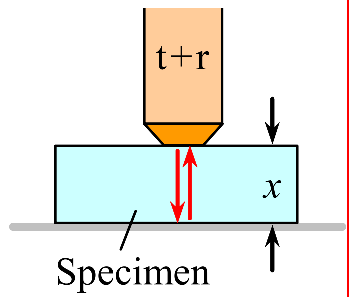
Thus, $V_o = V_s \frac{x}{L} \rightarrow \boxed{x = \frac{V_o}{V_s} L}$ (See interactive potentiometer on the website)

- Numbers: $x = \frac{V_o}{V_s} L = \frac{2.25\text{ V}}{5.00\text{ V}}(10\text{ cm}) = \boxed{4.50\text{ cm} = x}$

Note that the output is linearly proportional to the input.
$V_o$ $x$

**Example: Displacement measurement**

**Given**: The pulse-echo ultrasonic transducer in the ME 345 lab is used to measure the thickness of a piece of aluminum. The transmitted and reflected signals are read by an oscilloscope. The speed of sound in the aluminum is $a$ = 6300 m/s.

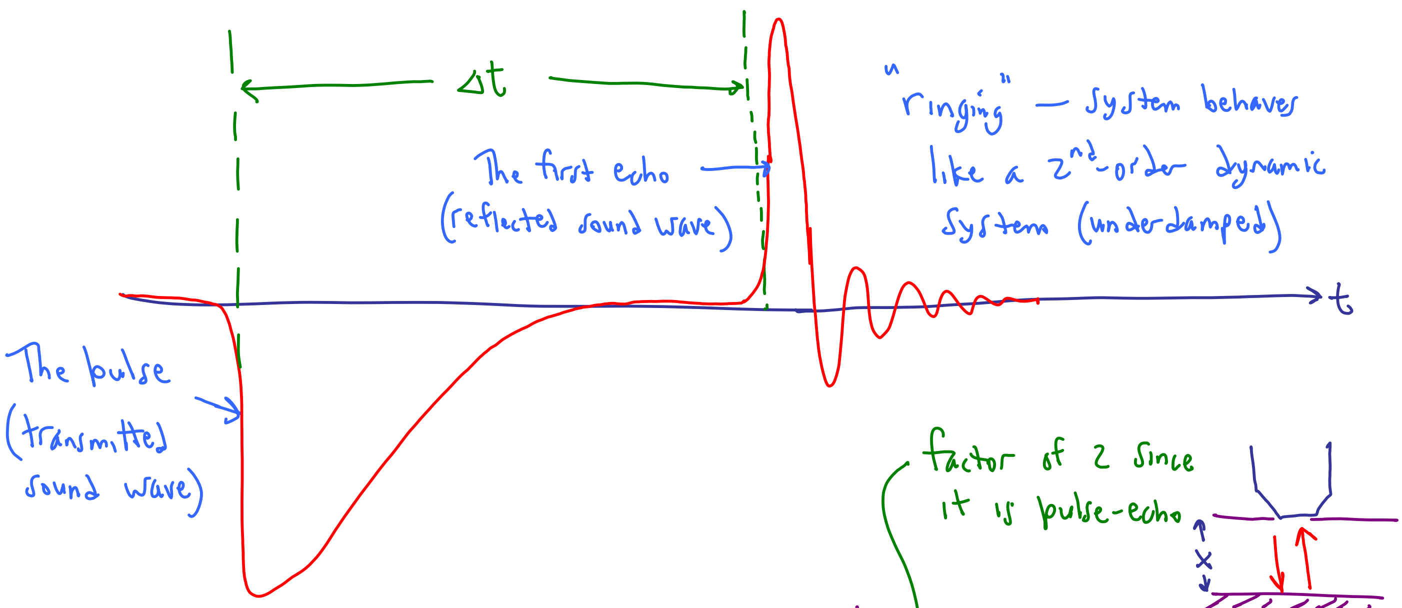



**To do**: Sketch the oscilloscope trace. Also, for $\Delta t$ = 4.1 μs, calculate the thickness of the specimen.

**Solution**: ["Ultrasonic" means that the frequency is greater than the limit of human hearing. Typically $f \gtrsim 20$ kHz is ultrasonic.]

- The instrument transmits a pulse of high frequency periodically & captures both the pulse and its reflection (echo) on an oscilloscope

- Typical oscilloscope trace (what it will look like in the lab):




factor of 2 since it is pulse-echo

- For $\Delta t$ = measured = 4.1 μs, $X = \dfrac{a\Delta t}{2}$

- Numbers: $X = \dfrac{a\Delta t}{2} = \dfrac{(6300\ \text{m/s})(4.1 \times 10^{-6}\ \text{s})}{2}\left(\dfrac{1000\ \text{mm}}{\text{m}}\right) = 12.9\ \text{mm}$

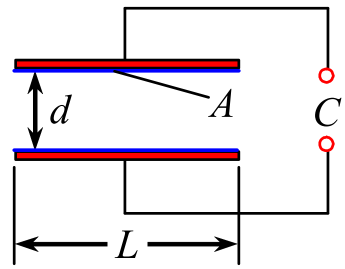
$$\boxed{X \cong 13.\ \text{mm} \quad (2\ \text{digits})}$$

Note: Can do this experiment "backwards" for a known $x$ to measure the speed of sound & determine the alloy of the metal sample

**Example: Displacement measurement**

**Given**: A home-made capacitance displacement sensor is constructed of two metal plates with air in the gap:

- initial gap distance is $d = 0.100$ mm
- the plate surface dimensions are $L = 1.00$ cm × $W = 1.00$ cm (where $W$ is the plate dimension into the page)




**(*a*) To do**: Calculate the initial capacitance in units of picofarads.

**(*b*) To do**: If the plates move apart *vertically*, calculate the *sensitivity* of the sensor.

**(*c*) To do**: If the plates move apart *horizontally*, calculate the *sensitivity* of the sensor.

**Solution**:

(a) For air, $K = 1$ = dielectric coefficient (K is dimensionless)

$\therefore$ $C = K \varepsilon_0 \dfrac{A}{d}$ ($\varepsilon_0$ = Permittivity of free space)

Here, $A = L \cdot W = 1.00 \text{ cm}^2$ (W = width into page)

• Good lesson in unit conversions!

Everything from here on is a unity conversion factor →

$$C = \underbrace{(1)}_{K}\underbrace{\left(8.854\times10^{-12}\frac{C^2}{N\cdot m^2}\right)}_{\varepsilon_0}\underbrace{\frac{1.00 \text{ cm}^2}{0.100 \text{ mm}}}_{A/d}\left(\frac{1000 \text{ mm}}{m}\right)\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^2 \cdot$$

(Coulomb)

$$\left(\frac{F\cdot V}{C}\right)\left(\frac{N\cdot m}{s\cdot W}\right)\left(\frac{W}{V\cdot A}\right)\left(\frac{A\cdot s}{C}\right)\left(\frac{10^{12} \text{ pF}}{F}\right) = 8.854 \approx \boxed{8.85 \text{ pF} = C}$$

(Farad) (Volt) (Watt) (ampere)

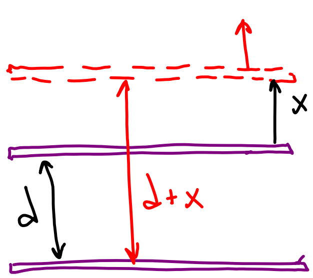
picofarad → "pico" = $10^{-12}$

(b) Plates move vertically — calculate the sensitivity

• Recall, Sensitivity $\equiv \dfrac{\Delta \text{ output}}{\Delta \text{ input}} = \dfrac{d(\text{output})}{d(\text{input})}$ when we have an equation to use

Here, output = C = capacitance
Input = x = distance moved

$\boxed{\text{Sensitivity} = \dfrac{dC}{dx}}$

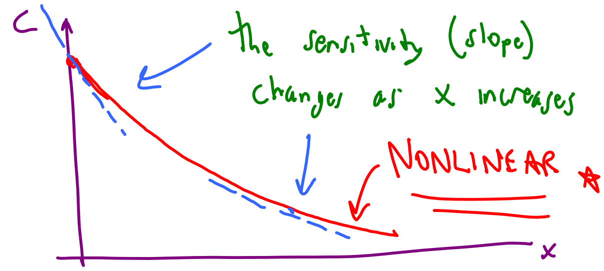



→ In our equation, d becomes d + x as sketched

(d in the equation for C)

- Eq. $\boxed{C = K\varepsilon_0 \dfrac{A}{d+x}}$ (Area A remains constant as plates move away from each other vertically, but d increases $\rightarrow$ d becomes $d+x$)

- Sensitivity $= \boxed{\dfrac{dC}{dx} = \dfrac{-K\varepsilon_0 A}{(d+x)^2}}$




- At $x=0$, the <u>initial</u> sensitivity is

$$\text{Initial Sensitivity} = \left.\frac{dC}{dx}\right)_{x=0} = \frac{-K\varepsilon_0 A}{d^2} = \frac{(-1)\left(8.854\times10^{-12}\,\frac{C^2}{N\cdot m^2}\right)\left(1.00\text{ cm}^2\right)}{(0.100\text{ mm})^2}\left(\frac{1000\text{ mm}}{m}\right)^2\left(\frac{1\text{ m}}{100\text{ cm}}\right)^2\left(\frac{F\cdot V}{C}\right)\cdot$$

$$\cdot\left(\frac{N\cdot m}{S\cdot V\cdot A}\right)\left(\frac{A\cdot S}{C}\right)\left(\frac{10^{12}\text{ pF}}{F}\right) = \boxed{-88.5\ \frac{\text{pF}}{\text{mm}}}$$

Initial sensitivity @ $x=0$

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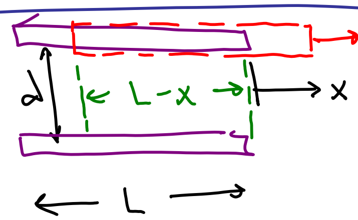
(c) Plates move <u>horizontally</u>




- In this case, the distance d between plates remains constant, but area A decreases with x. Here $\boxed{A = (L-x)W}$ $\rightarrow$ $C = K\varepsilon_0 \dfrac{A}{d}$ — A decreases; d = constant

- Sensitivity $= \boxed{\dfrac{dC}{dx} = -\dfrac{K\varepsilon_0 W}{d}}$ $\leftarrow$ $= K\varepsilon_0 \dfrac{(L-x)W}{d}$

- In this case, the sensitivity is constant since C varies linearly with X. <u>But</u>, C goes to zero when $x=L$, then $C=0$ from then on

- Numbers:

$$\frac{dC}{dx} = \frac{(-1)\left(8.854\times10^{-12}\,\frac{C^2}{N\cdot m^2}\right)(1.00\text{ cm})}{0.100\text{ mm}}\left(\frac{1\text{ m}}{100\text{ cm}}\right)\left(\frac{F\cdot V}{C}\right)\left(\frac{N\cdot m}{s\cdot V\cdot A}\right)\left(\frac{A\cdot s}{C}\right)\left(\frac{10^{12}\text{ pF}}{F}\right)$$

$$\boxed{\frac{dC}{dx} = -0.885\,\frac{\text{pF}}{\text{mm}}} = \text{sensitivity}$$

Comment: This is 100 times smaller sensitivity than the vertical case @ $x=0$ !!