Today, we will:

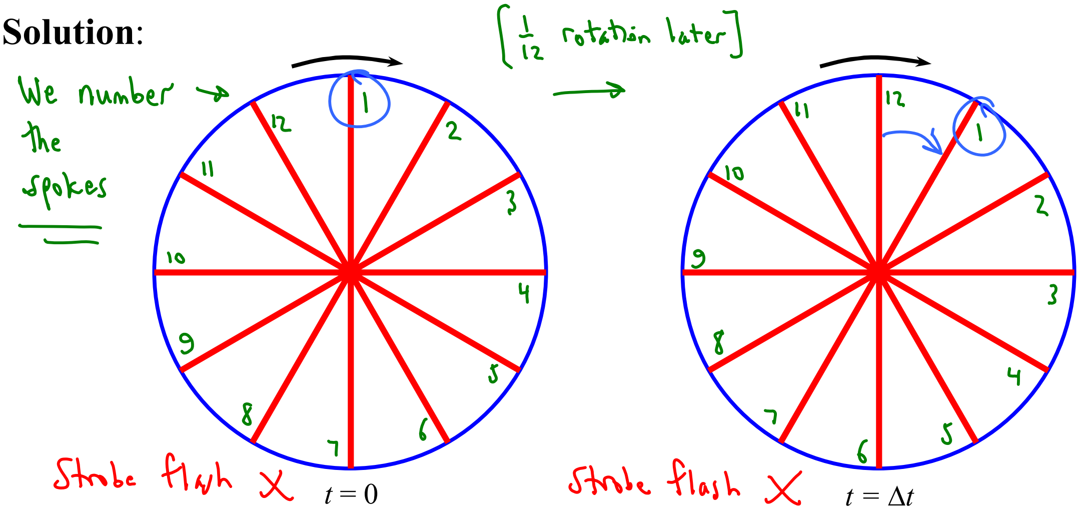
- Do some more example problems – rotation rate and stroboscopic tachometers
- Finish the pdf module: **Mechanical Measurements – Torque**, and do some examples

Example: RPM measurement

Given: A wagon wheel has 12 identical spokes, and rotates at 600 rpm. The rpm is measured with a stroboscopic tachometer in a room where it is dark except when the strobe light flashes. There are no painted dots anywhere, and *there is no way to distinguish one spoke from another.* → All spokes are identical & no painted dots on the wheel

To do: Calculate the maximum strobe flashing frequency at which you could be fooled. In other words, calculate the maximum strobe flashing frequency at which you would see a wagon wheel that appears to be frozen (not rotating), and therefore you could be fooled into thinking that this is the correct rpm. *Give your answer as in integer in units of rpm.*

Solution:



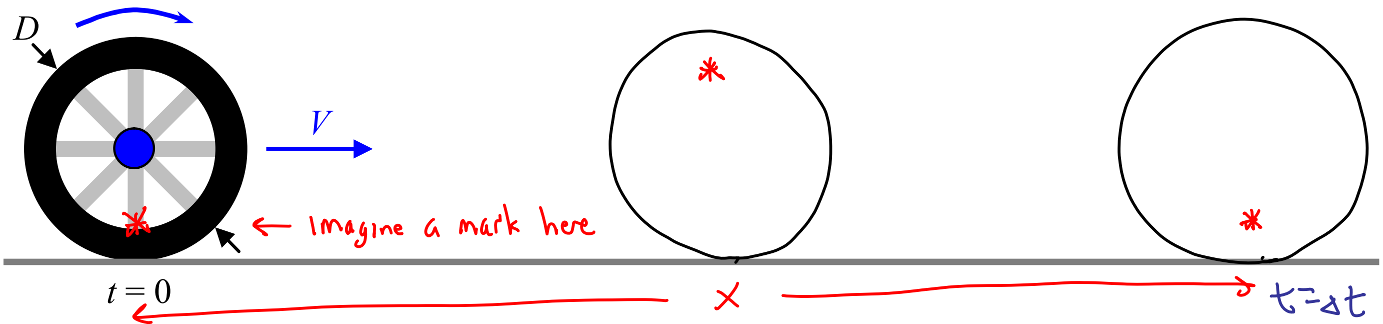
- At $t = \Delta t$, the wheel has rotated $\frac{1}{12}$ of a rotation, so that all spokes are aligned the same as at $t=0$, but rotated by one spoke, as sketched
- But we see NO DIFFERENCE between a flash @ $t=0$ & a flash @ $t=\Delta t$
- $\Delta t = \dfrac{\frac{1}{12}\text{ rot}}{600\ \frac{\text{rot}}{\text{min}}} = \underline{\underline{\dfrac{1}{7200}}}\text{ min}$ → If we flash the strobe every $\frac{1}{7200}$ min, we are fooled
- This corresponds to a strobe flash rate of $\boxed{7200\text{ rpm}}$

Comments: • This is the max. Any flash rate > 7200 rpm would show more than 12 spokes. E.g. 14400 rpm → would see 24 spokes

• We can be fooled at several flash rates < 7200 rpm, however.
E.g., At 3600 rpm, each spoke moves 2/12 of a rotation, & we would be fooled

Example: RPM of car tire

Given: A car travels at 40 miles per hour. The outer diameter of its tires is 28 inches.



To do: Calculate the rotation rate of the wheel in units of rpm (rotations per minute).

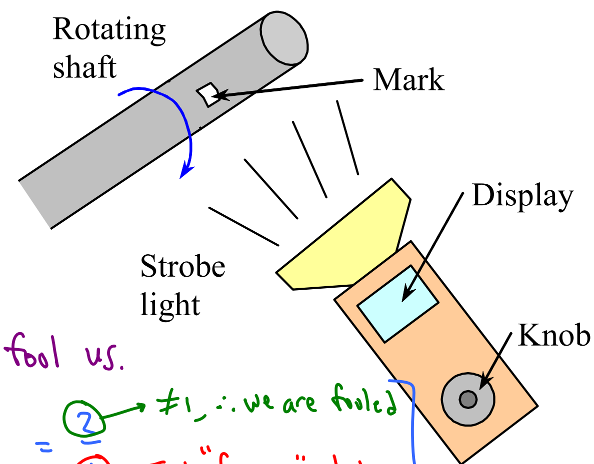
Solution:

- One complete rotation occurs when the car travels $\boxed{x = \pi D}$
  (πD = outer circumference)
- But $\boxed{x = V\Delta t}$ in this same time period
- Equate → $x = \pi D = V \Delta t$ → $\boxed{\Delta t = \frac{\pi D}{V}}$ = time required for one rotation
- Since $\Delta t = \frac{\text{time}}{\text{rot}}$, frequency (rpm) = $\frac{\text{rot}}{\text{time}} = \boxed{\frac{1}{\Delta t} = f}$
- Numbers: $f = \frac{1}{\Delta t} = \frac{V}{\pi D}$ = frequency

$$N_{rpm} = f = \frac{40 \text{ mph}}{\pi (28 \text{ in})} \left(\frac{12 \text{ in}}{\text{ft}}\right)\left(\frac{88 \text{ ft/s}}{\text{mph}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right) = 480.193 \frac{\text{rot}}{\text{min}}$$

$$\boxed{N_{rpm} \approx 480. \text{ rpm}}$$

Example: RPM measurement



Given: A white dot is painted on the shaft of a turbine. The shaft starts rotating at 860 rpm. Its rotational rate is measured by a stroboscopic tachometer.

(*a*) To do: Name at least two strobe settings (rpm) at which the strobotachometer would give a false reading of rotation rate (you are fooled).

Solution:

Any integer fraction of 860 rpm will fool us.

e.g. $\frac{860}{2} = \boxed{430}$ $\left[\frac{\#\text{ rot}}{\text{flash}} = \frac{860 \text{ rot/min}}{430 \text{ flash/min}} = \frac{2}{1} \right]$ 2 → ≠1, ∴ we are fooled; 1 — 1 "frozen" dot

$\frac{860}{3} = \boxed{286.7}$

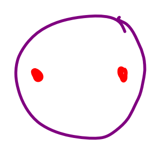
etc.

(*b*) To do: For the setup shown here, are there any strobe settings greater than 860 rpm at which the strobotachometer would give a false reading of rotation rate? Explain.

Solution:

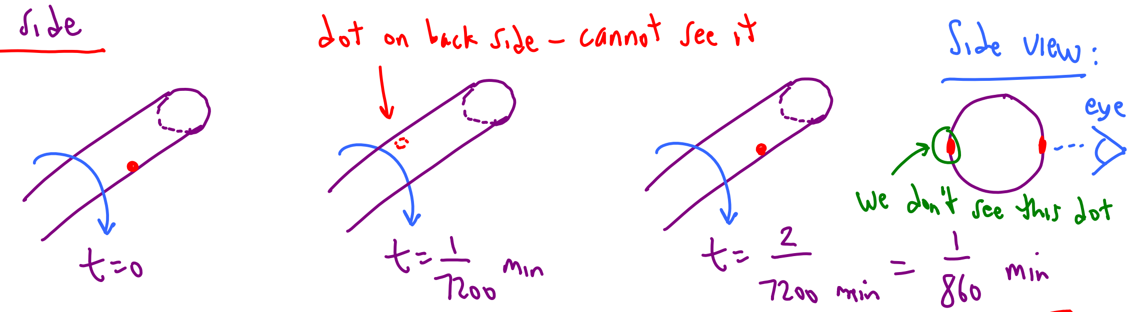
- If the dot were on the end of the shaft, we would never see only one "frozen" dot for any rpm > 860.

  Eg., At rpm = 2(860) = 1720, we would see 2 dots



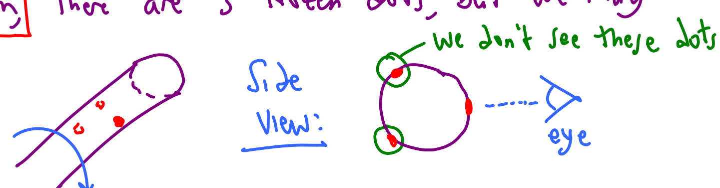
- But, here the dot is on the side of the shaft, so we cannot see the back side

- At 1720 rpm flash rate,



$\boxed{\text{At 1720 flash/min, we are fooled} \rightarrow \text{There are two "frozen" dots, but we see only one (cannot see the one on the back side)}}$ ☆

- Similarly, @ strobe = 860 * 3 = $\boxed{2850 \text{ rpm}}$, there are 3 frozen dots, but we may see only one, and would be fooled.



Bottom line – Be careful w/ a strobe if you cannot see the whole shaft ☆

Example: Dynamometer measurement

Given: Gerry is building a prony brake dynamometer to measure the torque output from a small (100 mL displacement) motorcycle engine. The maximum expected torque is 7.0 N·m, at a rotation rate of around 7500 rpm. Gerry has a fish-scale-type force gage with a range of 0 to 50 N. He would like to use this scale in his dynamometer.

(*a*) To do: Calculate the moment arm that Gerry should build for his dynamometer.

Solution:

$$T = F \cdot r \rightarrow r = \frac{T}{F}$$

At max torque, we want the scale to read max F

So, set $r = \frac{T_{max}}{F_{max}} = \frac{7.0 \text{ Nm}}{50 \text{ N}} = 0.14 \text{ m}$

$$\boxed{r = 0.14 \text{ m} = 14. \text{ cm}}$$

(*b*) To do: Gerry tests the engine and finds that its maximum power output is 8.1 hp at 8500 rpm. Calculate the torque at this maximum-power operating condition.

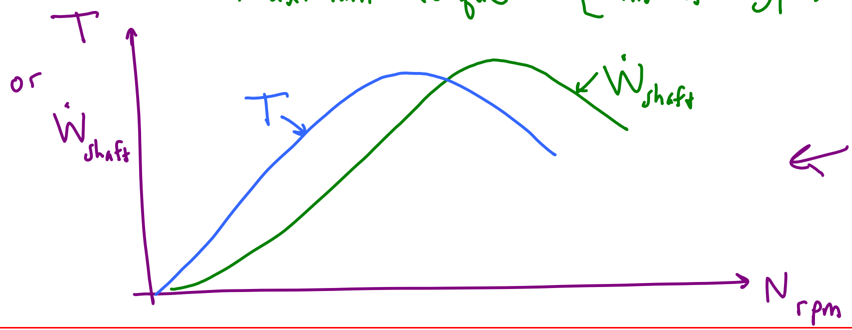
Solution:

- $\dot{W}_{shaft} = T \cdot \omega \rightarrow \boxed{T = \frac{\dot{W}_{shaft}}{\omega}}$

- Numbers: $T = \frac{8.1 \text{ hp}}{8500 \frac{\text{rot}}{\text{min}}} \left(\frac{1 \text{ rot}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right)\left(\frac{745.7 \text{ W}}{\text{hp}}\right)\left(\frac{\text{N}\cdot\text{m}}{\text{s}\cdot\text{W}}\right) = 6.79 \text{ N}\cdot\text{m}$

$\boxed{T \approx 6.8 \text{ N}\cdot\text{m}}$ = torque @ max. power

COMMENT: Notice that @ max power, the torque is lower than the maximum torque [this is typical of engines]



Typical performance curves for an engine

Example: Dynamometer measurement [Good review problem, RSS uncertainty]

Given: A prony brake dynamometer is used to test the output of a small engine:

- moment arm r = 11.30 cm ± 0.05 cm (measured with a ruler)
- force F = 36.5 ± 0.5 N (measured with a fish scale)
- rotation rate of the engine shaft N_{rpm} = 2012 ± 1 rpm (measured with a strobotach)

All measurements are to standard engineering (95%) confidence level.

To do: Calculate the engine shaft power (in units of watts) and its uncertainty.

Solution:

- First convert N_{rpm} into rad/s (ω)

$$\omega = \left(2012 \frac{\text{rot}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{\text{rot}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = \boxed{210.696 \frac{\text{rad}}{\text{s}} = \omega}$$

- Do the same for the uncertainty in ω, call it u_ω

$$u_\omega = \pm\left(1 \frac{\text{rot}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{\text{rot}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = \boxed{\pm 0.1047 \frac{\text{rad}}{\text{s}} = u_\omega}$$

- So, we write $\omega = \bar{\omega} \pm u_\omega = \boxed{210.696 \pm 0.1047 \text{ rad/s} = \omega}$

[keep extra digits to avoid round-off errors]

- $\dot{W}_{shaft} = T \cdot \omega = F \cdot r \cdot \omega$

$$= (36.5 \text{ N})(0.113 \text{ m})\left(210.696 \frac{\text{rad}}{\text{s}}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{N}\cdot\text{m}}\right) = \underline{\underline{869.02 \text{ W}}}$$

- Uncertainty in $\dot{W}_{shaft}$ is obtained from the RSS uncertainty method

$\dot{W}_{shaft} = F^1 r^1 \omega^1$ = "simple" exponent form, so we can use the simple formula for RSS uncertainty:

$$\frac{u_{\dot{W}}}{\dot{W}} = \sqrt{\left(1\frac{u_F}{F}\right)^2 + \left(1\frac{u_r}{r}\right)^2 + \left(1\frac{u_\omega}{\omega}\right)^2} = \sqrt{\left(\frac{0.5}{36.5}\right)^2 + \left(\frac{0.05}{11.30}\right)^2 + \left(\frac{0.1047}{210.696}\right)^2}$$

$$= 0.0144 \text{ W}$$

$$\therefore u_{\dot{W}} = \frac{u_{\dot{W}}}{\dot{W}}(\dot{W}) = (0.0144)(869.0 \text{ W}) = 12.5 \text{ W}$$

Answer:

$$\boxed{\dot{W}_{shaft} = 869. \pm 12.5 \text{ W}}$$