Today, we will:

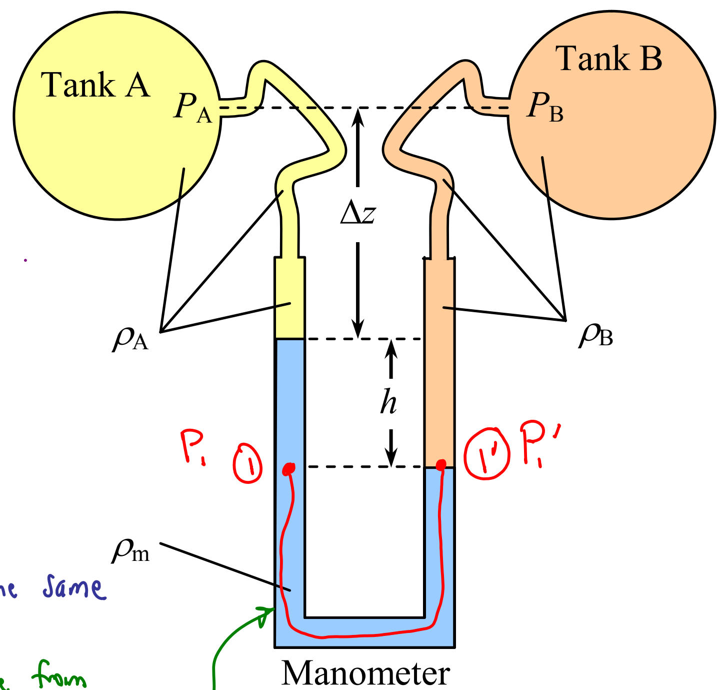
- Review the pdf module: **Pressure Measurement**, and do some example problems
- If time, begin the pdf module: **Linear Velocity Measurement**

Example: Pressure measurement

Given: A U-tube manometer is used as a differential pressure measurement instrument to measure the pressure difference between two tanks. The two tanks are at the same elevation.

(*a*) To do: Calculate the pressure difference $P_B - P_A$ for the general case in which ρ_A is not the same as ρ_B (they are different fluids.

(*b*) To do: Simplify for the case in which $\rho_A = \rho_B$ (they are the same fluid).



Solution:

(a) At ① & ①' the pressures must be the same

[same elevation & can draw a continuous line from ① to ①' through the same fluid in hydrostatics]

- Left leg: $P_1 = P_A + \rho_A g \Delta z + \rho_m g h$ ← Using $P_{below} = P_{above} + \rho g |\Delta z|$
- Right leg: $P_1' = P_1 = P_B + \rho_B g \Delta z + \rho_B g h$
- Equate the RHS of the above two equations:

$$P_B + \rho_B g \Delta z + \rho_B g h = P_A + \rho_A g \Delta z + \rho_m g \Delta h$$

or $$\boxed{P_B - P_A = (\rho_m - \rho_B) g h + (\rho_A - \rho_B) g \Delta z} \qquad (1)$$

ALTERNATIVE WAY: Work around the loop from A to B, adding pressure when go down & subtracting pressure when go up:

(many people prefer this method, and find it easier)

$$P_A + \rho_A g \Delta z + \rho_m g h - \rho_B g h - \rho_B g \Delta z = P_B$$

$$\boxed{P_B - P_A = (\rho_m - \rho_B) g h + (\rho_A - \rho_B) g \Delta z}$$

[We get the same answer, which is reassuring]

(b) If the fluids are the same (A & B), then $\rho_A = \rho_B$ & Eq. (1) simplifies to

$$\boxed{P_B - P_A = (\rho_m - \rho_B) g h} \qquad (2)$$

Note: If in addition to $\rho_A = \rho_B$, we also know that $\rho_A \ll \rho_m$,

[e.g., A = air & $\rho_A \approx 1.2\ kg/m^3$
m = mercury & $\rho_m = 13600\ kg/m^3$ } $\rho_m \gg \rho_A$]

then (2) reduces further to

$$\boxed{P_B - P_A \simeq \rho_m g h}$$

This h is called the "head" = pressure expressed as an equivalent column height of the manometer liquid

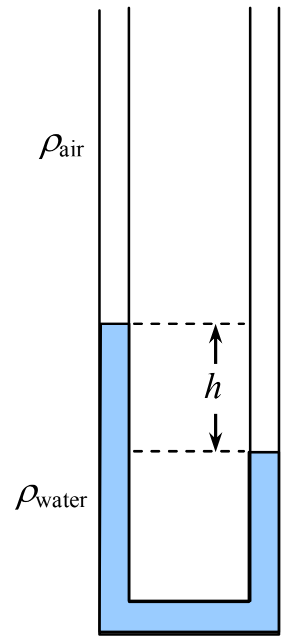
Note: This is just approximate, and should be used only if $\rho_m \gg \rho_A$

Caution: People like to say $\Delta P = \rho g h$ since it is so simple & easy to remember, but keep in mind that this is only ★ an approximation, valid if $\rho_{fluid} \ll \rho_{manometer}$

★ I recommend (strongly) that you always use the exact equations for these hydrostatic problems to avoid potential errors due to faulty approximations.

Example: U-tube manometer

Given: A 4.0-ft tall U-tube manometer is used with water as the manometer fluid to measure a pressure difference in air.

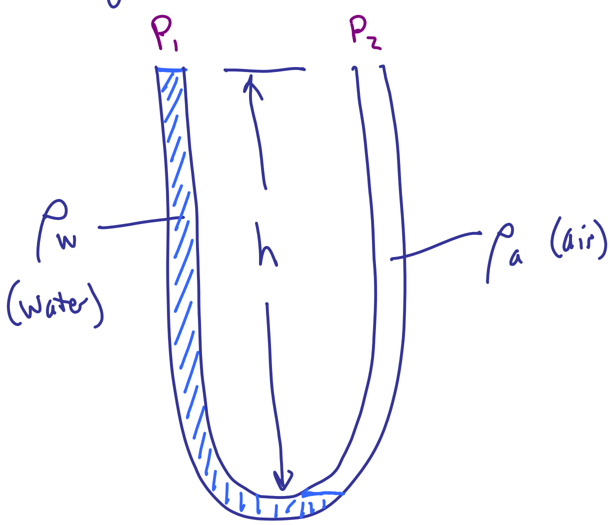


Manometer

To do: Calculate the maximum pressure difference ΔP that can be measured with this manometer and these two fluids.

Solution:

- Max P that can be measured is when the left column is ready to overflow & the right column is at the bottom as sketched:



- Equations: (hydrostatics) →

$$P_1 + \rho_w g h - \rho_a g h = P_2$$

$$\boxed{\Delta P = P_2 - P_1 = (\rho_w - \rho_a) g h} \qquad (1)$$

- Numbers:

$$\Delta P_{max} = (1000 - 1.2)\frac{kg}{m^3}\left(9.807\frac{m}{s^2}\right)(4.0\ ft)\left(\frac{0.3048\ m}{ft}\right)\left(\frac{N \cdot s^2}{kg \cdot m}\right)\left(\frac{kPa \cdot m^2}{1000\ N}\right)$$

(As always, be careful with your units!)

$$\boxed{\Delta P_{max} = 11.9\ kPa}$$

Additional Question: How tall (in ft) would the manometer need to be in order to measure $\Delta P = 1$ atmosphere (101.3 kPa)?

Solution: Solve Eq. (1) for h:

$$h = \frac{\Delta P}{(\rho_w - \rho_a) g} = \frac{101{,}300\ N/m^2}{(1000 - 1.2)\ kg/m^3\ (9.807\ m/s^2)}\left(\frac{kg \cdot m}{s^2 \cdot N}\right)\left(\frac{1\ ft}{0.3048\ m}\right)$$

$$h = 33.9297\ ft \rightarrow \boxed{h \cong 33.9\ ft}$$

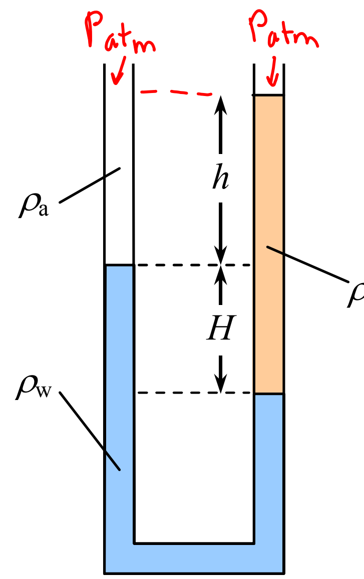
[Wow! Very tall! — This is why a denser liquid (like mercury) is preferred when measuring large pressures]

Example: Density measurement with a U-tube manometer

Given: Brett claims that he can use a U-tube manometer to measure the density of an oil. He sets up the manometer as sketched, with both sides open to the atmosphere, with water (ρ_w) on the left leg, and with both water and oil (ρ_o) on the right leg. He knows the density of the air (ρ_a) and the water, and he carefully measures H and h.

To do: Calculate the density of the oil in terms of the given variables.

Solution:



Manometer

- Apply our eq. of hydrostatics:

$$P_{below} = P_{above} + \rho g |\Delta z|$$

- Let's start on the top left and work around to the top right:

$$\cancel{P_{atm}} + \rho_a g h + \rho_w g H \underbrace{- \rho_{oil} g H - \rho_{oil} g h}_{\text{collect } \rho_{oil} \text{ terms \& solve:}} = \cancel{P_{atm}}$$

$$\rho_{oil}\, g (H+h) = \rho_a g h + \rho_w g H$$

$$\rho_{oil} = \frac{\rho_a \cancel{g} h + \rho_w \cancel{g} H}{\cancel{g}(H+h)}$$

$$\boxed{\rho_{oil} = \frac{\rho_a h + \rho_w H}{H+h}}$$