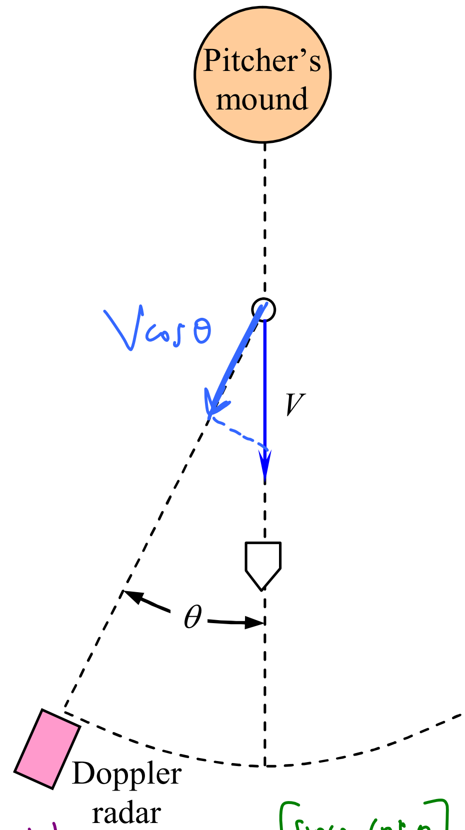
Today, we will:

- Review the pdf module: **Linear Velocity Measurement** and do some example problems

Example: Velocity measurement

Given: Doppler radar is used to measure the speed of a baseball as sketched (not to scale). The radar frequency is 12,000 MHz.

To do: Calculate the maximum angle θ that will yield less than 1% error in measuring the speed of the baseball.



Solution:

- You will read $V\cos\theta$, not V since you are at an angle
- So, $\boxed{V_{incorrect} = V\cos\theta}$
- For 1% error, let

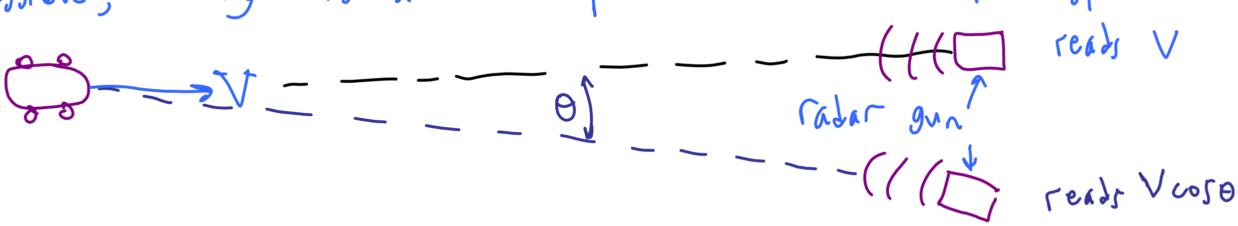
$$0.01 = \left|\frac{V_{incorrect} - V}{V}\right| = \left|\frac{V\cos\theta - V}{V}\right| = \left|\frac{\cos\theta - 1}{1}\right| = 1 - \cos\theta$$

Since $\cos\theta$ is always ≤ 1

Solve $\longrightarrow$ $\cos\theta = 1 - 0.01 = 0.99 \rightarrow \theta = \arccos(0.99) = \pm 8.11°$

- Answer $\longrightarrow$ must keep θ within $\pm 8.11°$ to avoid errors of more than 1% in velocity

This is why policemen with radar guns like to point as directly at cars as possible, or they read a lower speed than the car's true speed.



Example: Velocity measurement

Given: The LDV system in the M E 325 fluids lab uses a red helium-neon laser with wavelength λ = 632.8 nm. The angle between the two beams is α = 13.53°. The system is used in a small water tunnel with a speed range of -1 to 1 m/s.

To do: Calculate the maximum expected frequency (in units of kHz) of the LDV output.

Solution:

- See notes on the learning module → $V = \dfrac{f\lambda}{2\sin(\alpha/2)} \Rightarrow \boxed{f = \dfrac{2V\sin(\alpha/2)}{\lambda}}$ (1)

- Numbers (Watch units!): @ $V = V_{max} = 1$ m/s,

$$f_{max} = \frac{2(1\ \text{m/s})\sin(13.53°/2)}{632\times10^{-9}\ \text{m}}\left(\frac{1\ \text{kHz}}{1000\ \text{Hz}}\right)\left(\frac{\text{Hz}}{1/\text{s}}\right) = 372.305\ \text{kHz}$$

[1 nanometer = 10^{-9} m]

or $\boxed{f_{max} \cong 372\ \text{kHz} \ @\ V = 1\ \text{m/s}}$

Comment: Need a very fast DAQ to measure this very high frequency!

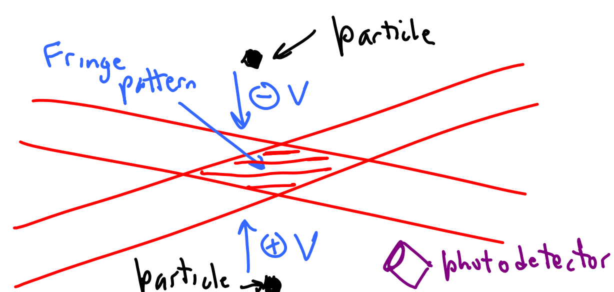
Additional Question: What frequency will we observe when V = -1 m/s?

Answer → If plug V = -1 m/s into Eq. (1), get $f = -372$ kHz ✗

No! Cannot have a negative frequency!!

Answer → $\boxed{f = 372\ \text{kHz}}$

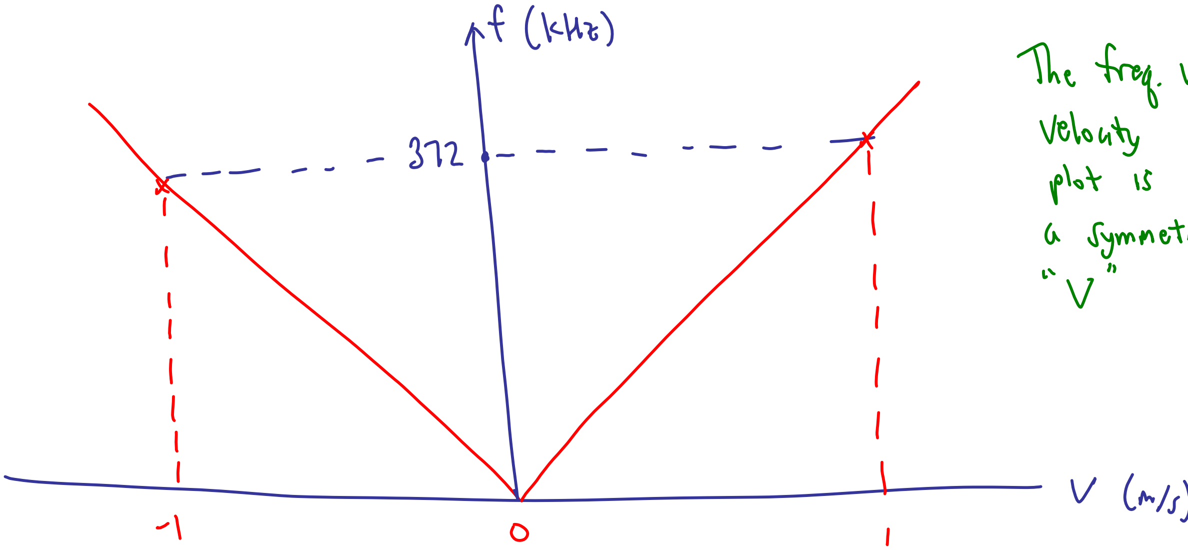
☆ ☆ The LDV (without Bragg shifting) cannot distinguish between ⊕ & ⊖ velocities ☆☆



(both ⊕ & ⊖ V yield same scattering freq.)

BRAGG CELL SHIFTING

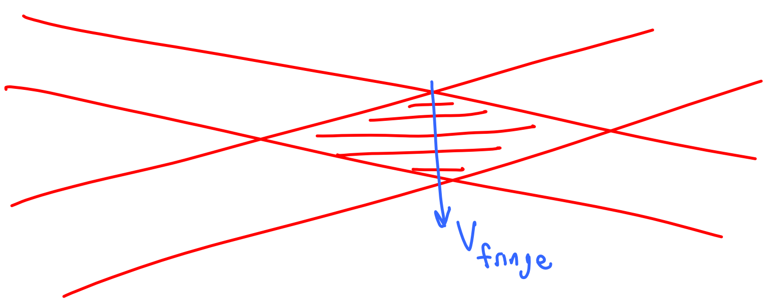
- This leads us to a discussion of Bragg Cell shifting



The freq. vs Velocity plot is a symmetric "V"

[We cannot tell from the frequency whether V is ⊕ or ⊖]

A Bragg cell shifts the frequency of one of the beams, causing the fringe pattern to move (instead of standing still)



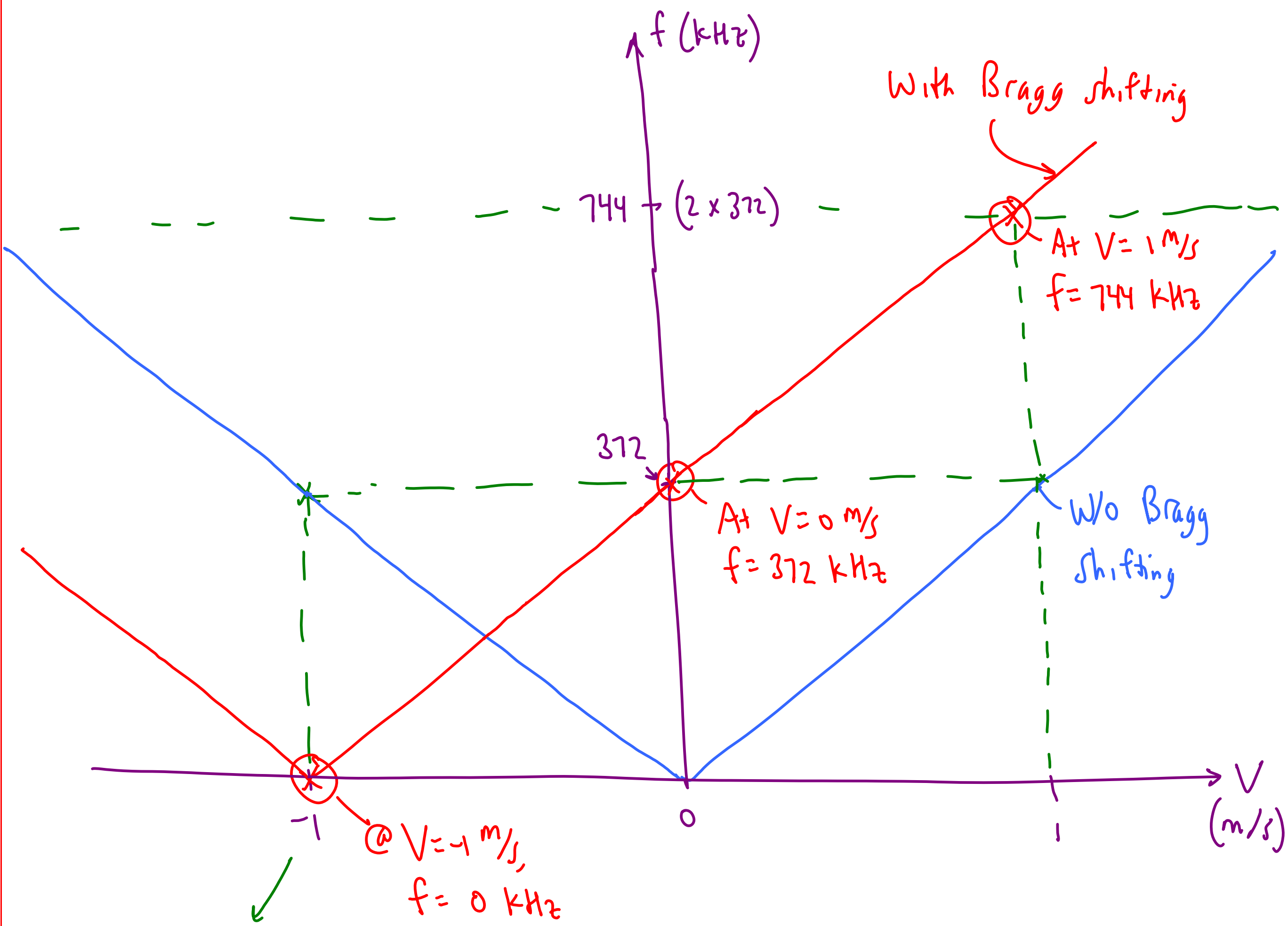
Here, we shift the frequency such that the fringe pattern moves down (⊖ V) at a magnitude of 1 m/s $\rightarrow$ $\boxed{V_{fringe} = -1 \text{ m/s}}$

$\therefore$ $\boxed{V_{LDV\ optics} = V_{actual} - V_{fringe}}$

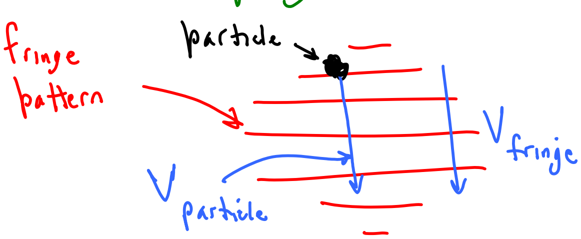
- With Bragg cell @ $V_{fringe} = -1$ m/s,

$$\boxed{\begin{array}{l} \bullet\ @\ V_{actual} = -1 \text{ m/s}, \quad V_{LDV\ optics} = -1 - (-1) = 0 \text{ m/s} \\ \bullet\ @\ V_{actual} = 0 \text{ m/s} \quad V_{LDV\ optics} = 0 - (-1) = 1 \text{ m/s} \\ \bullet\ @\ V_{actual} = 1 \text{ m/s} \quad V_{LDV\ optics} = 1 - (-1) = 2 \text{ m/s} \end{array}}$$

We have shifted the whole freq. vs. Velocity plot to the left.



particle moves @ exactly the same velocity as the fringe pattern when $V = -1$ m/s & $V_{fringe} = -1$ m/s. Thus, the observed frequency measured by the LDV optics is zero



(the particle does not cross the light-dark-light fringe pattern, ∴ $f = 0$)

(The scattered light is now at a frequency proportional to $V - V_{fringe}$ instead of just V itself — we have shifted the frequency vs. velocity plot)

★ Thus, with a Bragg cell, we can distinguish between ⊕ & ⊖ velocities!