Today, we will:

- Finish the pdf module: **Linear Velocity Measurement** and do some example problems
- Additional notes: **hydraulic jacks, tire gages, strain gage pressure cells**

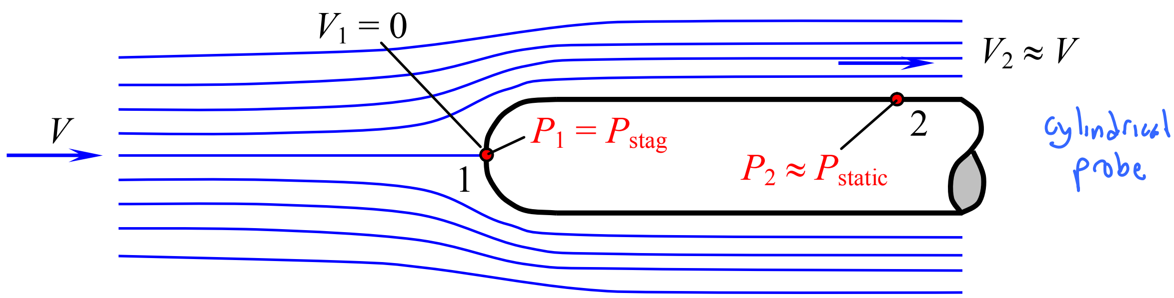
Pressure Measurement in Moving Fluids

We define three types of pressure used in moving fluids:

- **Stagnation pressure** P_{stag} = pressure at a stagnation point where the velocity is slowed down to zero nearly isentropically. This is the pressure at the *nose* (*stagnation point*) of a probe in the flow.
- **Static pressure** P = pressure that would be measured by an infinitesimal pressure sensor moving with the flow. This is the pressure upstream of a probe in the flow.

> It turns out that the pressure at point 2 in the sketch below is approximately equal to the static pressure, since the velocity at point 2 is approximately equal to V and the streamlines are straight (not curved, which leads to pressure changes) at point 2.

- **Dynamic pressure** $\rho V^2/2$ = difference between stagnation and static pressure = $P_{stag} - P$. This is the "extra" pressure that is felt at the stagnation point at the nose of a probe in the flow.



We combine these definitions in a practical application – measurement of velocity.

Comments:

- P (static pressure) is the same P we use in thermodynamics = the thermodynamic pressure of the fluid (the pressure moving with the fluid)
- "Static" is confusing → means that the probe is static with respect to the fluid, i.e., it is moving with the fluid.
- How to measure static pressure? — Imagine a small neutrally buoyant pressure sensor moving with the fluid, with a wireless transmitter that we monitor remotely [easy to imagine, but extremely hard to actually make]
- Fortunately, it turns out that the pressure at point ② in the above sketch, after the flow has become parallel to the probe is approximately equal to the static pressure since $V_2 \simeq V$

$$\boxed{P_2 \simeq P_{static} = P}$$

Equations:

$$\boxed{P_{stagnation} = P + \frac{1}{2}\rho V^2}$$ (from Bernoulli eq.)

$P_{stagnation} = P_1$ in our sketch

Static pressure

$P = P_2$ in our sketch

$\frac{1}{2}\rho V^2 =$ dynamic pressure

So, $$\frac{1}{2}\rho V^2 = P_{stagnation} - P$$

$$= P_1 - P_2$$

or, solving for V,

$$\star \quad \boxed{V = \sqrt{\frac{2(P_{stagnation} - P)}{\rho}} = \sqrt{\frac{2(P_1 - P_2)}{\rho}}}$$

☆ Pitot formula

So, if we can measure $P_{stagnation}$ & P_{static} at ≈ the same point in the flow, we can calculate V

- ☆ Pitot probe — measures only stagnation pressure
- ☆ Static probe — measures only static pressure
- ☆ Pitot-static probe — measures both stagnation and static pressure

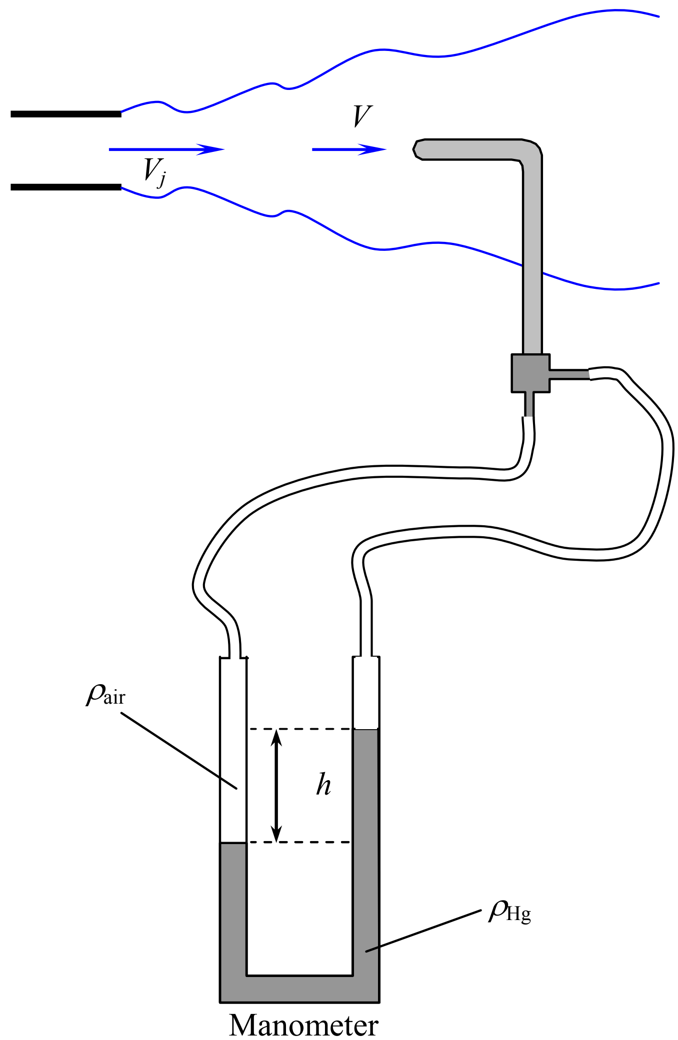
Most useful

Example: Velocity measurement

Given: A Pitot-static probe is placed in an air jet to measure the air speed. The differential pressure is measured with a U-tube manometer that uses mercury (ρ = 13,600 kg/m^3) as the manometer fluid. The difference in column height between the two legs of the manometer is h = 1.20 cm. The air density is 1.204 kg/m^3.

To do: Calculate the air speed at the location of the Pitot-static probe.

Solution:



- $P_1 = P_{stag}$
- $P_2 = P$ (static pressure)
- manometer (hydrostatics):

$$P_1 - P_2 = \boxed{P_{stag} - P = (\rho_{Hg} - \rho_{air})\, g h}$$

- Pitot formula:

$$V = \sqrt{\frac{2(P_{stag} - P)}{\rho}}$$

use ρ of the fluid, not $\rho_{manometer}$!!
(a common mistake)

or
$$V = \sqrt{\frac{2g(\rho_{Hg} - \rho_{air})h}{\rho_{air}}}$$

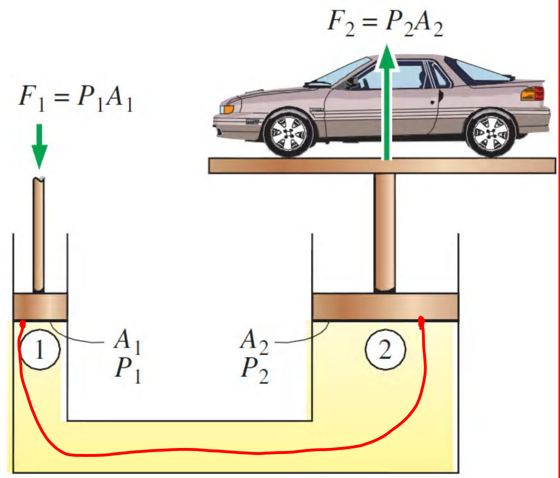
$$= \sqrt{\frac{2(9.807\ m/s^2)(13{,}600 - 1.204)\ kg/m^3\ (0.0120\ m)}{1.204\ kg/m^3}}$$

$$\boxed{V = 51.6\ m/s}$$

Additional Notes: **Hydraulic Jack (how does it work?)**

① = Small diameter pipe
② = large diameter pipe

$\Big\}\ A_2 \gg A_1$



• Hydrostatics → $P_1 = P_2$ (same fluid, same elevation, & can draw a curve from ① to ② through the same fluid)

• But $P_1 = F_1/A_1$ & $P_2 = F_2/A_2$

equate:

$$\boxed{\frac{F_2}{F_1} = \frac{A_2}{A_1} = \text{ideal mechanical advantage}}$$ ★ → can be huge!

Example: Pressure measurement and hydrostatics

Given: A hydraulic jack is constructed with the large piston diameter equal to 25.4 cm (10 inch) and the small piston diameter equal to 0.635 cm (1/4 inch).

To do: How much weight (in lbf) can a person lift with the jack if he exerts a force of 20.0 lbf on the small piston? *Give your answer to three significant digits.*

Solution:

• $$\frac{F_2}{F_1} = \frac{A_2}{A_1} = \frac{\pi D_2^2}{\pi D_1^2} = \left(\frac{D_2}{D_1}\right)^2 \rightarrow \boxed{F_2 = F_1\left(\frac{D_2}{D_1}\right)^2}$$ (Answer in variable form)

• Numbers: $$F_2 = (20.0 \text{ lbf})\left(\frac{10 \text{ in}}{1/4 \text{ in}}\right)^2 = 32{,}000 \text{ lbf}$$

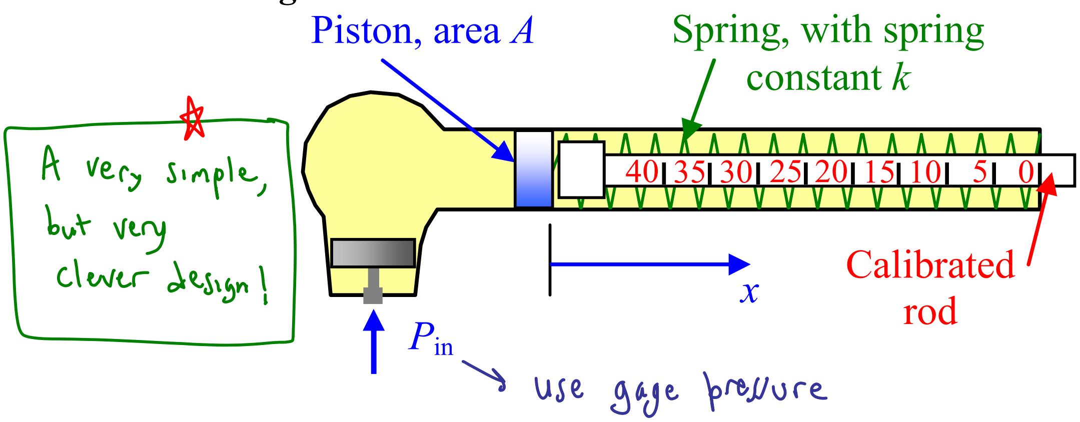
$$\boxed{F_2 = 32{,}000 \text{ lbf}}$$!! Huge!

Comments: • Oil density does not matter

• To conserve mass, piston 2 moves up much less than piston 1 moves down. [factor of A_2/A_1]

[That is why you have to pump the jack handle so many times to move the car up by a small amount.]

Tire Pressure Gauge – How does it work?

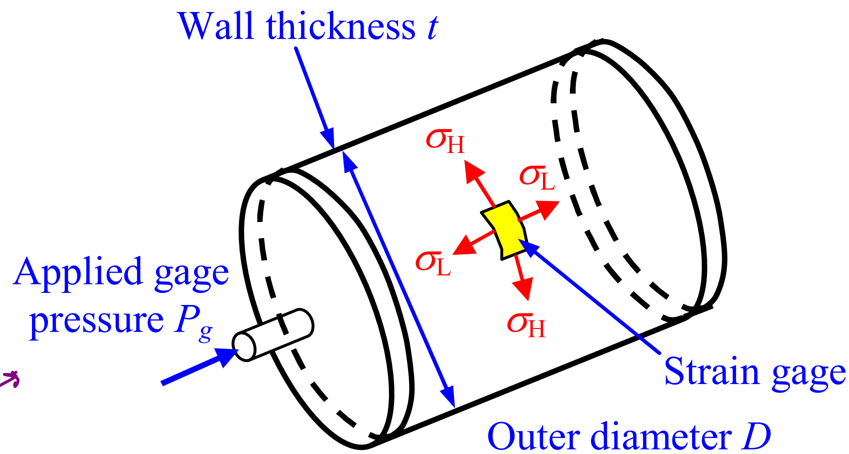


- The key to understanding this is that the rod is <u>loose</u> (it is not attached to the piston)
- When P_{in} is applied, the piston moves out, forcing the rod to also move out.
- But since the rod is loose, it does not move back in when P_{in} is removed — it stays where it was, and we read P_{in}
- Equations — Spring: $F = kx$ = linear; Piston: $F = P_{in,\,gage} \cdot A$ } equate:

$$P_{in,\,gage} \cdot A = kx \rightarrow P_{in,\,gage} = \frac{kx}{A} = \text{linear}$$

- Then, after reading, you have to push the rod in manually

Strain Gage Pressure Cell – How does it work?

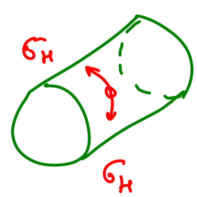


$P_g = P_{in} - P_{atm} =$ gage pressure being measured

- This is similar to the soda can / strain gage experiment we did in the lab
- Recall from E. Mech. class, for a thin-walled cylinder under internal gage pressure P_g,

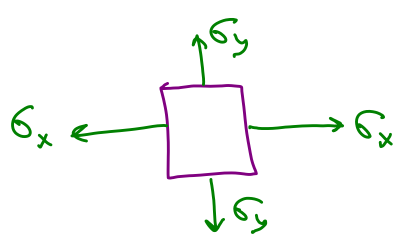
$$\boxed{\begin{aligned} \sigma_H &= \text{hoop stress} = \frac{P_g D}{2t} \\ \sigma_L &= \text{longitudinal stress} = \frac{P_g D}{4t} \end{aligned}}$$

see sketch above



σ_H is the one encircling the cylinder, like a "hoop"

- Recall, at a surface,



- If σ_x & σ_y are principal stresses then we had previously derived these relationships for principal strains:

$$\boxed{\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right)} \qquad \boxed{\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x\right)}$$

- Here, let's define the directions as $\boxed{\begin{matrix} x = L \\ y = H \end{matrix}}$ (longitudinal) (hoop)

• Thus,

Hoop strain → $$\boxed{\varepsilon_H = \frac{1}{E}\left(\sigma_H - \nu\sigma_L\right)}$$

$$\left[\vdots \quad \varepsilon_L = \frac{1}{E}\left(\sigma_L - \nu\sigma_H\right)\right]$$

here, since σ_H & σ_L are principal stresses on the surface of the cylinder

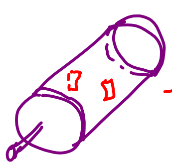
• Plug in the equations for σ_H & σ_L above:

$$\varepsilon_H = \frac{1}{E}\frac{P_g D}{t}\left(\frac{1}{2} - \frac{1}{4}\nu\right) = \boxed{\frac{P_g D}{2Et}\left(1 - \frac{\nu}{2}\right) = \varepsilon_H} \quad \star \quad (1)$$

• To use this device, we measure ε_H (hoop strain), then we use Eq. (1) to calculate P_g (gage pressure being measured)

COMMENTS:

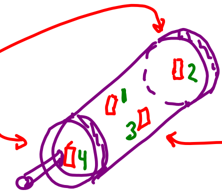
• The above eq's are for one strain gage. We may use two strain gages to double the sensitivity (half bridge instead of quarter bridge)



Hoop strain gage on either side

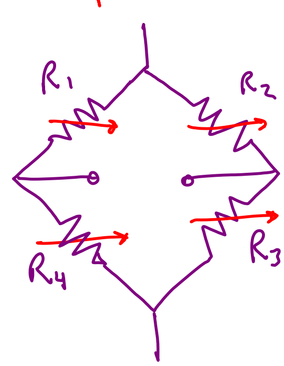
• It is also common to add two "dummy" temperature-compensating strain gages that are mounted on the thick end plate(s) (negligible strain)

temperature-compensating strain gages (2 & 4)



Hoop strain gages (1 & 3)

[now use a full bridge!]



• R_1 & R_3 = hoop strain gages

• R_2 & R_4 = temperature-compensating strain gages (dummy gages)

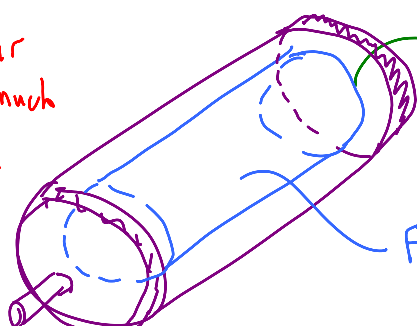
Finally, let's discuss the time response of a cylindrical pressure cell.

- Problem: If we suddenly increase P_{in}, it can take some time to fill up the cylindrical can with air. (must wait until the whole volume of air comes to equilibrium pressure inside the can)

  So → the time response is poor (too slow)

- Solution: Add a "filler" → a solid plug (another smaller cylinder that fits inside the pressure cylinder, but does not touch the walls of the cylinder (thus it does not affect the pressure or the strain gage reading).

This annular volume is much smaller than the volume of the whole cylinder.



The filler cylinder is typically mounted to the (thick) end wall so it stays in place

→ Therefore the time response is greatly improved!

- Another option is to flatten the cylinder, but then it must be calibrated since our hoop stress equations are no longer valid (not round any more)