Today, we will:

- Review the pdf module: **Volume and Mass Flow Rate Measurement** (last one!) ☺
- Do some example problems – pressure, velocity, and volume flow rate measurement
- Discuss some additional items about pressure and velocity measurement not in the notes

Example: Pressure measurement

Given: Jeri decides to use a soda can as a simple pressure cell. She mounts a strain gage on an empty soda can, much like was done in the strain gage lab. The soda can diameter is 66.0 mm and the wall thickness of the cylindrical portion is 0.105 mm. The aluminum has a Young's modulus of 70. GPa and a Poisson ratio of 0.35. The strain gage has a no-load resistance of 120.1 Ω and a strain gage factor of 2.10. Jeri constructs a quarter bridge circuit using resistor R_3 as the strain gage. She sets the bridge supply voltage to 5.00 V. She balances the bridge with zero gage pressure by setting the other three resistors in the Wheatstone bridge to 120.1 Ω. The opening to the can is then connected to a pressure line subjected to the pressure Jeri wishes to measure.

To do: When the measured output voltage from the Wheatstone bridge is 3.89 mV, calculate the applied gage pressure in units of kPa. Also calculate the hoop strain in units of microstrain.

Solution:

- Recall from our analysis, $\boxed{\frac{P_g D}{2Et}\left(1-\frac{\nu}{2}\right) = \varepsilon_H}$ = hoop strain

- We also know from our discussion of strain gages & Wheatstone bridges,

$$\boxed{\varepsilon_H = \frac{4}{S}\frac{V_o}{V_s}}$$

(quarter bridge)

Equate & solve for P_g:

$$\boxed{P_g = \frac{8Et}{SD\left(1-\frac{\nu}{2}\right)}\frac{V_o}{V_s}}$$

(Answer in variables)

- Numbers:

$$P_g = \frac{8\left(70\times10^{9}\ \text{N/m}^2\right)\left(0.105\times10^{-3}\ \text{m}\right)}{(2.10)(0.066\ \text{m})\left(1-\frac{0.35}{2}\right)}\frac{3.89\times10^{-3}\ \text{V}}{5.00\ \text{V}}\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) = 400.07\ \text{kPa}$$

So, $\boxed{P_g = 400\ \text{kPa}}$ (good to only 2 significant digits because of the given information)

- Hoop strain: $\varepsilon_H = \frac{4}{S}\frac{V_o}{V_s} = \frac{4\left(3.89\times10^{-3}\ \text{V}\right)}{(2.10)(5.00\ \text{V})} = 0.00148$, $\boxed{\varepsilon_H = 1480\ \mu\text{strain}}$

Example: Volume flow rate measurement

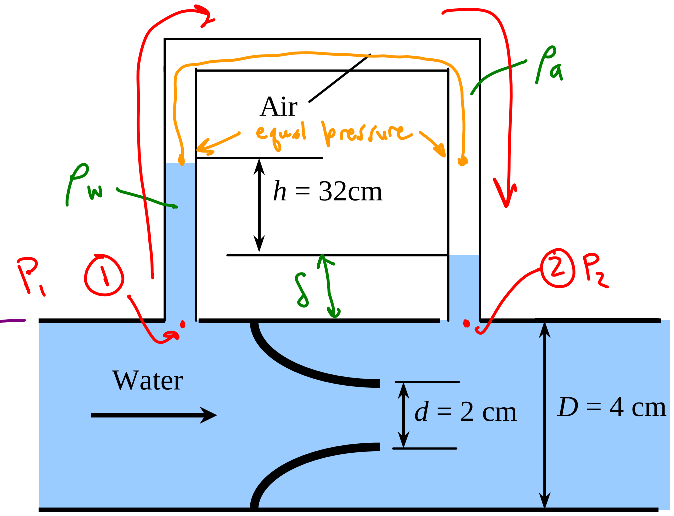
Given: Water at 20°C (ρ_w = 998 kg/m^3 and μ_w = 1.002 × 10^{-3} kg/m·s) flows through a 4.0-cm pipe equipped with a flow nozzle and an *inverted* air-water manometer. ρ_{air} = 1.204 kg/m^3.

To do: Calculate the volume flow rate.

Solution:

hydrostatics

everything from here up is static (not moving) ⇒ hydrostatics



• First, do a manometer problem:

$$\boxed{P_{below} = P_{above} + \rho g |\Delta z|}$$

Start at P_1 & work clockwise to P_2

$$P_1 - \rho_w g \delta - \cancel{\rho_w g} h + \rho_a g h + \cancel{\rho_w g} \delta = P_2$$

Solve for $\Delta P = P_1 - P_2$ → $\boxed{P_1 - P_2 = (\rho_w - \rho_a) g h}$

• Now apply the volume flow rate equation from the learning module

For an obstruction flow meter, $\dot{V} = A_o C_d \sqrt{\dfrac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$

Plug in $P_1 - P_2$ from manometer

plugging in $P_1 - P_2$, we get

$$\boxed{\dot{V} = A_o C_d \sqrt{\frac{2(\rho_w - \rho_a) g h}{\rho_w (1-\beta^4)}}} \quad (1)$$

☆ Note: Use the density of the flowing fluid (water) here, not the manometer fluid (air)

orifice area

• We can calculate: $\beta = \dfrac{d}{D} = \dfrac{2 \text{ cm}}{4 \text{ cm}} = 0.5$ $A_o = \dfrac{\pi d^2}{4} = \dfrac{\pi (0.0200 \text{ m})^2}{4} = 3.142 \times 10^{-4} \text{ m}^2$

• We also know that the mean flow speed through the pipe (not the orifice) is

$$\boxed{V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4}} \quad (2)$$

& we define (ρ_w) $\boxed{Re = \frac{\rho V D}{\mu} = \text{Reynolds \#}} \quad (3)$

Discharge coeff.

• From the notes, for a standard flow nozzle, $\boxed{C_d = 0.9975 - \dfrac{6.53 \beta^{0.5}}{Re^{0.5}}} \quad (4)$

- The problem is that we need Re to get C_D, we need C_D to get $\dot{\forall}$, but we need $\dot{\forall}$ to get V, & we need V to get Re!

Need to iterate

- Here is a good way to iterate: Start by guessing C_d

	C_d	$\dot{\forall}$ (m^3/s) (Eq 1)	V $(\frac{m}{s})$ (Eq 2)	Re (Eq 3)	C_d (Eq 4)
GUESS →	0.96	7.789×10^{-4}	0.62062	24726	0.9681
	0.9681	7.8650×10^{-4}	0.62587	24935	0.96826
	0.96826	7.86599×10^{-4}	0.62596	24938	0.96826

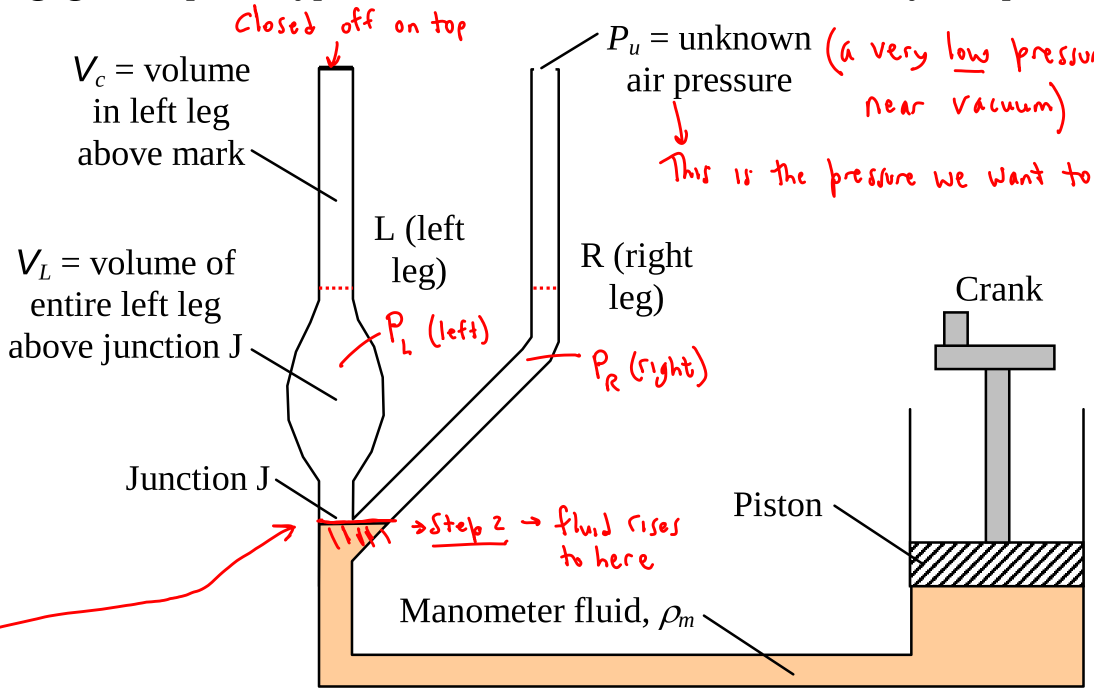
★ Converged! ☺

- This technique converges rapidly — in 3 iterations we nail C_d to 5 digits

Final answer: $\boxed{\dot{\forall} = 7.87 \times 10^{-4}\ m^3/s}$ (to 3 digits)

Additional Notes: The McLeod Gage

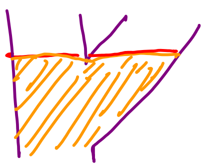
A **McLeod gage** is a special type of U-tube manometer to measure very low pressure:



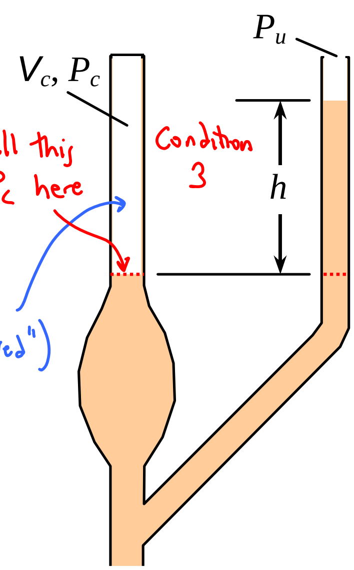
[Ignore p_a elevation effects since p_a is so small]

Procedure (do all this at constant temperature – *isothermally*):

1. Start with $P_L = P_R = P_u$ as sketched (the manometer fluid is just below junction J).
   Air pressure in left & right legs must all be the same, since all connected
2. Crank down the piston until the manometer fluid seals off the air at junction J in the left leg.



→ At this point, the air in the left leg is trapped – it cannot escape
[$P_L = P_R = P_u$ at this point]



3. Keep cranking until the manometer fluid reaches the red mark on the left leg – the manometer fluid in the right leg will rise higher, because the air in the left leg is *compressed* (higher air pressure). The manometer fluid acts like a piston. The air in the left leg is compressed to known volume V_c, & $P = P_c$ (c = "compressed")
4. Measure column height h.
   (as sketched)
5. Calculate the unknown pressure, P_u.

How? Use ideal gas law & equation of hydrostatics

(We ignore variations of air pressure with elevation)

pretty clever, eh?

- Ideal gas law $\boxed{P\forall = mRT}$

Specific gas constant for air = constant

air temperature is constant (we let everything cool down such that T = constant)

[As we crank from condition 2 to 3, the air is compressed & P & T rise. But we wait for T to go back to room temperature]

[mass of air in left leg = constant from condition 2 to condition 3 since all that air is trapped]

So, since everything on RHS = constant, $\boxed{P\forall = \text{constant}}$

i.e., $\boxed{P_c \forall_c = P_u \forall_L}$ $\Rightarrow$ $\boxed{P_c = \frac{P_u \forall_L}{\forall_c}}$ (1)

condition 3   condition 2

- Manometer (hydrostatic) equations @ condition 3:

$$\boxed{P_u + \rho_m g h = P_c} \quad (2)$$

- Combine (1) & (2) & do some algebra:

$$P_u + \rho_m g h = P_u \frac{\forall_L}{\forall_c}$$

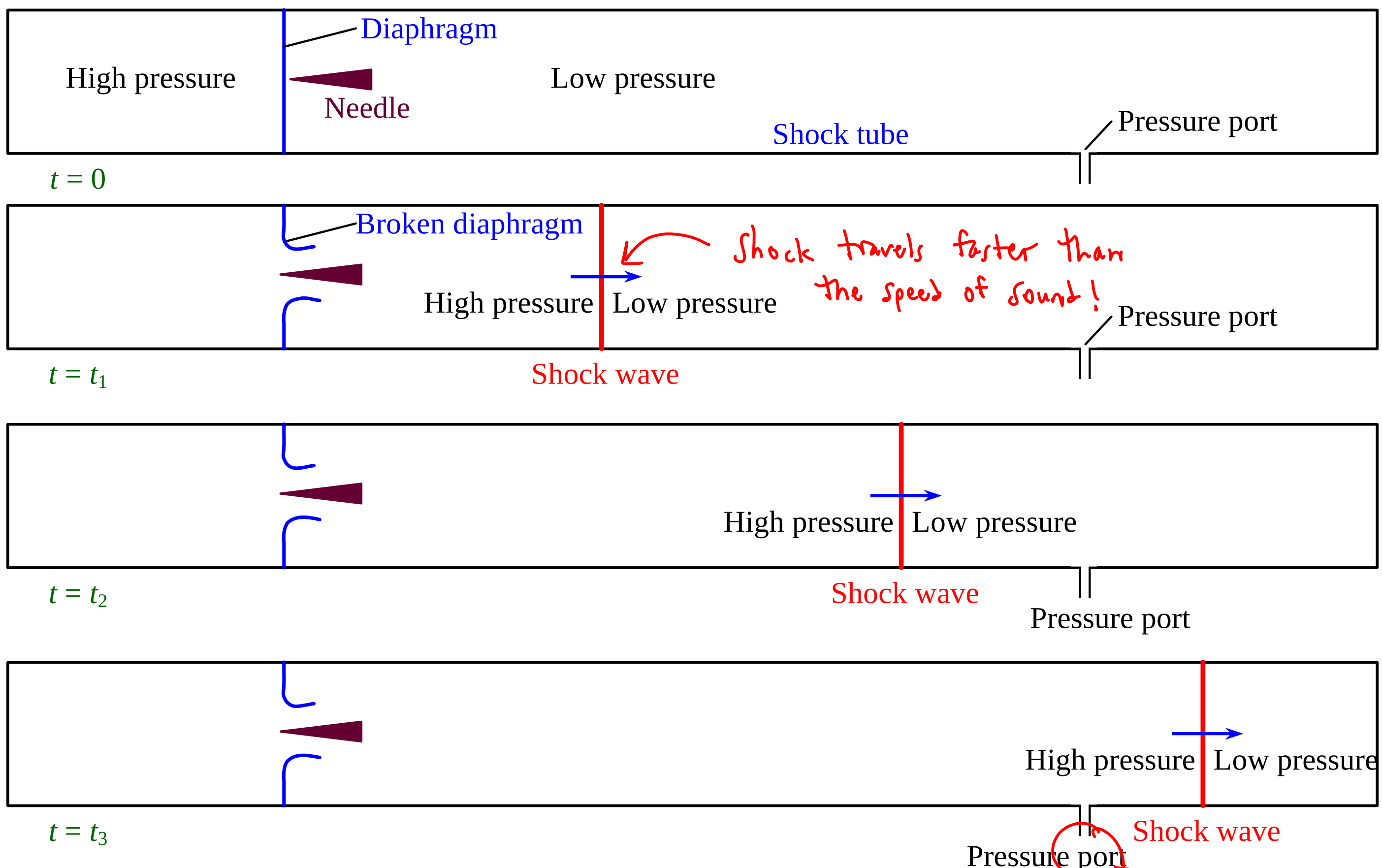
$$P_u\left(1 - \frac{\forall_L}{\forall_c}\right) = -\rho_m g h \rightarrow P_u(\forall_L - \forall_c) = \rho_m g h \forall_c$$

or
$$\boxed{P_u = \frac{\rho_m g h \forall_c}{\forall_L - \forall_c}} \quad ☆$$

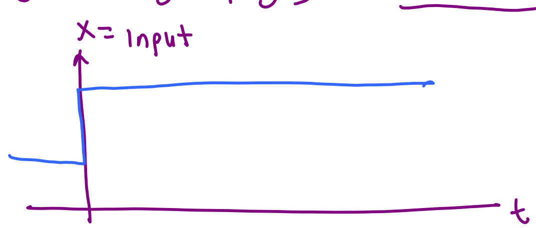
Final result → we calculate P_u, the unknown pressure, from the known values of g, $\forall_c$, $\forall_L$, & ρ_m plus the measured value of h.

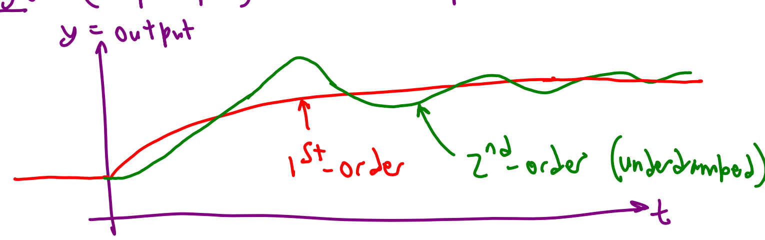
Additional Notes: Dynamic calibration of a pressure transducer

Shock tube: [Designed to send a shock wave (sudden increase in pressure) down a tube]



· Recall, we test the time response of a first- or second- order dynamic system by applying a sudden change (step input) of the input





· How to do this for pressure? (x = input = pressure) (y = output = transducer voltage) (or other type of output)

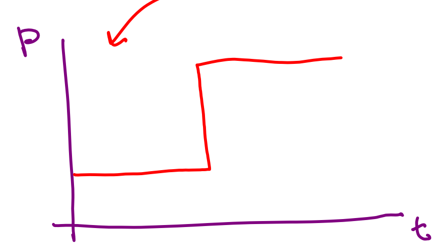
· One way is to use a shock tube ☆

· pump it up to high pressure

· break the diaphragm

· a shock wave travels down the tube as sketched

· When the shock passes the pressure port, P ↑ suddenly



[the air to the right of the shock does not feel anything until the shock passes by]