Equation Sheet for ME 405

Print out for homework, quizzes, exams, and future reference.

Author: John M. Cimbala, Penn State University. Latest revision, 10 August 2021

General and conversions: $g = 9.807\ \dfrac{\text{m}}{\text{s}^2}$, $\dfrac{0.3048\ \text{m}}{1\ \text{ft}}$, $\dfrac{1\ \text{mile}}{1.6093\ \text{m}}$, $\dfrac{1\ \text{kPa}\cdot\text{m}^2}{1\ \text{kN}}$, $\dfrac{1\ \text{kN}\cdot\text{m}}{1\ \text{kJ}}$, $\dfrac{1\ \text{kW}\cdot\text{s}}{1\ \text{kJ}}$, $\dfrac{1\ \text{Btu}}{1.055056\ \text{kJ}}$,

$\dfrac{1\ \text{kg}}{2.205\ \text{lbm}}$, $\dfrac{1\ \text{ton}}{2000\ \text{lbm}}$, $\dfrac{1\ \text{tonne (metric ton)}}{1000\ \text{kg}}$, $\dfrac{1\ \text{g}}{10^6\ \mu\text{g}}$, $\dfrac{1\ \text{m}}{10^6\ \mu\text{m}}$, $\dfrac{1\ \text{m}}{10^9\ \text{nm}}$, $V_{\text{sphere}} = \dfrac{4}{3}\pi\left(R_p\right)^3 = \dfrac{1}{6}\pi\left(D_p\right)^3$.

HERP risk: $\text{HERP risk} = \dfrac{N \cdot ESS \cdot HERP}{HSS}$, where N = number of servings, ESS = exposed serving size, $HERP$ = HERP risk from table (%), HSS = HERP serving size from table.

Molecular weights and mols: $m = nM$, $M_{\text{air}} = 28.97\ \text{g/mol}$, $M_{\text{water}} = 18.02\ \text{g/mol}$, Avagadro's number: 6.02214×10^{23}.

Air at SATP: $P_{\text{SATP}} = 101.325\ \text{kPa},\ T_{\text{SATP}} = 298.15\ \text{K}$, $\rho = 1.184\ \text{kg/m}^3$, $\mu = 1.849 \times 10^{-5}\ \text{kg/(m s)}$, $\lambda = 0.06704\ \mu\text{m}$.

Air at *P* & *T*: $\rho = \dfrac{P}{R_{\text{air}} T}$, $\mu \approx \mu_s \left(\dfrac{T}{T_{s,0}}\right)^{3/2} \dfrac{T_{s,0} + T_s}{T + T_s}$, $T_{s,0} = 298.15\ \text{K}$, $T_s = 110.4\ \text{K}$, $\mu_s = 1.849 \times 10^{-5}\ \dfrac{\text{kg}}{\text{m}\cdot\text{s}}$, $\lambda = \dfrac{\mu}{0.499}\sqrt{\dfrac{\pi}{8\rho P}}$

Ideal gas: $PV = nR_uT$, $R = R_u / M$, $PV = mRT$, $P = \rho RT$, $R_u = 8.314\ \dfrac{\text{kJ}}{\text{kmol}\cdot\text{K}}$, $R_{\text{air}} = 0.287\ \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}} = 287.0\ \dfrac{\text{J}}{\text{kg}\cdot\text{K}}$.

Volume and mass flow rate: $Q = \dot{V} = UA_c$, $\dot{m} = \rho Q = \rho\dot{V}$, $Q_{\text{standard}} = Q_{\text{actual}} \dfrac{P}{P_{\text{SATP}}} \dfrac{T_{\text{SATP}}}{T}$, $P_{\text{SATP}} = 101.325\ \text{kPa} = 760\ \text{mm Hg}$, $T_{\text{SATP}} = 25^\circ\text{C} = 298.15\ \text{K}$.

Ideal gas mixture: $m_t = \sum_{j=1}^{J} m_j$, $n_t = \sum_{j=1}^{J} n_j$, $P = \sum_{j=1}^{J} P_j$, $V = \sum_{j=1}^{J} V_j$, $f_j = \dfrac{m_j}{m_t} = y_j \dfrac{M_j}{M_t}$, $y_j = \dfrac{n_j}{n_t} = \dfrac{P_j}{P} = \dfrac{V_j}{V}$, $M_j = \dfrac{m_j}{n_j}$,

$PV = n_t R_u T$, $P_j V = n_j R_u T$, $PV_j = n_j R_u T$, $M_t = \sum_{j=1}^{J}\left(y_j M_j\right)$, $c_j = \dfrac{m_j}{V}$, $c_{\text{molar},j} = \dfrac{n_j}{V} = \dfrac{c_j}{M_j}$, $c_j = y_j \dfrac{M_j}{R_u}\dfrac{P}{T}$, $\dot{m}_j = c_j Q$.

Relative humidity & vapor pressure: $RH = \Phi = \dfrac{P_{H_2O}}{P_{v,H_2O}} \times 100\% = \dfrac{P_{H_2O}}{P_{\text{sat},H_2O}} \times 100\%$, $y_{H_2O} = \dfrac{P_{H_2O}}{P_{\text{atm}}}$, $P_v = P_{\text{sat}}$ for any VOC.

Water loss from breathing: $\dot{m}_{H_2O\ \text{loss}} = Q_t \dfrac{M_{H_2O}}{R_u}\left[\left(\dfrac{P_{v,H_2O} RH}{T}\right)_{\text{exhale}} - \left(\dfrac{P_{v,H_2O} RH}{T}\right)_{\text{inhale}}\right]$, $Q_{H_2O\ \text{loss}} = \dfrac{\dot{m}_{H_2O\ \text{loss}}}{\rho_{\text{liq. } H_2O}} = \dfrac{\dot{m}_{H_2O\ \text{loss}}}{1000\ \text{kg/m}^3}$.

Dose and dose rate: $D_{\min} = \text{dose rate} = (Qc)_{\min}\ [\text{mg per minute}]$, $D_t = \text{total dose} = \int_0^T (Qc)\,dt\ [\text{mg}]$

First-order ODE: $\dfrac{dy}{dt} = B - Ay$. For a step function change and constant A and B, $t_{1/2} = \dfrac{-\ln(1/2)}{A} = -\ln(1/2)\tau$, $y_{ss} = \dfrac{B}{A}$,

$\tau = 1/A$, and the solution at any time t is $y(t) = y_{ss} - \left[y_{ss} - y(0)\right]\exp(-At)$.

Mass body burden ODE and solution: $\dfrac{dm_{bb}}{dt} = FQc_{\text{inspired}} - k_r m_{bb}$, $m_{bb,ss} = \dfrac{FQc_{\text{inspired}}}{k_r}$, $\tau = \dfrac{1}{k_r}$, $t_{1/2} = \dfrac{-\ln(1/2)}{k_r}$, and the solution at any time t is $m_{bb}(t) = m_{bb,ss} - \left[m_{bb,ss} - m_{bb}(0)\right]\exp(-k_r t)$.

Exposure parameter for gas mixtures: For J toxins, each with its own PEL: $E_n = \sum_{j=1}^{J} \dfrac{y_j}{\text{PEL}_j}$, $E_n > 1$ = violation.

Hearing and noise: $L_P = 20\log_{10}\left(\dfrac{P}{P_0}\right)$, $P_0 = 2 \times 10^{-5}\ \dfrac{\text{N}}{\text{m}^2}$, $L_W = 10\log_{10}\left(\dfrac{W}{W_0}\right)$, $W_0 = 1 \times 10^{-12}\ \text{watt}$, $r_0 = 1\ \text{m}$,

$L_P(r) = L_W + 10\log_{10} Q - 11.0 - 20\log_{10}\left(\dfrac{r}{r_0}\right)$, Q = 1 (free space), 2 (hard floor), 4 (hard 2-D corner), 8 (hard 3-D corner).

For J sound sources: $L_P = 10\log_{10}\left[\sum_{j=1}^{J} 10^{\left(L_{P,j}/10\right)}\right]$. For J exposures: $E_n = \sum_{j=1}^{J} \dfrac{t_j}{t_{j,\text{permitted}}}$, $E_n > 1$ = violation.

Mifflin St. Jeor equation for BMR: $\dot{M}_b = \text{BMR} = \left(\frac{10.0m}{1\text{ kg}} + \frac{6.25h}{1\text{ cm}} - \frac{5.0a}{1\text{ yr}} + s\right)\frac{\text{kcal}}{\text{day}}$ = Engr. equation where m = mass [kg], h = height [cm], a = age [yr], s = +5 male, s = -161 female.

Heat stress: *Note*: These are **engineering equations** with units built in. All $\dot{Q}$ values have units of [kcal/min].

$\dot{Q}_{\text{evap,req}} = -\left[\dot{M} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{res}} + \dot{Q}_{\text{rad}}\right]$, $\dot{Q}_{\text{conv}} = KA_s\left(0.0325 + 0.1066U_a^{0.67}\right)\left(T_a - T_s\right)$, $\dot{Q}_{\text{rad}} = 0.0728A_sK\left(T_w - T_s\right)$,

$\dot{Q}_{\text{res}} = -\dot{M}\left[0.0014\left(307.15 - T_a\right) + 0.0173\left(5.87 - RH_aP_v(\text{at }T_a)\right)\right]$, $\dot{Q}_{\text{evap,max}} = 0.198KA_sU_a^{0.63}\left[RH_aP_v(\text{at }T_a) - P_v(\text{at }T_s)\right]$,

with T in K, RH as a ***number***, not a %, $T_w = \left[T_G^4 + \left(0.248\times10^9U_a^{0.5}\right)\left(T_G - T_a\right)\right]^{1/4}$, $HSI = \frac{\dot{Q}_{\text{evap,req}}}{\dot{Q}_{\text{evap,max}}}\times100\%$.

***Note* (confusing!)**: P_v in **kPa** for $\dot{Q}_{\text{res}}$, but P_v in **mm Hg** for $\dot{Q}_{\text{evap,max}}$. Conversion: $(760\text{ mm Hg}) / (101.325\text{ kPa})$.

Emission factors (EPA AP-42): $EF = \frac{\dot{m}_{\text{pollutant}}\ (\text{or } m_{\text{pollutant}})}{\text{some appropriate denominator}}$, $\dot{m}_{\text{d (discharged)}} = (1-\eta)\dot{m}_{\text{g (generated)}}$ or $m_d = (1-\eta)m_g$ where η = APCS removal efficiency.

Flux chamber: $\frac{dm_j}{dt} = V\frac{dc_j}{dt} = c_{j,a}Q_a + S_j - c_jQ_a$, $\dot{m}_{j,\text{ generated}} = S_j = \left(c_{j,\text{ss}} - c_{j,a}\right)Q_a$, $c_{j,\text{ss}} = c_{j,a} + \frac{S_j}{Q_a}$, $\tau = \frac{V}{Q_a}$,

$t_{1/2} = -\ln(1/2)\frac{V}{Q_a}$, and the solution at any time t is $c_j(t) = c_{j,ss} - \left[c_{j,ss} - c_j(0)\right]\exp\left(-\frac{Q_a}{V}t\right)$.

Tank filling: $m_{j,\text{ displaced}} = f\frac{M_jP_{v,j}}{R_uT}V_{\text{liquid in}}$, $\dot{m}_{j,\text{ displaced}} = f\frac{M_jP_{v,j}}{R_uT}Q_{\text{liquid in}}$, filling factor $f = \frac{P_j}{P_{v,j}}$, Loading factor $L_r = \frac{Q_{\text{liquid in}}}{V}$.

When a liquid puddle of species k sits at the bottom of a tank being filled with species j, emissions come from *both* j & k,

$\dot{m}_{\text{total}} = \dot{m}_{j,\text{ displaced}} + \dot{m}_{k,\text{ displaced}} = f_j\frac{M_jP_{v,j}}{R_uT}Q_{\text{liquid in}} + f_k\frac{M_kP_{v,k}}{R_uT}Q_{\text{liquid in}}$, where $f_j = \frac{P_j}{P_{v,j}}$ & $f_k = \frac{P_k}{P_{v,k}} = 1$ since k is saturated.

Gradient diffusion of *A*: $J_A = -b\frac{da}{dz}$, where $a = \frac{A}{V}$ and A = mass, energy, momentum, … For mass, $M_jJ_j = -D_{aj}\frac{dc_j}{dz}$.

Evaporation, pure liquid into stagnant air: $y_j + y_a = 1$. For z_1 = liquid surface and z_2 = top of container,

$N_j = k_G\left(P_{j,1} - P_{j,2}\right)$, $k_G = \frac{D_{aj}P}{R_uT\left(z_2 - z_1\right)P_{am}} = \frac{D_{aj}}{R_uT\left(z_2 - z_1\right)y_{am}}$, $y_{am} = \frac{P_{am}}{P}$, $y_{am} = \frac{y_{a,2} - y_{a,1}}{\ln\left(y_{a,2}/y_{a,1}\right)}$, $P_{am} = \frac{P_{a,2} - P_{a,1}}{\ln\left(P_{a,2}/P_{a,1}\right)}$,

and since $\dot{m}_j = \dot{m}_{j,\text{evap}} = N_jM_jA$, $\dot{m}_j = \dot{m}_{j,\text{evap}} = \frac{D_{aj}P\left(P_{j,1} - P_{j,2}\right)M_jA}{R_uT\left(z_2 - z_1\right)P_{am}}$ where A is the area of the evaporating surface.

Evaporation, pure liquid into moving air: $y_j + y_a = 1$. For z_1 = liquid surface and $z_2 = z_\infty$ = freestream (above BL),

$N_j = k_G\left(P_{j,1} - P_{j,\infty}\right)$, where $k_G = \text{Nu}\frac{D_{aj}}{L}\left(\frac{\text{Sc}}{\text{Pr}}\right)^{0.33}\frac{P}{P_{am}}\frac{1}{R_uT}$, $\text{Re} = \frac{LU_\infty\rho}{\mu} = \frac{LU_\infty}{\nu}$, $\text{Sc} = \frac{\mu}{D_{aj}\rho} = \frac{\nu}{D_{aj}}$, $\text{Pr} = \frac{\mu c_p}{k} = \frac{\nu\rho c_p}{k}$,

$\text{Nu} = \frac{Lh}{k}$ = Nusselt number, $h = \frac{q}{\Delta T}$ = *heat transfer coefficient*; Nu depends on geometry, Re, Pr, etc. (look up Nu eq. in tables), $P_{am} = \frac{P_{a,\infty} - P_{a,1}}{\ln\left(P_{a,\infty}/P_{a,1}\right)}$, and finally $\dot{m}_j = \dot{m}_{j,\text{evap}} = N_jM_jA$ where A is the area of the evaporating surface.

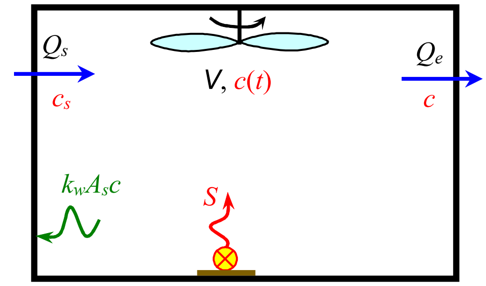
Evaporation, two film: Same equations as above, but at the liquid interface ($z_1 = z_i$), use $x_j = n_j / n_t$ for liquid mol fraction, and at the interface use: **Raoult's law**: $P_{j,i} = x_{j,i}P_{v,j}$ or **Henry's law**: $P_{j,i} = x_{j,i}H'$ (H' looked up in tables).

If liquid at bottom of a confined space for a long time, $P_{j\text{ in the air}} = P_{j,i}$ and use either Raoult or Henry for $P_{j,i}$.

Thermodynamics of evaporation: Category 1: $T_{\text{room}} < T_C$ and $P_{\text{room}} < P_C$, $y_{j,\text{ss}} = P_{v,j} / P$, $y_{j,\text{max}} = n_j / \left(n_j + n_a\right)$,

Category 2: $T_{\text{room}} > T_C$ and $P_{\text{room}} < P_C$, *all* the liquid evaporates and thus $y_{j,\text{ss}} = n_j / \left(n_j + n_a\right)$.

Room ventilation: *Note*: No j subscript, well-mixed conditions. E.g., for the simple room sketched here with a source and wall adsorption,



$V\dfrac{dc}{dt} = Q_s c_s + S - Q_e c - k_w A_s c \rightarrow \dfrac{dc}{dt} = B - Ac$, $A = \dfrac{Q_e + k_w A_s}{V}$, $B = \dfrac{S + Q_s c_s}{V}$,

$c_{ss} = \dfrac{B}{A}$, $\dfrac{c_{ss} - c(t)}{c_{ss} - c(0)} = \exp(-At)$. If also **desorption**: $\dot{m}_{\text{wall loss}} = k_w A_s (c - c_d)$.

Modify as necessary for other configurations. *Students must be able to generate equations for A and B for **any** given room ventilation configuration. Examples:*

Infiltration: $N_{\text{inf}} = 0.315 + 0.0273U + 0.0105|T_{\text{outside}} - T_{\text{inside}}|$ $[U \text{ in mph, } T \text{ in °F, } N \text{ in 1/h}]$

Recirculated and make-up air: $N = Q/V$, $Q_r = (1-f)Q$, $Q_m = fQ$, $Q = Q_r + Q_m$. **Air cleaners**: $c_{\text{out}} = (1-\eta)c_{\text{in}}$.

Effectiveness coefficient: $e = t_N / t_{\text{age,P}}$, $t_N = V/Q$, $t_{\text{age,P}}$ = time for a fluid particle to go from air supply to point P.

Room effectiveness coefficient: $e_{\text{room}} = \dfrac{t_N}{t_{\text{room,avg}}}$, $t_N = \dfrac{V}{Q}$, $t_{\text{room,avg}} = \dfrac{\int_0^\infty t\left(1 - \left[c_E / c_{E,ss}\right]\right)dt}{\int_0^\infty \left(1 - \left[c_E / c_{E,ss}\right]\right)dt}$ where E is at the room exhaust.

Clean rooms: Same equations as above, but specify maximum particle concentrations according to *Class* of clean room.

Make-up air operating costs: <u>heating</u> $DD_h = (1 \text{ day}) \sum_{365 \text{ days}} \left(T_{\text{bal}} - \overline{T_{\text{outdoor}}}\right)^+$, <u>cooling</u> $DD_c = (1 \text{ day}) \sum_{365 \text{ days}} \left(\overline{T_{\text{outdoor}}} - T_{\text{bal}}\right)^+$,

Engineering equation (be careful to use these units): $\$_{\text{heating}} = 0.154\dfrac{DD_h t_{\text{operating}} C_{fu} Q}{q_{fu}}$, where DD_h = [°F heating days],

$t_{\text{operating}}$ = [h/wk], C_{fu} = unit fuel cost [\$/unit], q_{fu} = unit fuel energy [BTU/unit], Q = make-up air [ACFM, ft^3/min].

Tunnel ventilation: *Note*: We consider only ***balanced***, steady-state, uniformly distributed transverse tunnel ventilation.

Source: $S = (EF)_c n_c v_c L$, where $(EF)_c$ = emission factor per car [mg/(car·km)], n_c = traffic density [cars/km], v_c = car speed [km/hr], and L = tunnel length [km] (sometimes [m] – must convert; be careful of units, as always!)

Concentration: We get a first-order ODE as a function of x (distance down the tunnel): $dc/dx = B - Ac$, with solution

$c_{\max} = \dfrac{B}{A}$, $\dfrac{c_{\max} - c(x)}{c_{\max} - c(0)} = \exp(-Ax)$, where $A = \dfrac{k + q_m}{U}$, $B = \dfrac{s + q_m c_m}{U}$, $q_m = \dfrac{Q_m}{A_c L}$, $q_e = \dfrac{Q_e}{A_c L} = q_m$, $s = \dfrac{S}{A_c L}$, $k = \dfrac{k_w A_s}{A_c L}$.

Hood design: **Particles** – match *capture velocity* to actual velocity. **Vapors** – use *control velocity* and tables as needed.

Canopy hoods with periodic surging: V_h = hood volume, $Q_s(t)$ and $Q_w(t)$ are source and hood volume flow rates, respectively, and T is the time period between surges. $V_s = \int_0^T Q_s dt$, $V_w = \int_0^T Q_w dt$ To avoid spillover, $V_s < V_h$ *and* $V_s < V_w$.

Gaseous air cleaners in series and parallel: *Note*: Some books use E instead of η for air cleaner removal efficiency.

Parallel: $\eta_{\text{overall}} = 1 - \sum_{j=1}^{m} f_j \left[1 - \eta_j\right]$ for m cleaners, where f_j = volume fraction through cleaner j, $f_j = \dfrac{Q_j}{Q_{\text{total}}}$.

Series: $\eta_{\text{overall}} = 1 - \prod_{j=1}^{m} \left[1 - \eta_j\right]$ for m cleaners, where the volume flow rate of air through each cleaner is the same.

Exhaust Duct System Design: For **major losses** (long, straight sections of duct), use the Darcy friction factor, f,

$h_{L,\text{major}} = f\dfrac{L}{D}\dfrac{V^2}{2g}$, $f = 8\left[(8/\text{Re})^{12} + (A+B)^{-1.5}\right]^{1/12}$, where $A = \left\{-2.457 \cdot \ln\left[\left(\dfrac{7}{\text{Re}}\right)^{0.9} + 0.27\dfrac{\varepsilon}{D}\right]\right\}^{16}$ and $B = \left(\dfrac{37530}{\text{Re}}\right)^{16}$.

Non-circular ducts: **hydraulic diameter**, $D_h = \dfrac{4A_c}{p}$, where A_c = cross-sectional area of the duct, p = wetted perimeter.

For **minor losses** (elbows, transition sections, …), $h_{L,\text{minor}} = \sum C_0 V^2 / 2g$ ($C_0 = K_L$). Use tables and charts provided.

Energy equation in pressure form: $P_1 + \alpha_1 (VP)_1 + \rho g z_1 + \delta P_{\text{fan,u}} = P_2 + \alpha_2 (VP)_2 + \rho g z_2 + \rho g h_L$, where $VP = \rho V^2/2$ and Δz is negligible in air. **Operating point** is volume flow rate $Q = VA_c$ where ***required*** **fan pressure =** ***available*** **fan pressure**.

Particles: $c_{\text{number},j} = \frac{c_j}{m_{p,\text{mean}}}$, $m_{p,\text{mean}} = \rho_p \frac{1}{6}\pi\left(D_{p,am}(\text{mass})\right)^3$, $\vec{F}_{\text{gravity}} = \left(\rho_p - \rho\right)\frac{\pi}{6}D_p{}^3\vec{g}$, $\vec{F}_{\text{drag}} = -\frac{\rho}{8}\frac{C_D}{C}\pi D_p{}^2\vec{v}_r\left|\vec{v}_r\right|$,

where v_r = **relative particle velocity**, $\vec{v}_r = \vec{v} - \vec{U}$, where $\vec{v}$ is the particle velocity and $\vec{U}$ is the air velocity.

Kn is the **Knudsen number**, λ is the **mean free path** of air molecules, and C is the **Cunningham correction factor**,

$\text{Kn} = \frac{\lambda}{D_p}$, $\lambda = \frac{\mu}{0.499}\sqrt{\frac{\pi}{8\rho P}}$, $C = 1 + \text{Kn}\left[2.514 + 0.80\exp\left(-\frac{0.55}{\text{Kn}}\right)\right]$, and $C_D = C_D(\text{Re})$, where $\text{Re} = \frac{\rho\left|\vec{v}_r\right|D_p}{\mu}$,

Stokes: $C_D = \frac{24}{\text{Re}}$ for Re < 0.1, **Morrison**: $C_D \approx \frac{24}{\text{Re}} + \frac{2.6\left(\frac{\text{Re}}{5.0}\right)}{1+\left(\frac{\text{Re}}{5.0}\right)^{1.52}} + \frac{0.411\left(\frac{\text{Re}}{2.63\times10^5}\right)^{-7.94}}{1+\left(\frac{\text{Re}}{2.63\times10^5}\right)^{-8.00}} + \frac{0.25\left(\frac{\text{Re}}{10^6}\right)}{1+\left(\frac{\text{Re}}{10^6}\right)}$ for $\text{Re} < 10^6$.

Terminal settling speed: $V_t = \sqrt{\frac{4}{3}\frac{\rho_p - \rho}{\rho}gD_p\frac{C}{C_D}}$, $\text{Re} = \frac{\rho V_t D_p}{\mu}$. **Stokes flow *approx.*** (Re < 0.1), $V_t = \frac{\rho_p - \rho}{18}D_p{}^2 g\frac{C}{\mu}$.

Grade efficiency for particulate APCS: V_t (or v_r) = fnc(D_p), so η = fnc(D_p) & **Grade efficiency**: $\eta\left(D_p\right) = 1 - \frac{c}{c(\text{in})}$.

Settling in box, room, container of height *H*: $t_c = H/V_t$ = critical time; **laminar and well-mixed are two extremes**:

Laminar: $\frac{c_{\text{avg}}}{c_0} = 1 - \frac{t}{t_c}$, $\eta\left(D_p\right) = \frac{t}{t_c}$ if $t \le t_c$; $\frac{c_{\text{avg}}}{c_0} = 0$, $\eta\left(D_p\right) = 1$ if $t > t_c$ Well-mixed: $\frac{c}{c_0} = \exp\left(-\frac{t}{t_c}\right)$, $\eta\left(D_p\right) = 1 - \exp\left(-\frac{t}{t_c}\right)$

Settling in duct: $L_c = \frac{HU}{V_t}$, Laminar: $\eta\left(D_p\right) = \frac{L}{L_c}$ if $L \le L_c$; $\eta\left(D_p\right) = 1$ if $L > L_c$, Well-mixed: $\eta\left(D_p\right) = 1 - \exp\left(-\frac{L}{L_c}\right)$.

Non-spherical: Aero: $V_t = \sqrt{\frac{4}{3}\frac{\rho_0 - \rho}{\rho}gD_{ae}\frac{C}{C_D}}$, $\rho_0 = 1000\ \frac{\text{kg}}{\text{m}^3}$, Spherical: $V_t = \sqrt{\frac{4}{3}\frac{\rho_p - \rho}{\rho}gD_{se}\frac{C}{C_D}}$, Volume: $V_p = \frac{\pi D_{ve}{}^3}{6}$.

Inertial separation devices:

Terminal radial speed, inertial separation: $v_r = \sqrt{\frac{4}{3}\frac{\rho_p - \rho}{\rho}\frac{U_\theta{}^2}{r_m}D_p\frac{C}{C_D}}$, $\text{Re} = \frac{\rho v_r D_p}{\mu}$, where $\frac{U_\theta{}^2}{r_m}$ replaces g in the

equations, r_m = mean radius, $x = r_m\theta$, $L_c = \frac{WU_\theta}{v_r}$, $\theta_c = \frac{L_c}{r_m}$. For **Stokes flow *approx.*** (Re < 0.1), $v_r = \frac{\rho_p - \rho}{18}D_p{}^2\frac{U_\theta{}^2}{r_m}\frac{C}{\mu}$.

Laminar settling: $\eta\left(D_p\right) = \frac{L}{L_c}$ if $L < L_c$; $\eta\left(D_p\right) = 1$ if $L > L_c$. Well-mixed settling: $\eta\left(D_p\right) = 1 - \exp\left(-\frac{L}{L_c}\right)$.

Standard Lapple cyclone: $\eta\left(D_p\right) = \frac{1}{1+\left(D_{p,\text{cut}}/D_p\right)^2}$, where $D_{p,\text{cut}} = \sqrt{\frac{3\mu D_2{}^3}{128\pi Q\left(\rho_p - \rho\right)}}$, D_2 = overall cyclone diameter.

Pressure drop and required power: $\Delta P = 40.96\rho\left(\frac{Q}{WH}\right)^2 = 2621.44\rho\frac{Q^2}{D_2{}^4}$, $\dot{W}_{\text{blower}} = \frac{Q\Delta P}{\eta_{\text{blower}}}$, where $W = \frac{D_2}{4}$ & $H = \frac{D_2}{2}$.

Particle air cleaners in series and parallel: *Note*: Same as for gaseous contaminants except now a ***grade*** efficiency.

Parallel: $\eta\left(D_p\right)_{\text{overall}} = 1 - \sum_{j=1}^{m} f_j\left[1 - \eta\left(D_p\right)_j\right]$, where f_j = volume fraction through cleaner j, $f_j = \frac{Q_j}{Q_{\text{total}}}$.

Series: $\eta\left(D_p\right)_{\text{overall}} = 1 - \prod_{j=1}^{m}\left[1 - \eta\left(D_p\right)_j\right]$, where the volume flow rate of air through each cleaner is the same.

Air filters: (ε = porosity, U_0 = air speed upstream of filter, L = filter thickness, $\eta_f(D_p)$ = single-fiber collection efficiency)

$Stk = \frac{\left(\rho_p - \rho\right)D_p{}^2\left(U_0/\varepsilon\right)}{18\mu D_f}$, $\eta_f\left(D_p\right) = \left(\frac{Stk}{Stk + 0.425}\right)^2$, $L_c = \frac{\pi}{4}\frac{\varepsilon}{1-\varepsilon}\frac{D_f}{\eta_f\left(D_p\right)}$, $\eta\left(D_p\right) = 1 - \exp\left(-\frac{L}{L_c}\right)$.