Today, we will:

- Finish discussing the filling of tanks with VOCs
- Discuss gradient diffusion and the Reynolds analogy
- Do Candy Questions for Candy Friday

**Tank filling (continued):**

From the previous lecture, we derived equations for the emission (mass) and the emission rate (mass/time) for filling a tank with a liquid volatile organic compound (VOC):

\[
\dot{m}_j, \text{displaced} = f \frac{M_j P_{v,j}}{R_u T} \dot{V}_{\text{liquid in}} \quad \text{and} \quad \dot{m}_j, \text{displaced} = f \frac{M_j P_{v,j}}{R_u T} Q_{\text{liquid in}}
\]

where \( f \) is called the filling factor, 

\[
f = \frac{P_j}{P_{v,j}}
\]

is kind of like relative humidity, but with some species other than water.

If any amount of VOC has been sitting at the bottom of the tank for a long time, the partial pressure will equal the vapor pressure (the tank is saturated with the VOC vapors), and for this case \( f = 1 \).

But what is \( f \) for other cases?

E.g., filling an empty tank (no liquid initially):

\[
P_{v,j} = 0 \quad \text{at start of filling}
\]
Figure 4.3 Methods to fill vessels with liquids; (a) splash filling, (b) submerged filling, and (c) bottom filling (redrawn from AWMA Handbook on Air Pollution Control, 2000).

* THIS REDUCTION OF $f$ IS ONLY TRUE FOR EMPTY TANKS
Summary: When a liquid puddle of species $k$ sits (for a long time) at the bottom of a tank being filled with species $j$, there are emissions from both $j$ and $k$,

$$
\dot{m}_k = f_k \frac{P_{v, k} M_k}{R_a T} Q_{\text{liquid in}}
$$

$$
\dot{m}_j = f_j \frac{P_{v, j} M_k}{R_a T} Q_{\text{liquid in}}
$$

where $f_j = \frac{P_j}{P_{v, j}}$ and $f_k = \frac{P_k}{P_{v, k}} = 1$ since $k$ is saturated.
Evaporation & Diffusion

Diffusion

Disperse into air

$p_j \uparrow$

$p_i = p_{ji}$

Liquid evaporation

$\nu$

Gradient Diffusion (1-D analysis)

Let $a = \text{some concentration of a property}$

$a = \frac{A}{A}$

Let $a = a(t)$ (one-D)

Gradient = slope $\frac{da}{dt} < 0$

A diffusion from high conc. to low conc. region
\[ J_A = \text{net amt. of property A transported (diffused) per unit time per unit area in the z direction} \]

\[ \{ \overline{a} \} = \left\{ \frac{A}{L^3} \right\} \]

Let \( b \) = a diffusion coefficient -> determining how rapidly A diffuses.

**ONE-D DIFFUSION EQ FOR ANY PROPERTY A**

\[ J_A = -b \frac{\partial a}{\partial t} \]

\[ \{ a \} = \left\{ \frac{A}{L^3} \right\} \]

\[ \{ b \} = \left\{ \frac{L^2}{t} \right\} \text{ for any property A} \]
### Examples of the One-Dimensional Diffusion Equation

<table>
<thead>
<tr>
<th>Property with a Gradient</th>
<th>Amount Diffused per Unit Area per Unit Time</th>
<th>Diffusion Coefficient</th>
<th>Diffusion Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Energy</strong> $A$</td>
<td>$J_A = q = \text{rate of heat (energy)}$</td>
<td>$\kappa = \text{thermal diffusivity}$</td>
<td>$J_A = -b \frac{da}{dz}$</td>
</tr>
<tr>
<td>$a = \rho C_p T$</td>
<td>$q = \text{rate of heat (energy)}$ transfer per unit area</td>
<td>$\kappa = \frac{k}{\rho C_p}$</td>
<td>$q = -\kappa \frac{d}{dz} \left( \rho C_p T \right)$</td>
</tr>
<tr>
<td><strong>Temperature</strong> $A$</td>
<td>${ q } = \left{ \frac{\text{energy}}{\text{area} \cdot \text{time}} \right}$</td>
<td>${ \kappa } = \left{ \frac{\text{length}^2}{\text{time}} \right}$</td>
<td></td>
</tr>
<tr>
<td><strong>Momentum</strong> $A$</td>
<td>$J_A = -\tau = \text{shear stress}$</td>
<td>$\nu = \text{kinematic viscosity}$</td>
<td>$-\tau = -\nu \frac{d}{dz} \left( \rho U \right)$</td>
</tr>
<tr>
<td>$a = \rho U$</td>
<td>$-\tau = \frac{d}{dt} \left( \text{momentum} \right)$</td>
<td>${ \nu } = \left{ \frac{\text{length}^2}{\text{time}} \right}$</td>
<td>or $-\tau = -\mu \frac{dU}{dz}$</td>
</tr>
<tr>
<td><strong>Species</strong> $j$</td>
<td>$J_j = \text{rate of transfer of mols of species } j \text{ per unit area}$</td>
<td>$D_{aj} = \text{binary diffusion coefficient between air and species } j$</td>
<td>$J_j = -D_{aj} \frac{d}{dz} \left( c_{\text{molar}, j} \right)$</td>
</tr>
<tr>
<td>$a = c_{\text{molar}, j}$</td>
<td>${ J_j } = \left{ \frac{\text{mols}}{\text{area} \cdot \text{time}} \right}$</td>
<td>${ D_{aj} } = \left{ \frac{\text{length}^2}{\text{time}} \right}$</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** The negative sign on $\tau$ is due to the fluid mechanics sign convention.
Non-dimensional ratios of diffusion coefficients

\[ \theta = \frac{L^2}{k} \quad \text{for any diffusion} \]

\[ S_c = \text{Schmidt} \# = \frac{\nu}{D_{aj}} = \frac{\text{momentum diff}}{\text{speew diff}} \]

\[ Pr = \text{Prandtl} \# = \frac{\nu}{\kappa} = \frac{\text{mom. diff}}{\text{heat energy diff}} \]

\[ Le = \text{Lewis} \# = \frac{\kappa}{D_{aj}} = \frac{\text{heat energy diff}}{\text{mass speew diff}} \]
Reynolds Analogy – Energy, momentum, and mass, all diffuse in similar fashion. Compare:

Suddenly heated wall \([T = T_0 = 0^\circ \text{C} \text{ everywhere, then suddenly } T = T_i \text{ at the wall.}]\)

Suddenly moving wall \([U = U_0 = 0 \text{ m/s everywhere, then suddenly } U = U_i \text{ at the wall.}]\)

Sudden removal of a membrane \([c_{\text{molar}} = c_{\text{molar},0} = 0 \text{ mol/m}^3 \text{ everywhere, then suddenly } c_{\text{molar}} = c_{\text{molar},i} \text{ at the location of the membrane, and the membrane disappears suddenly.}]\)
Reynolds Analogy for Turbulent Flow

Laminar  \[ \text{Diffusion is slow} \]

Turbulent

Define turbulent Pr and Schmidt-Levis

\[ \frac{\nu_T}{K_T} \gg K \]

\[ \nu_T \gg \nu \]

\[ D_{aj,T} \gg D_{aj} \]

Turbulent Prandtl

\[ Pr_T = \frac{\nu_T}{K_T} \]

Turb. Schmidt

\[ Sc_T = \frac{\nu_T}{D_{aj,T}} \]

Turb. Lewis

\[ Le_T = \frac{K_T}{D_{aj,T}} \]

\[ Pr_T \approx Sc_T \approx Le_T \approx 1 \]

Exam 1 Material Ends Here