Today, we will:

- Discuss **Dilution Ventilation with Unsteady Properties**
- Do some example problems – unsteady dilution ventilation
- Discuss **Removal by Solid Surfaces (Adsorption)**
- Do **Candy Questions for Candy Friday**

\[ \text{Well-mixed model, but with unsteady parameters} \]

\[ \frac{dC(t)}{dt} = Q(t) C_a(t) + S(t) - Q(t) C(t) \]  

When \( Q, C_a, S, i, \) and \( t \) are constant – analytical soln.

- More general case – \( Q = Q(t) \) and/or \( C_a(t) \) and/or \( S(t) \)

\[ \text{1st order ODE with non-constant coefficient} \]

\[ \text{Do not have an analytical soln} \]

How to solve?
TWO TYPES OF UNSTEADY PROBLEMS

(1) Piece-wise function of time
   
   \[ m_{bb} \] problem
   
   \[ m_{bc} \]

   \[ m_{bc} \]
   
   Piece 1 | Piece 2

   Analytical soln in each segment of time

(2) Arbitrary flow of time — need numerical soln
   
   I use Runge-Kutta
   
   \[ \text{See R-K description on website} \]

Example of piece-wise problem first
Example (Example 5.4 in text – Conference Room with 100% Make-up Air)

**Given:** A conference room of volume 33.3 m$^3$ contains 6 people who smoke at irregular times as follows, over a 90 minute period:

<table>
<thead>
<tr>
<th>$t$ (minutes)</th>
<th>number of smokers ($n$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; t &lt; 10$</td>
<td>2</td>
</tr>
<tr>
<td>$10 &lt; t &lt; 20$</td>
<td>4</td>
</tr>
<tr>
<td>$20 &lt; t &lt; 30$</td>
<td>0</td>
</tr>
<tr>
<td>$30 &lt; t &lt; 40$</td>
<td>6</td>
</tr>
<tr>
<td>$40 &lt; t &lt; 50$</td>
<td>4</td>
</tr>
<tr>
<td>$50 &lt; t &lt; 90$</td>
<td>0</td>
</tr>
</tbody>
</table>

Assume that the smoke particles are emitted at a rate of 1100 µg/min while each cigarette is smoked (Repace and Lowery, 1980). A ventilation system removes air from the room at a rate of 6.3 m$^3$/min and replaces it with ambient air at the same rate. Such a condition corresponds to 100% make-up air and 0% recirculated air. The smoke concentration in the ambient air and the initial smoke concentration in the room are both 20. µg /m$^3$.

**To do:** Assuming well-mixed conditions, compute and plot smoke concentration as a function of time for the 90-minute period. Also compute the maximum concentration and the average concentration. Compare the average concentration to the ACGIH standard of 3 mg/m$^3$ (3000 µg /m$^3$) for “not otherwise classified” (NOC) respirable particles (particles with diameter $D_p$ less than about 2 or 3 µm).

**Solution:** I used Excel, splitting the time into six different time periods as per the given table (number of smokers). The Excel file for this problem is also available on the course website. The maximum smoke concentration is around 920 µg /m$^3$, and the average smoke concentration over the period is around 330 µg /m$^3$, both of which are within the ACGIH standard.
"Tee" in HVAC ductwork

\[ Q_2 = 1 \text{ m}^3/\text{min} \]
\[ C_2 = 15 \text{ m}^3/\text{min} \]
\[ Q_1 = 2 \text{ m}^3/\text{min} \]
\[ C_1 = 10 \text{ m}^3/\text{min} \]

\[ Q_3 = Q_1 + Q_2 \]
\[ C_3 = 3 \text{ m}^3/\text{min} \]

Small flow rate of species j (dropped j)

\[ \dot{m}_3 = \dot{m}_1 + \dot{m}_2 \] of species j

\[ C_3 Q_3 = C_1 Q_1 + C_2 Q_2 \]
\[ 35 \text{ m}^3/\text{min} \]

\[ \dot{m}_3 = \frac{C_3 Q_3}{Q_3} = 35 \text{ m}^3/\text{min} = 11.7 \text{ m}^3/\text{min} \]

General case example

Recall our hotel room fire example

Let \( S \) be unsteady: 
\[ S(t) = 50,000 \left( 1 + \sin \frac{2\pi t}{2} \right) \]

See Excel on website
First order ODE → \[ B = \frac{\omega^2 c_0 + S}{A} \]
\[ A = \frac{Q}{x} \]

This S is now unstable.
Example (Example 5.5 in text – the Clever Outdoorsman)

**Given:** A man sleeps overnight in a cabin with volume $V = 32.65 \, \text{m}^3$. The rate of “fresh” air entering the cabin is 0.30 air changes per hour. The concentration of CO in the outside “fresh” air is $c_a = 10 \, \text{PPM} (11.4 \, \text{mg/m}^3)$. A kerosene space heater emits CO according to $S(t)=1500[1+\sin(0.80\pi t)] \, \text{mg CO per hour}$, where $t$ is in hours.

**To do:** Calculate the concentration of CO in the cabin as a function of time.

**Solution:**

$$Q = \frac{Q}{A} = \frac{N \cdot A}{4} = (0.3 \, \frac{\text{air changes}}{\text{hr}}) \left( \frac{32.65 \, \text{m}^3}{4} \right) = 9.79 \, \text{m}^3/\text{hr}$$

When $t=0$ (no heater on yet): $C(0) = c_a = 11.4 \, \text{mg/m}^3$

See Excel or Matlab file on website:

$Y_{CO} = 130 \, \text{PPM} \quad \text{or} \quad 150 \, \text{mg/m}^3 \quad \text{average, but oscillate}$

He will wake up with a headache but be alive

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**Figure 2.41** Response to carbon monoxide as a function of concentration (PPM) and exposure time (hr). The OSHA 8-hr PEL is 35 PPM and the EPA Primary Air Quality Standard is 9 PPM (redrawn from Seinfeld, 1986).
Removal by Solid Surfaces (Sec 5.5)

- Walls, furniture, carpet, etc. adsorb contaminant
- Terminology — "plate out" or "wall loss" — a "sink" of the contaminant

Equations:

\[ \frac{dM_{\text{wall loss}}}{dt} = k_w A_s C \]

\[ \text{mass conc. of contaminant} \left( \text{g m}^{-3} \right) \]

\[ \text{mass of contaminant} \left( \text{g m}^{-3} \right) \]

\[ \text{wall loss coefficient} \left( \text{m s}^{-1} \right) \]

\[ \text{surface area of adsorbing material} \left( \text{m}^2 \right) \]
\[ \langle k_w \rangle = \langle \frac{L}{t} \rangle \] (like a velocity)

\[ k_w = \text{Wall loss coefficient} \]

or deposition velocity

or adsorption coefficient

See text for empirical eqs. for \( k_w \)

- for vapor
- for particle

Room ventilation eq.:

\[ \frac{dC}{dt} = \frac{Qc_A}{V} + S - Qc - k_wA_xc \]

In standard form \( \frac{dC}{dt} = B - Ac \)

\[ A = \frac{Q + k_wA_x}{V} \]

\[ B = \frac{S + Qc_A}{V} \]

Everything is same as previously except for the addition of this will loss term (in \( A \))