Today, we will:

- Continue to discuss how to match required fan pressure to available fan pressure
- Do an example problem – hood/exhaust duct fan selection
- If time, begin to discuss **Particle Motion** [Chapter 8]

\[ \text{CAUTION: Fan manufacturers like to advertise their fans like:}
\]

- Maximum pressure \( \Delta p_{\text{max}} = \) 
- Minimum flow rate \( Q = \)

\[ \text{Problem – cannot achieve both max P and max Q}
\]

\[ \begin{align*}
\Delta p_{\text{fan}} &\quad \text{AVAILABLE} \\
\Delta p_{\text{max}} - &\quad \text{REQUIRED} \\
&\quad n_{\text{fan}} \quad n_{\text{max}} \\
&\quad \text{Actual operating condition} \\
&\quad \text{Actual } \Delta p < \Delta p_{\text{max}} \\
&\quad Q < Q_{\text{max}} \\
&\quad n_{\text{fan, typ}} < n_{\text{max}}
\end{align*} \]
Example

Given: Air is drawn into a 45° tapered hood, and then goes through a damper, several long sections of pipe, and three elbows, as sketched. The air is exhausted by a fan.

- duct length = 100 ft
- duct dia. = 6 in
- duct roughness = 0.006 in
- $C_{o, \text{hood}} = 0.15$
- $C_{o, \text{damper}} = 0.50$ when fully open
- elbows are 5-gore, 90° CD3-9 with $R/D = 1.5$
- $\rho_{\text{air}} = 1.2 \text{ kg/m}^3$
- $\nu_{\text{air}} = 1.5 \times 10^{-5} \text{ m}^2/\text{s}$
- $Q = 1200 \text{ CFM}$

(a) To do: Calculate the required pressure rise across the fan

(b) To do: Select an appropriate fan from a family of available fans (shown below).

Solution:

Convert all to SI

$D = 6'' = 0.1524 \text{ m} \approx 150 \text{ mm}$

$L = 100' = 30.48 \text{ m}$
\[ \varepsilon = \frac{0.006''}{6''} = 0.001 \quad Q = 1200 \frac{\text{ft}^3}{\text{min}} = 0.56634 \frac{\text{m}^3}{\text{s}} \]

Look up minor loss coeff for elbow CD3.9
\[ @ D = 150 \text{ mm} \]
\[ C_{0, \text{ elbow}} = 0.28 \]

Apply energy eq. for our CV from 1 to 2
\[ [V_P = \frac{1}{2} PV^2] \]
\[ (\delta P_{\text{fan}})_{\text{req.}} = (P_2 - P_1) + \left[ \alpha_2 (V_P)^2 - \alpha_1 (V_P)^2 \right] + p g h_{L, \text{turb}} + \text{gravity} \]
\[ \text{dem} \quad V_1 = 0 \quad \text{(wir. cv)} \]
\[ \text{V}(\text{turb. flow}) \]
\[ (\delta P_{\text{fan}})_{\text{req.}} = \alpha_2 (V_P)^2 + p g h_{L, \text{turb}} \]
\[ \alpha_2 = 1.05 \]

Major loss \[ V = \frac{Q}{A} = \frac{4Q}{\pi D^2} = 31.046 \frac{\text{m}^3}{\text{s}} = V = V_2 \]
\[ Re = \frac{VD}{\nu} = 3.1544 \times 10^5 \]
\( \varepsilon_0 = 0.001 \)

Use Moody Chart or Chart II, eq \( \rightarrow f = 0.020695 \)

\[ \Sigma \Delta P_{e, \text{major}} = \left( f \frac{L}{D} \frac{V^2}{2} \right) \rho g = \frac{f L}{D} \rho \frac{V^2}{2} \]
\[ = \frac{f L}{D} (VP) \]

Where \( VP = \frac{1}{2} \rho V^2 = \frac{1}{2} (1.2 \text{ kg/m}^3)(31.046 \text{ m/s})^2 \)

\[ \Rightarrow VP = 578.3 \text{ Pa} \]

\[ \Sigma \Delta P_{e, \text{major}} = \frac{f L}{D} (VP) = 2394 \text{ Pa} \]

\[ \approx 4 \sqrt{VP} \]

\[ \begin{align*}
\text{Minor Loss} & \quad \Sigma \Delta P_{e, \text{minor}} = \Sigma C_0 (VP) \\
\text{hood} & \quad \text{use bigger } V \\
& \quad \text{same as our } V \\
\text{elbow} & \quad \text{and fitting} - \text{same } V
\end{align*} \]
\[ \sum C_0 = 0.15 + 0.50 + 3(0.28) = 1.49 \]

\[ \sum \Delta P_{L, min} = \sum C_0 \cdot VP = 862 \text{ Pa} \]

\[ 2 \times 1.5 \cdot VP \]

Energy eq.:

\[ (\delta P_{fan})_{eq} = \alpha_2(\text{VP}) + \sum \Delta P_{L, maj} + \sum \Delta P_{L, min} \]

\[ = 1.05(578.3 \text{ Pa}) + 2394 \text{ Pa} + 861.7 \text{ Pa} \]

\[ \delta P_{fan, eq} = 8363 \text{ Pa} = 3900 \text{ Pa} \]

\[ @ Q_{design} = 1200 \text{ CFM} \]
Now, match \((\delta P)_{req}\) to \((\delta P)_{avail}\)

3500 Pa 3 chain

Generate \((\delta P)_{req}\) as a fnc. of Q — repeat above @ var Q values

(b) To do: Select an appropriate fan from a family of available fans (shown below).

Solution:

\[
\begin{align*}
\delta P_{fan} (\text{Pa}) & \quad \text{Fan C} \\
& \quad \text{Fan B} \\
& \quad \text{Fan A}
\end{align*}
\]

\[
\begin{align*}
0 & \quad 2000 \\
4000 & \quad 6000
\end{align*}
\]

\[
\begin{align*}
0 & \quad 500 \\
1000 & \quad 1500
\end{align*}
\]

\[
\text{Actual oper. pt. of Fan C} \\
\text{Actual oper. pt. of Fan B}
\]

\[
\text{Levin Q} = 1200 \text{ CFM}
\]

\[
\begin{align*}
\text{Fan A} & \quad \text{too wimpy} \quad \text{too small} \\
\text{Fan C} & \quad \text{too hefty} \quad \text{too big} \\
\text{Fan B} & \quad \text{almost just right}
\end{align*}
\]

Choose Fan B
Fundamental difference between vapors and particles.

Vapors follow the same air streamlines.

Particles are heavier than air — have inertia.

We assume one-way coupling:
- Particle path is influenced by the air.
- Air flow is not affected by the particles.

Particle path (trajectory) depends on:
- Particle size
- Density + air properties ($\mu, \rho, ...$)
- Shape + air flow field (streamlines)