

Sphere Drag ☆

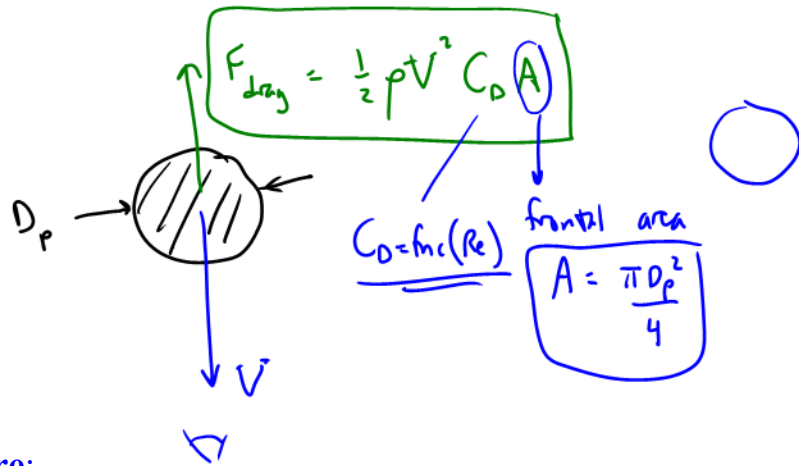
Today, we will:

- Discuss **Drag on Spheres** and various equations for sphere **Drag Coefficient**
- Do an example problem

Drag on Spheres:

$$\frac{\mu}{\rho} = \nu$$

$$Re = \frac{\rho V D_p}{\mu} = \frac{V D_p}{\nu}$$



Drag coefficient C_D on a sphere:

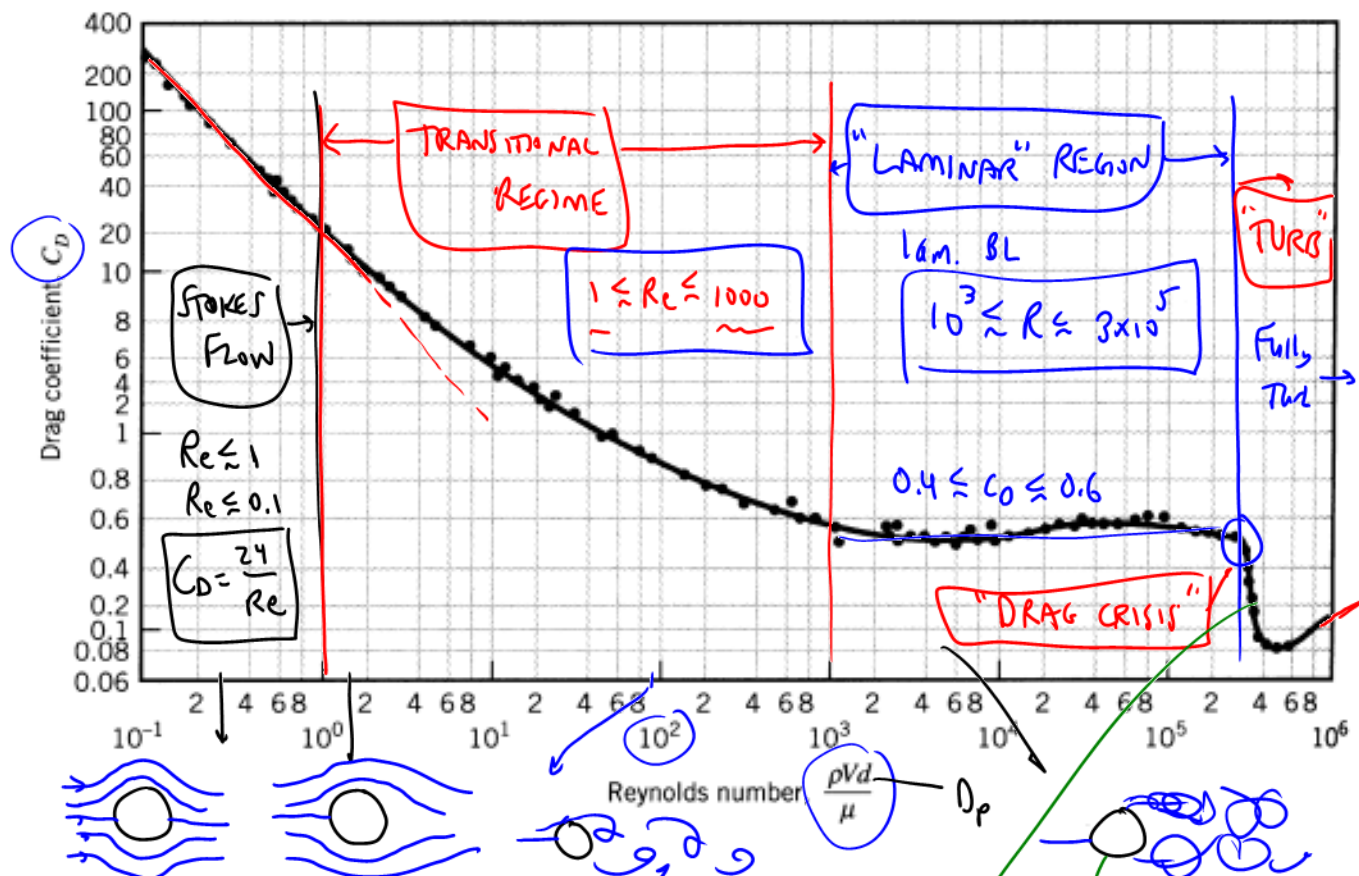
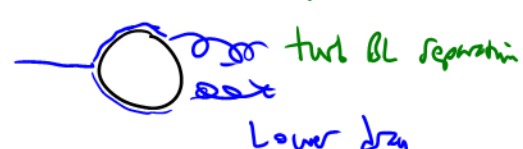


Fig. 2: The effect of Reynolds number on the drag coefficient C_D of a smooth sphere of diameter d .



Some empirical equations for sphere drag coefficient, $C_D = C_D(\text{Reynolds number})$:

Stokes flow Approx ONLY FOR VERY SMALL RE

• $C_D = \frac{24}{\text{Re}}$ for $\text{Re} < 0.1$ where for all equations, $\text{Re} = \frac{\rho V D_p}{\mu} = \frac{V D_p}{\nu}$

• $C_D = \frac{24}{\text{Re}}(1 + 0.0916\text{Re})$ for any $\text{Re} < 5$

• $C_D = \frac{24}{\text{Re}}(1 + 0.158\text{Re}^{2/3})$ for $5 < \text{Re} < 1000$

• $C_D = 0.4 + \frac{24}{\text{Re}} + \frac{6}{1 + \sqrt{\text{Re}}}$ for $1000 < \text{Re} < 2 \times 10^5$

• $C_D \approx 0.2$ for $\text{Re} > 2 \times 10^6$

CAN COMBINE
SEGMENTED EQ'S
Use IF statements

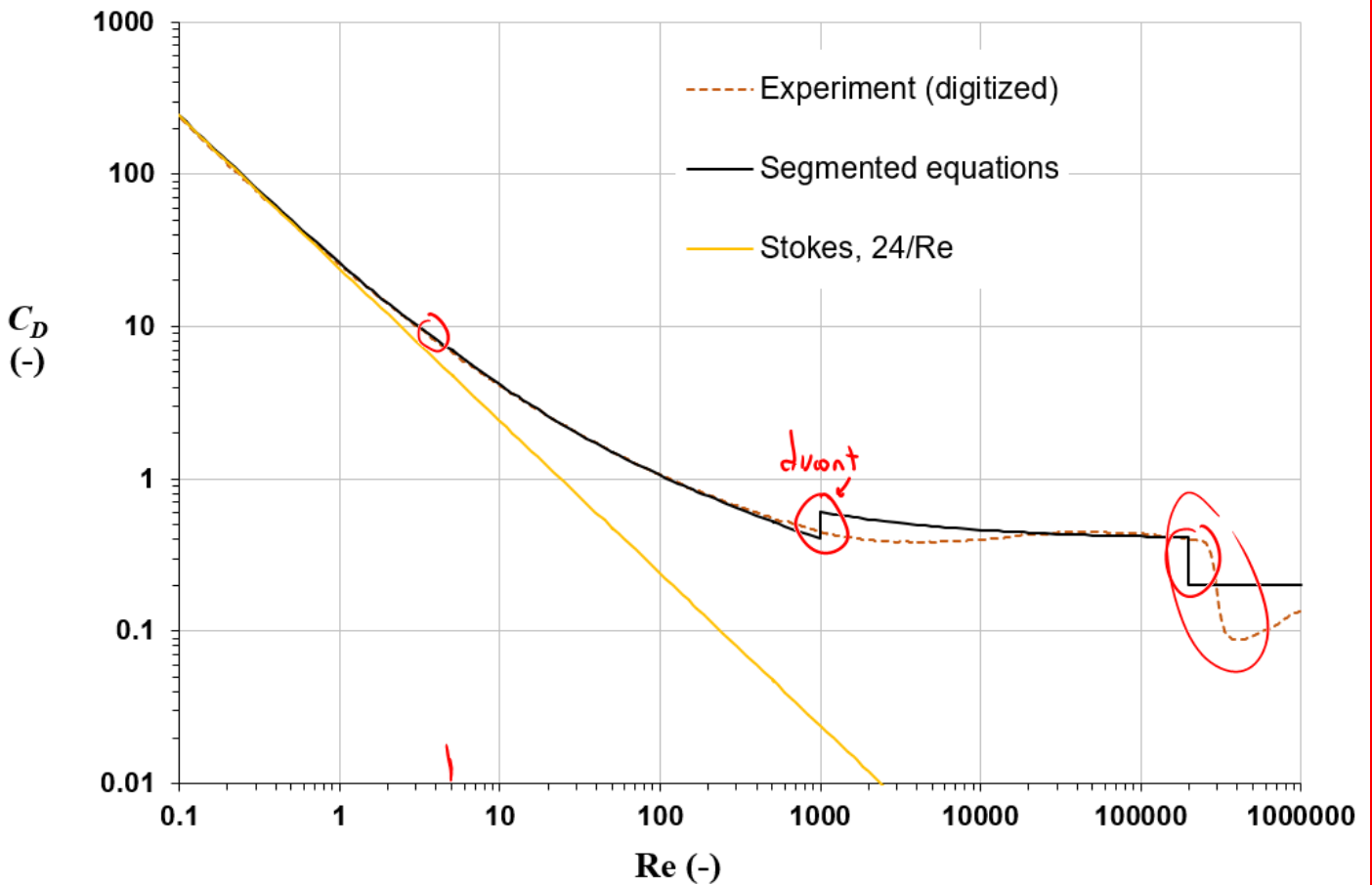
e.g., in Excel

Re	C_D
8.8	

= IF (B5 < 5, 24 * (1 + 0.0916 * B5) / B5, IF (B5 < 1000, 24 * (1 + 0.158 * B5^(2/3)) / B5, IF (B5 < 200000, 0.4 + 24 / B5 + 6 / (1 + SQRT(B5)), 0.2)))

Nested IF

Problem: If use these in an IF statement, there are discontinuities.



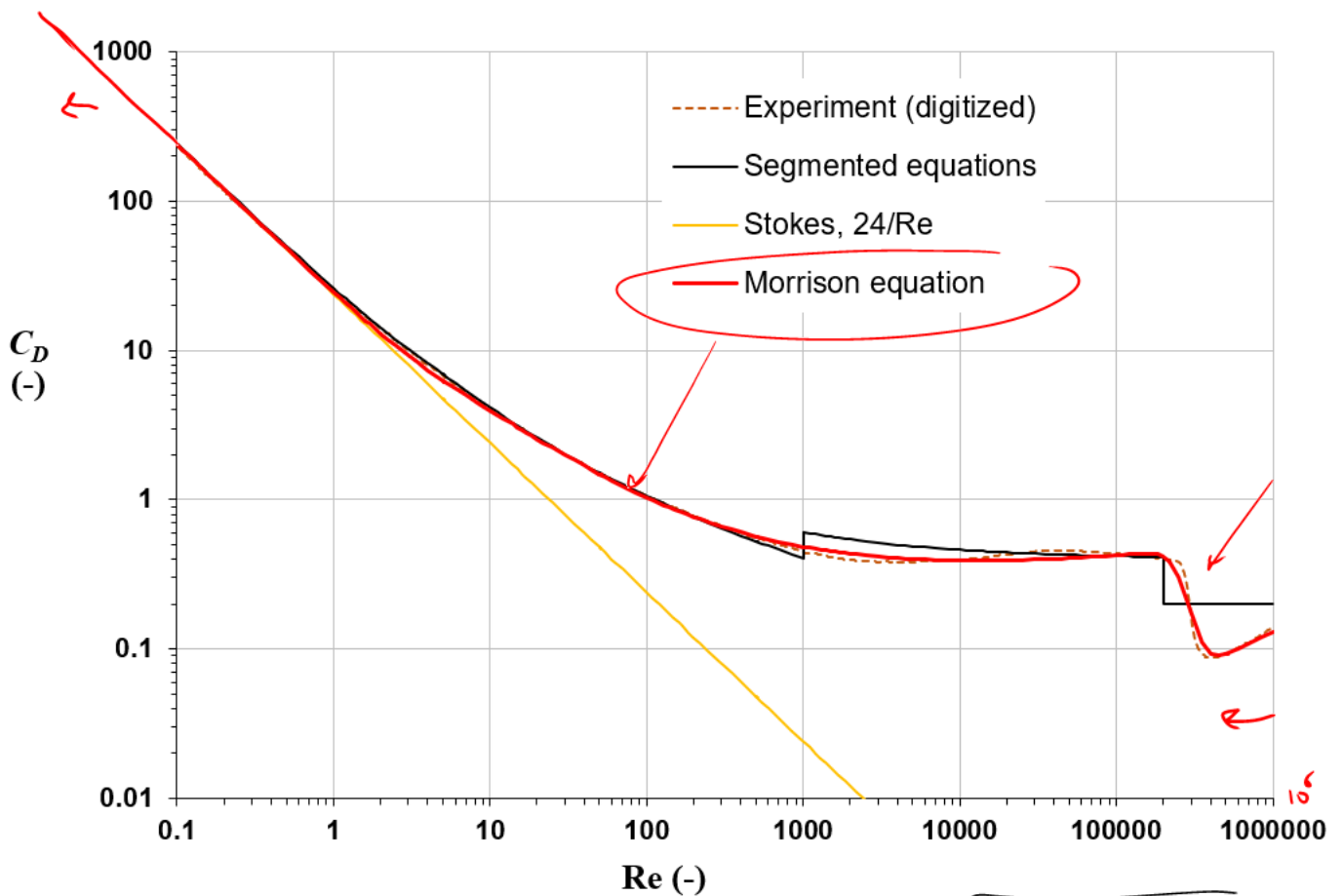
Fortunately, there is a 2016 paper by **Faith A. Morrison** where she created a curve fit equation that spans the entire range of Reynolds number up to 10^6 . Here is the equation:

$$C_D \approx \frac{24}{Re} + \frac{2.6 \left(\frac{Re}{5.0} \right)}{1 + \left(\frac{Re}{5.0} \right)^{1.52}} + \frac{0.411 \left(\frac{Re}{2.63 \times 10^5} \right)^{-7.94}}{1 + \left(\frac{Re}{2.63 \times 10^5} \right)^{-8.00}} + \frac{0.25 \left(\frac{Re}{10^6} \right)}{1 + \left(\frac{Re}{10^6} \right)} \quad \text{for } Re < 10^6$$

Explicit Accurate

WE WILL ALWAYS USE THIS MORRISON EQ FOR CD

Here is a plot of $C_D(Re)$ comparing Stokes, our segmented equations, and Morrison:



Compare Stokes : Morrison:

Re	CD (Stokes)
0.01	2400
0.1	240
1.0	24
10.0	2.4

CD (Morrison)
2400.15
240.22
24.673
3.9676

↓ never use Stokes

Example: Drag coefficient on a sphere

Given: A 1.55 mm sphere is moving in air at a speed of 1.25 m/s. The air properties are:

- $\rho = 1.246 \text{ kg/m}^3$
- $\nu = 1.426 \times 10^{-5} \text{ m}^2/\text{s}$

$$\left[\nu = \frac{\mu}{\rho} \right]$$

To do: Calculate the Reynolds number and the drag coefficient for this sphere.

Solution:

using Morrison,

$$Re = \frac{VD_p}{\nu}$$

$$C_D \approx \frac{24}{Re} + \frac{2.6 \left(\frac{Re}{5.0} \right)}{1 + \left(\frac{Re}{5.0} \right)^{1.52}} + \frac{0.411 \left(\frac{Re}{2.63 \times 10^5} \right)^{-7.94}}{1 + \left(\frac{Re}{2.63 \times 10^5} \right)^{-8.00}} + \frac{0.25 \left(\frac{Re}{10^6} \right)}{1 + \left(\frac{Re}{10^6} \right)} \text{ for } Re < 10^6$$

$$Re = \frac{(1.25 \text{ m/s})(1.55 \text{ mm})}{1.426 \times 10^{-5} \text{ m}^2/\text{s}} \left(\frac{1 \text{ m}}{1000 \text{ mm}} \right) = 135.869 \text{ Re}$$

unity conv. factor

$C_D =$ Put this in Excel, Matlab, ... software

$$C_D = 0.90149$$

IN EXCEL,

Re	C_D
35.869	

$= 24/35 + (2.6 \times (35/5.0)) / (1 + ((35/5.0)^{1.52})) +$

cell B5 → Fill down → Copy & paste →

Re	C_D
0	0