IAQE Lesson 14D: Stokes Terminal Particle Settling Speed J.M. Cimbala

Today, we will:

- Discuss **Terminal Settling Speed** in quiescent air due to gravity, also called **Gravimetric Settling**
- Discuss an **Approximation** for terminal settling speed for the case of **Stokes Flow**
- Do an example problem

Review of Air Properties and Equations of Particle Motion:

Air property equations: $\boxed{\rho = \dfrac{P}{R_{\text{air}}T}}$, where $\boxed{R_{\text{air}} = 0.2870\dfrac{\text{kJ}}{\text{kg}\cdot\text{K}} = 287.0\dfrac{\text{J}}{\text{kg}\cdot\text{K}} = 287.0\dfrac{\text{m}^2}{\text{s}^2\cdot\text{K}}}$

and viscosity comes from the **Sutherland Equation** $\boxed{\mu \approx \mu_{\text{s}}\left(\dfrac{T}{T_{\text{s},0}}\right)^{3/2}\dfrac{T_{\text{s},0}+T_{\text{s}}}{T+T_{\text{s}}}}$,

where $T_{\text{s},0} = 298.15\text{ K}$, $T_{\text{s}} = 110.4\text{ K}$, $\mu_{\text{s}} = 1.849\times10^{-5}\dfrac{\text{kg}}{\text{m}\cdot\text{s}}$.

Relative particle velocity = $\boxed{\vec{v}_r = \vec{v} - \vec{U}}$, where $\vec{v}$ = particle velocity and $\vec{U}$ = air velocity.

Drag force on a spherical particle = $\boxed{\vec{F}_{\text{drag}} = -\dfrac{1}{8}\rho\dfrac{C_D}{C}\pi D_p^{\,2}\vec{v}_r\left|\vec{v}_r\right|}$, where C_D comes from the

Morrison Eq. $$\boxed{C_D \approx \frac{24}{\text{Re}} + \frac{2.6\left(\dfrac{\text{Re}}{5.0}\right)}{1+\left(\dfrac{\text{Re}}{5.0}\right)^{1.52}} + \frac{0.411\left(\dfrac{\text{Re}}{2.63\times10^5}\right)^{-7.94}}{1+\left(\dfrac{\text{Re}}{2.63\times10^5}\right)^{-8.00}} + \frac{0.25\left(\dfrac{\text{Re}}{10^6}\right)}{1+\left(\dfrac{\text{Re}}{10^6}\right)} \quad \text{for Re} < 10^6},$$

C is the **Cunningham Correction Factor** $\boxed{C = 1 + \text{Kn}\left[2.514 + 0.80\exp\left(-\dfrac{0.55}{\text{Kn}}\right)\right]}$, Kn is the

Knudsen number $\boxed{\text{Kn} = \dfrac{\lambda}{D_p}}$, λ is the **mean free path** of the air $\boxed{\lambda = \dfrac{\mu}{0.499}\sqrt{\dfrac{\pi}{8\rho P}}}$, and Re is

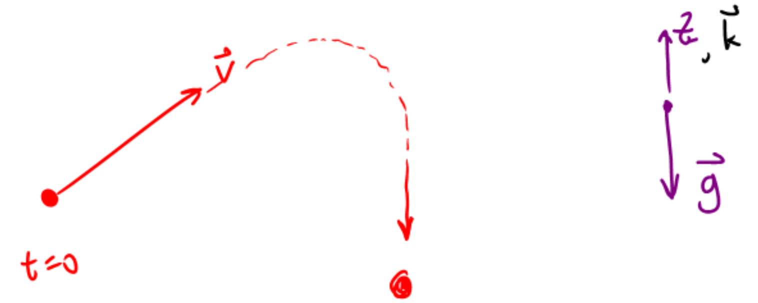
the **Reynolds number** relative to the moving particle $\boxed{\text{Re} = \dfrac{\rho\left|\vec{v}_r\right|D_p}{\mu}}$.

$m_p \vec{a}_p = \vec{F}_{grav} + \vec{F}_{drag}$

Equation of motion for a spherical particle: $$\boxed{\cancel{m_p\frac{d\vec{v}}{dt}}^{\,0} = \frac{\pi D_p^{\,3}}{6}\left(\rho_p - \rho\right)\vec{g} - \frac{1}{8}\rho\frac{C_D}{C}\pi D_p^{\,2}\vec{v}_r\left|\vec{v}_r\right|}.$$

The above equations are valid for the standard **continuum fluid approximation** (fluid is treated as a continuum, not worried about free molecular effects). But, for very small particles in air, the fluid is no longer a continuum, and **free molecular effects** are important. We have taken this into account by the Cunningham Correction Factor.

Simplest Application – Terminal Settling Speed in Quiescent Air:



Here $\vec{U}=0$

$\boxed{V_t = \text{Terminal Settling Speed}}$ V_t = const

$\vec{V} = -V_t \vec{k}$

@ steady settling speed $\rightarrow \frac{d\vec{V}}{dt} = 0$

$$\boxed{\left[\vec{V}_r \left|\vec{V}_r\right|\right] = -V_t^2 \vec{k}}$$

$$\boxed{0 = -(\rho_p - \rho) g + \frac{3}{4} \rho \frac{C_D}{C} \frac{1}{D_p} V_t^2}$$

Terminal settling speed: $\boxed{V_t = \sqrt{\frac{4}{3}\frac{\rho_p - \rho}{\rho} g D_p \frac{C}{C_D}}}$, but $C_D = C_D(\text{Re})$, where $\boxed{\text{Re} = \frac{\rho V_t D_p}{\mu}}$.

GENERAL CASE

IMPLICIT EQ

$C_D = C_D(Re)$, $Re = f(V_t)$

C_D depends on Re
Re .. .. V_t
V_t .. .. C_D

MUST ITERATE

Simplest Case of Terminal Settling Speed in Quiescent Air – Stokes Flow Approximation:

THIS IS AN APPROXIMATION!

★ STOKES FLOW → $Re \lesssim 0.1$ ∴ then $\boxed{C_D = \frac{24}{Re}} = \frac{24\mu}{\rho V_t D_p}$

• Plug this C_D into our general eq for V_t

$$V_t^2 = \frac{4}{3}\frac{(\rho_p-\rho)}{\cancel{\rho}} g D_p \frac{C \cancel{\rho} V_t D_p}{24\mu}$$

$$V_t = \frac{4}{3(24)}(\rho_p-\rho) g D_p^2 \frac{C}{\mu}$$

$$\boxed{V_t = \frac{\rho_p-\rho}{18} g D_p^2 \frac{C}{\mu}}$$

★ Terminal Settling Speed for Stokes Flow Approx.

VALID ONLY IF $Re \lesssim 0.1$

Example: Stokes Flow Settling Speed

Given: A spherical particle falls in quiescent air. Here are some properties:

- P = 101.325 kPa
- T = 25°C = 298.15 K] SATP
- D_p = 10.0 μm
- ρ_p = 1000 kg/m^3 (unit density)

To do: Calculate the Stokes flow terminal settling speed V_t. Repeat for D_p = 100 μm.

Solution:

First we need the air properties at this temperature and pressure:

At SATP → we prop. from Eq. st.

$\rho = 1.184\ \frac{kg}{m^3}$ $\mu = 1.849 \times 10^{-5}\ \frac{kg}{m \cdot s}$ $\lambda = 0.06704\ \mu m$

Next we need to calculate the CCF (Cunningham Correction Factor) C:

$$\boxed{C = 1 + \text{Kn}\left[2.514 + 0.80\exp\left(-\frac{0.55}{\text{Kn}}\right)\right]}, \text{ where } \boxed{\text{Kn} = \frac{\lambda}{D_p}} \rightarrow \frac{0.06704\ \mu m}{10.0\ \mu m} = 0.006704$$

$C = 1.0169$

Finally, we apply the Stokes flow terminal settling speed ***approximate*** equation:

$$\boxed{V_t = \frac{\rho_p - \rho}{18} D_p^{\,2} g \frac{C}{\mu}}$$

$$V_t = \frac{\left(1000 - 1.184\right)\frac{kg}{m^3}}{18} \frac{\left(10.0 \times 10^{-6}\, m\right)^2 \left(9.807\ \frac{m}{s^2}\right)\left(1.0169\right)}{1.849 \times 10^{-5}\ \frac{kg}{m \cdot s}}$$

$$\boxed{V_t = 0.00299\ \frac{m}{s}}$$

Notice how small!

e.g. in one minute

$$\Delta z = V_t \cdot \Delta t = \left(0.00299\ \frac{m}{s}\right)\left(1\ min\right)\left(\frac{60\ s}{min}\right)$$

$$\Delta z = 0.1796\ m \approx \underline{18\ cm}$$

CHECK

$$\star\ Re = \frac{\rho V_t D_p}{\mu} = \frac{\left(1.184\ kg/m^3\right)\left(0.002993\ \frac{m}{s}\right)\left(10.0 \times 10^{-6}\ m\right)}{1.849 \times 10^{-5}\ \frac{kg}{m \cdot s}} = \underline{\underline{0.00192}}$$

$< \underline{\underline{0.1}}$

$\star$ STOKES FLOW APPROX. IS VALID HERE

- REPEAT @ $\underline{D_p = 100\ \mu m}$

$$\boxed{V_t = \cancel{0.295\ m/s}}$$

$\rightarrow$ STOKES NOT VALID FOR THIS CASE $\star$

check $Re = 1.89 \lesssim \cancel{0.1}$

WHAT TO DO? Go back to our General Eq.

$$\boxed{V_t = \sqrt{\frac{4}{3} \frac{\rho_p - \rho}{\rho} g D_p \frac{C}{C_D}}}$$

$\star$ MUST ITERATE