8.7 Gravimetric Settling in a Room

Consider a room of volume V, height H, and horizontal cross-sectional area A as shown in Figure 8.18, which illustrates both models.

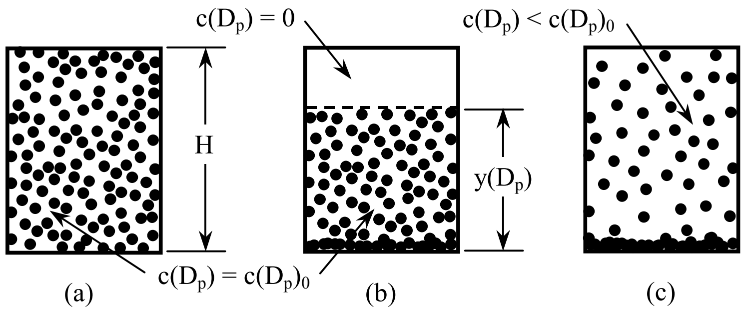

Figure 8.18 Gravimetric settling of a monodisperse aerosol in quiescent room air: (a) initial condition for both cases, $t = 0$, (b) laminar settling model for $t > 0$, (c) well-mixed model for $t > 0$.

Let $c(D_p)_0$ be the initial mass concentration of particles of diameter D_p in the room (Figure 8.18a). In the laminar model all particles of the same size fall uniformly at terminal velocity v_t such that those near the bottom settle to the floor. At any subsequent time there are *no* particles of that size above a certain height $y(D_p)$, and the concentration remains at $c(D_p)_0$ below this height (Figure 8.18b). The same argument applies to particles of other sizes, except the values of $y(D_p)$ are different because they settle at different velocities. For the well-mixed model, on the other hand, some particles settle, but those that remain are mixed throughout the room volume such that $c(D_p)$ decreases with time (Figure 8.18c).

<u>8.7.1 Laminar Settling Model</u>

…Some algebra yields …

$$\boxed{y(D_p) = H - v_t t}$$

Let the average mass concentration of particles be denoted by $\overline{c}\,(D_p)$. Thus,

$$\boxed{\overline{c}(D_p) = \frac{A y(D_p) c(D_p)_0}{V} = \frac{A\left(H - v_t t\right) c(D_p)_0}{V} = c(D_p)_0 \left(1 - \frac{v_t t}{H}\right)}$$

The average concentration of these particles decreases linearly with time until a time (t_c) called the ***critical time*** elapses, where

$$\boxed{t_c = \frac{H}{v_t}}$$

and all particles of the size whose terminal velocity is v_t have settled to the floor.

Note also that t_c varies with terminal velocity v_t, which depends on particle size, density, etc. – i.e, in a polydisperse aerosol, particles of different diameters settle at different speeds, and therefore have different critical times.

8.7.2 Well-Mixed Settling Model

On the other hand, suppose the well-mixed settling model is valid. As particles of a particular size fall to the floor with velocity v_t, an idealized mixer instantaneously redistributes the remaining particles throughout the room. The rate of change of mass of particles of this size suspended in the room air is equal to the rate of deposition onto the floor,

$$\frac{d\left(c(D_p)V\right)}{dt} = V\frac{dc(D_p)}{dt} = -v_t A c(D_p)$$

which can be integrated to yield

$$\boxed{\frac{c(D_p)}{c(D_p)_0} = \frac{\overline{c}(D_p)}{c(D_p)_0} = \exp\left(-\frac{v_t t}{H}\right)}$$

Since the well-mixed model presumes that the concentration is the same throughout the enclosure at any instant of time, the term $c(D_p)$ is also equal to the average concentration, $\overline{c}\,(D_p)$.

The well-mixed model predicts that the average mass concentration decreases exponentially, while the laminar model predicts that it decreases linearly. In the well-mixed model $\overline{c}\,(D_p)/c(D_p)_0 = 0.368$ at $t = t_c$, while in the laminar model it is zero. In the well-mixed model, the average mass concentration does not decrease to 0.001 (0.1%) of its initial value until nearly seven of these time constants, i.e. until $t \approx 7t_c$.

The laminar model leads one to believe that the dust will be removed too quickly because it ignores unavoidable thermal currents, drafts, diffusion, etc. that redistribute particles. The laminar model also predicts an infinite concentration gradient at an interface (see Figure 8.18b) that cannot exist in nature. The well-mixed model overestimates the time to clean the air because it exaggerates the mixing mechanisms. For small, inhalable particles, it is however the more realistic model to use. Certainly, it is the more *conservative* model. In either case, it has been assumed in the analyses above that any particle that hits the floor stays there. In reality, some of the particles can be re-entrained into the room by air currents; models of this re-entrainment process are beyond the scope of this text. An additional source of error is the fact that some particles are deposited or adsorbed on other surfaces in the room, including the side walls, as discussed in Chapter 5.

8.8 Gravimetric Settling in Ducts

Gravimetric settling in ducts can also be analyzed using the concepts of a laminar settling model and a well-mixed settling model. Figure 8.19 illustrates the laminar and well-mixed models for flow in a horizontal duct of rectangular cross section (A = WH).

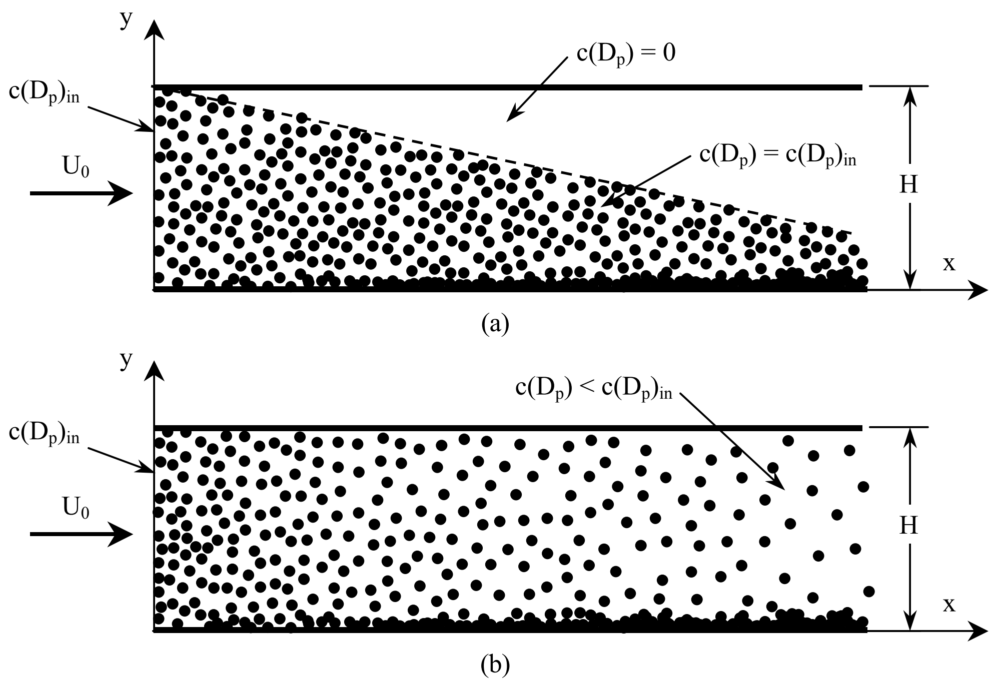

Figure 8.19 Gravimetric settling of a monodisperse aerosol in a horizontal duct with uniform air flow: (a) laminar conditions, (b) well-mixed conditions.

It is assumed that the horizontal velocity of the gas everywhere in the duct is equal to the average duct velocity (U_0), i.e. there is ***plug flow*** in the gas phase. At the duct inlet, the mass concentration of particles of diameter D_p is $c(D_p)_{in}$.

8.8.1 Laminar Settling Model

In the laminar settling model, all particles of the same size fall at their terminal velocity (v_t) and move with a horizontal velocity equal to that of the carrier gas, $v_x = U_0$. Thus at a downstream distance x, the *uppermost* particles have fallen a distance (H - y),

$$H - y = v_t t = v_t \frac{x}{U_0}$$

The average mass concentration of particles of a certain size $\bar{c}(D_p)$ at a distance x from the inlet can be written as

$$\bar{c}(D_p) = c(D_p)_{in} \frac{y}{H}$$

Combining the above two equations yields

$$\boxed{\frac{\bar{c}(D_p)}{c(D_p)_{in}} = 1 - \frac{x}{H}\frac{v_t}{U_0}}$$

Since particles settle to the floor of the duct, the duct can be thought of as a simple particle collector. The grade efficiency of the duct, $\eta(D_p)$, for particles of size D_p is defined as

$$\boxed{\eta(D_p) = 1 - \frac{\bar{c}(D_p)}{c(D_p)_{in}} = \frac{x}{H}\frac{v_t}{U_0}}$$

The above equation also applies to gravimetric settling in fully established flow between parallel plates. See Flagan and Seinfeld (1988) for the derivation. At a ***critical distance*** ($x = L_c$) downstream in the duct, the collection efficiency is 100% and the duct contains *no* particles of the size defined by the terminal velocity. The critical distance is defined by

$$\boxed{L_c = H\frac{U_0}{v_t}}$$

and

$$\boxed{\eta(D_p) = \frac{x}{H}\frac{v_t}{U_0} = \frac{x}{H}\frac{v_t H}{L_c v_t} = \frac{x}{L_c}}$$

Note that we can define a grade efficiency for the gravimetric settling process!

8.8.2 Well-Mixed Settling Model

…After some algebra – see text …..

$$\boxed{\eta(D_p) = 1 - \frac{\bar{c}(D_p)}{c(D_p)_{in}} = 1 - \exp\left(-\frac{v_t}{U_0}\frac{x}{H}\right) = 1 - \exp\left(-\frac{v_t}{U_0}\frac{x}{H}\frac{W}{W}\right) = 1 - \exp\left(-\frac{v_t A_s}{Q}\right)}$$

where

- A_s = area of lower collecting surface, $A_s = xW$
- Q = volumetric flow rate, $Q = U_0HW$

The grade efficiency can also be written in terms of the critical length, as defined above,

$$\boxed{\eta(D_p) = 1 - \exp\left(-\frac{x}{L_c}\right)}$$

Comparison of the collection efficiencies for laminar and well-mixed settling models shows differences similar to those concluded for settling in rooms:

(a) The laminar settling model overestimates deposition because it ignores turbulence and diffusion that mix and redistribute particles.
(b) The well-mixed model exaggerates mixing but nevertheless provides a more accurate and conservative design estimate.
(c) At the critical downstream distance, L_c, the well-mixed settling model predicts a collection efficiency of 63.2%, while the laminar settling model predicts 100%.
(d) At a downstream distance of approximately $7L_c$, the well-mixed settling model predicts a collection efficiency of 99.9%.

8.11 Inertial Deposition in Curved Ducts

Particles have *inertia*, and can cross air streamlines as sketched:

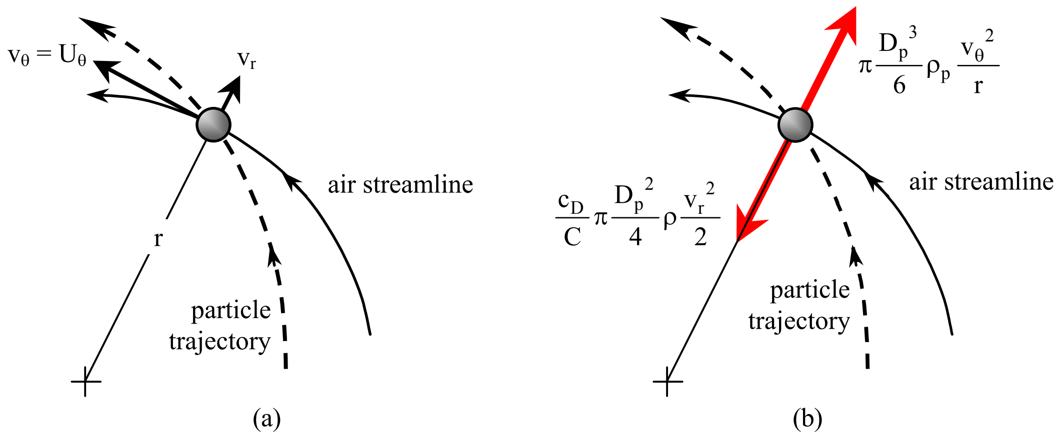

Figure 8.22 Quasi-static equilibrium of a particle of diameter D_p and density ρ_p in curvilinear flow; (a) particle velocity components, and (b) forces acting on the particle.

The particle's radial velocity component then simplifies to

$$\boxed{v_r = C\frac{U_\theta^2}{r}\tau_p}$$

The above equation is the same as that for gravimetric settling except that gravitational acceleration is replaced by centrifugal acceleration. Deposition of particles on the outer wall of the bend can therefore be modeled in a fashion similar to gravimetric deposition in horizontal ducts, through use of either the laminar (no mixing) settling model or the turbulent (well-mixed) settling model. The particles for which inertial separation is important are usually sufficiently large that the Cunningham slip factor (C) is close to unity, but for completeness C is included in the analysis which follows.

8.11.1 Laminar Settling Model

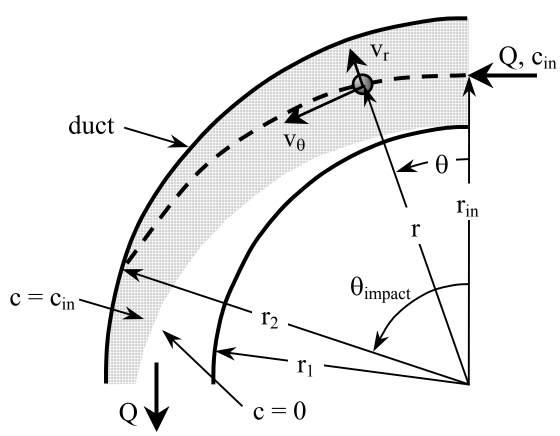

Figure 8.24 Particle trajectory (dashed line) and mass concentration in a curved duct of rectangular cross section for the laminar flow model; particle enters at $r = r_{in}$, $\theta = 0$, and impacts the outer wall at $r = r_2$, $\theta = \theta_{impact}$; particle shown at arbitrary time.

8.11.2 Well-Mixed Model

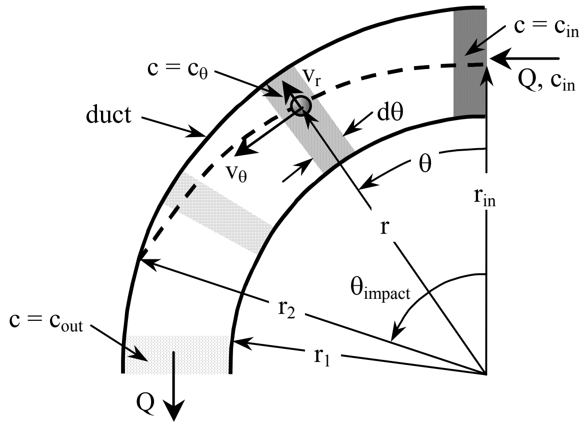

Figure 8.25 Particle trajectory (dashed line) and mass concentration in a curved duct of rectangular cross section for the well-mixed model; particle enters at $r = r_{in}$, $\theta = 0$, and impacts the outer wall at $r = r_2$, $\theta = \theta_{impact}$; particle shown at arbitrary time at location (r,θ). Four control volumes are shown with degree of shading indicating how mass concentration decreases with θ.

Example 8.9 - Particle Classifier

Given: A company processes agricultural materials, grains, corn, rice, etc. One of the processes is a milling operation. Significant fugitive dust is produced. Enclosures and exhaust air (Q, in CFM) are needed to capture the dust. A classifier is needed to separate no less than 50% of the particles larger than 100 μm ($D_p > 100$ μm) which are returned for reprocessing. The smaller particles are removed by filters (a baghouse – see Chapter 9). Your supervisor suggests constructing a simple device consisting of a 180-degree elbow of rectangular cross section containing louvers on the outside surface, as in Figure E8.9a. The volumetric flow rate of air in the elbow is Q. Centrifugal force sends large particles in the radial direction; the particles pass through the louvers and are drawn off by a slip stream and removed by other means.

To do: Compute the grade efficiency curves (similar to Figure 8.7) that will enable operators to select the proper volumetric flow rate Q to achieve a certain removal efficiency (η). Assume that the gas flow is irrotational and well mixed. Plot the results for a classifier whose dimensions are:

$r_1 = 0.30$ m $r_2 = 0.70$ m $W = 0.40$ m $s = 0.070$ cm

and which separates unit density particles ($\rho_p = 1{,}000$ kg/m^3) traveling in an air stream at 300 K.

Solution: From Eq. **Error! Reference source not found.**,

$$K = \frac{r_2 - r_1}{r_2\left[\ln\left(\frac{r_2}{r_1}\right)\right]^2} = \frac{0.70\text{ m} - 0.30\text{ m}}{(0.70\text{ m})\left[\ln\left(\frac{0.70\text{ m}}{0.30\text{ m}}\right)\right]^2} = 0.796$$

From Eq. **Error! Reference source not found.**,

$$\text{Stk}_{\text{avg}} = \frac{\tau_p Q}{r_2 W(r_2 - r_1)} = \frac{\tau_p Q}{(0.70\text{ m})(0.40\text{ m})(0.70\text{ m} - 0.30\text{ m})} = \frac{\tau_p Q}{0.112\text{ m}^3}$$

and from Eq. **Error! Reference source not found.**, for $\theta = \pi$ (180-degrees), the fractional efficiency is

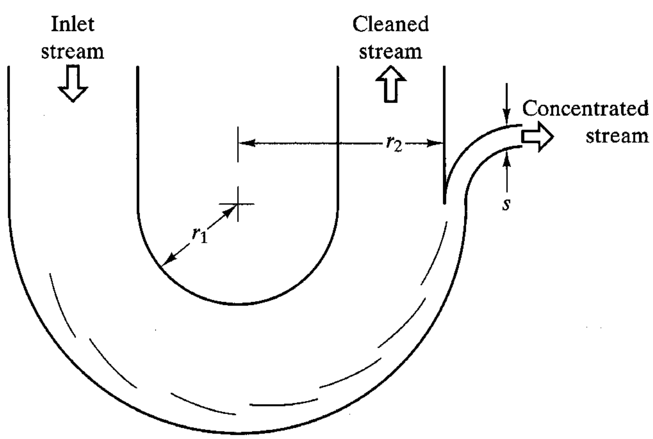

Figure E8.9a Centrifugal particle classifier (from Heinsohn & Kabel, 1999).

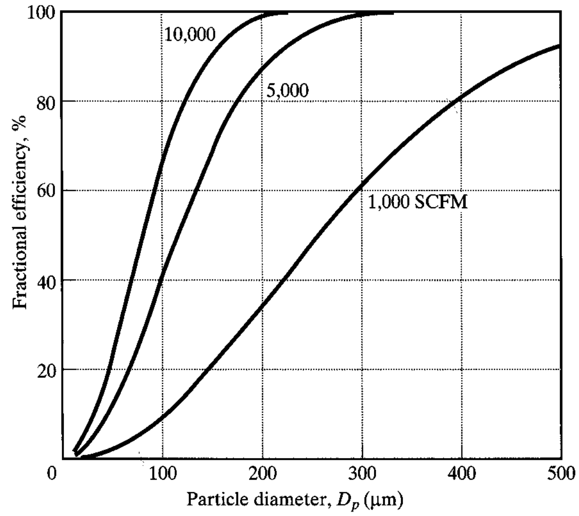

Figure E8.9b Fractional efficiency of a centrifugal particle classifier for three volumetric flow rates; classifier dimensions: r_1 = 0.3 m, r_2 = 0.7 m, W = 0.4 m, s = 0.07 cm (from Heinsohn & Kabel, 1999).

$$\eta\left(D_p\right) = 1 - \exp\left[-\left(Stk_{avg}\right)K\theta C\right] = 1 - \exp\left(-\frac{\tau_p Q}{0.112\ m^3} 0.796\pi(1.00)\right) = 1 - \exp\left(-22.3\tau_p Q\right)$$

where the Cunningham slip factor (C) is assumed to equal unity for such large particles, as was shown in Table 8.4. Figure E8.9b shows the fractional efficiency at three volumetric flow rates.

Discussion: Clearly the lowest volumetric flow rate classifies particles poorly. At the three higher flow rates, the device removes 200 and 300 μm particles efficiently. Use of the well-mixed model is a reasonable selection since Reynolds numbers in the elbow are surely large enough to establish turbulent flow. The assumption of irrotational flow, however, needs to be justified. The next level of sophistication is to use ***computational fluid dynamics*** (CFD) computer programs to predict the trajectories of particles in the three-dimensional velocity field of the 180-degree elbow. CFD is discussed in Chapter 10.