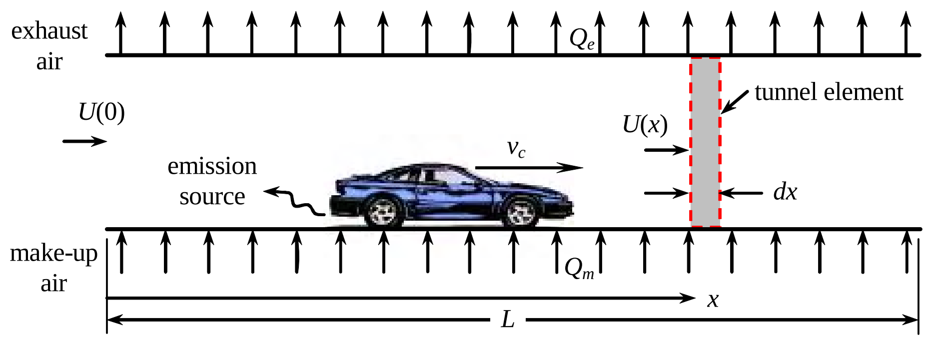
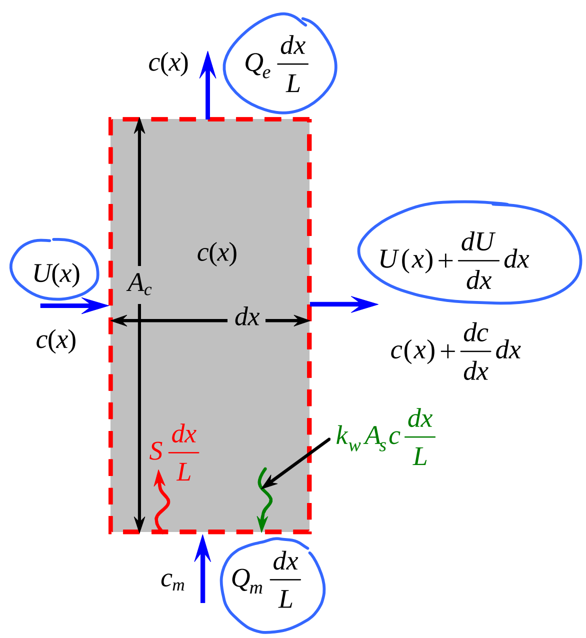
ME 405   Fall 2006   Professor John M. Cimbala   Lecture 24   11/01/2006

Today, we will:

- Continue our discussion of **tunnel ventilation** in **Section 5.14**
- Do an example problem – tunnel ventilation
- If time, begin to discuss **local ventilation** in **Section 6.1**

Recall, from last time, our analysis of an automobile tunnel:





We derived an equation for conservation of mass of the contaminant (for the control volume shown):

$$U\cancel{cA_c} + S\frac{dx}{L} + Q_m\frac{dx}{L}c_m = U\cancel{cA_c} + c\frac{dU}{dx}dxA_c + U\frac{dc}{dx}dxA_c + \cancel{\frac{dU}{dx}\frac{dc}{dx}dx^2A_c} + k_wA_s\frac{dx}{L}c + Q_e\frac{dx}{L}c$$

cancel

neglect dx^2 term

Rearrange: mult each term by $\frac{1}{dx A_c}$

$\Downarrow$

$$c\frac{dU}{dx} + U\frac{dc}{dx} = \underbrace{\frac{S}{A_c L}}_{=\,s} + \underbrace{\frac{Q_m}{A_c L}}_{=\,q_m} c_m - \underbrace{\frac{Q_e}{A_c L}}_{=\,q_e} c - \underbrace{\frac{k_w A_s}{A_c L}}_{=\,k} c$$

Define:

(lower case s)

$$\boxed{c\frac{dU}{dx} + U\frac{dc}{dx} = s + q_m c_m - q_e c - kc} \qquad (1)$$

what about this term? → Look @ cons. of mass of the bulk air flow

$$\cancel{U A_c} + Q_m \frac{dx}{L} = \left(\cancel{U} + \frac{dU}{dx}\right) A_c + Q_e \frac{dx}{L}$$

$$\frac{dU}{dx} = \underbrace{\frac{Q_m}{A_c L}}_{=\,q_m} - \underbrace{\frac{Q_e}{A_c L}}_{=\,q_e}$$

$$\boxed{\frac{dU}{dx} = q_m - q_e} \qquad (2)$$

Plug (2) into (1)

∴ rearrange:

$$\frac{dc}{dx} = \underbrace{\frac{s + q_m c_m}{U}}_{=\,B} - \underbrace{\frac{k + q_m}{U}}_{=\,A} c$$

$$\boxed{\frac{dc}{dx} = B - Ac} \quad \star$$

→ Standard 1st-order ODE in terms of x instead of t.

Two possibilities : 1) A & B constant → get analytical sol'n

not fnc. of x [exponential eq.]

2) A and/or B = fncs of x

For balanced transverse ventilation, U = constant

For unbalanced " " $U = U(x)$

$C_{max} = \frac{B}{A}$ = maximum possible value of c

$$\boxed{C_{max} = \frac{S + q_m C_m}{k + q_m}}$$

regardless of the value of U & whether U = fnc(x) or not !

analogous to C_{ss} for unsteady problems

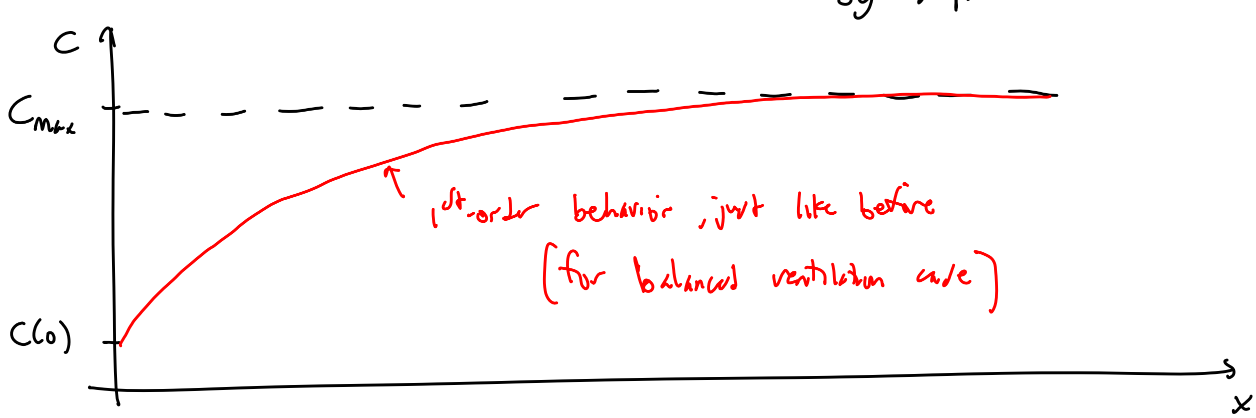
See text for details :

For balanced transverse ventilation,

$$\boxed{C(x) = C_{max} - \left[C_{max} - C(0)\right] \exp\left[-Ax\right]} \quad \star$$

For unbalanced " " see text for sol'n

[you can get an analytical sol'n by separation of variables]



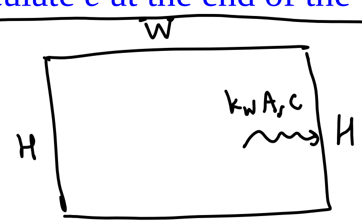
Example

Given: An engineering firm is designing a ***balanced transverse ventilation*** automobile tunnel. The design criteria are provided below:

$(EF)_c$ = emission factor for carbon monoxide = (5600 mg)/(auto km)	
$U(0)$ = inlet air speed into tunnel = 80. m/min	
c_m = mass concentration of make-up air = 1.0 mg/m^3	
k_w = wall adsorption coefficient = 0.013 cm/s	
→ Wall adsorption is on the side walls and ceiling only (not on the floor)	
Q_m = make-up air volumetric flow rate = 25,000 m^3/min	balanced
Q_e = exhaust air volumetric flow rate = 25,000 m^3/min	
PEL of carbon monoxide = 55 mg/m^3	
n_c = traffic density = 60. auto/km	
v_c = average automobile speed = 85. km/hr	
$c(0)$ = mass concentration at inlet = 2.0 mg/m^3	
W = tunnel width = 10.0 m	
H = tunnel height = 5.0 m	DIMENSIONS
L = tunnel length = 2000 m	

To do: Calculate the maximum CO mass concentration, $c_{\max}$ (in units of mg/m^3). Compare $c_{\max}$ to the PEL for CO. Calculate c at the end of the tunnel ($x = L$).

Solution: cross section of tunnel:



$$\boxed{k = \frac{A_s}{A_c L} k_w} = \frac{(2H+W)L}{HW\ L} k_w = \frac{2H+W}{HW} k_w$$

$$k = 0.00312\ \frac{1}{\text{min}}$$

$$\boxed{S = (EF)_c\, n_c\, v_c\, L} = 952{,}000\ \frac{\text{mg}}{\text{min}}$$

$$\boxed{s = \frac{S}{A_c L}} = 9.52\ \frac{\text{mg}}{\text{m}^3\,\text{min}}$$

$$\boxed{q_m = \frac{Q_m}{A_c L}} = 0.25\ \frac{1}{\text{min}}$$

$$\boxed{q_e = \frac{Q_e}{A_c L}} = 0.25\ \frac{1}{\text{min}}$$

(Balanced)

Plug into our ODE $\quad \frac{dc}{dx} = B - Ac$

$U = U(0) = \text{const.}$ since it is balanced

$$\boxed{A = \frac{k + q_m}{U}} = \underline{0.003164 \ \frac{1}{m}}$$

$$\boxed{B = \frac{S + q_m C_m}{U}} = \underline{0.122125 \ \frac{mg}{m^4}}$$

$$C_{max} = \frac{B}{A} = \underline{38.598 \ \frac{mg}{m^3}} \quad \text{or} \quad \boxed{C_{max} \approx 38.6 \ \frac{mg}{m^3}}$$

$$\underline{PEL = 55 \ \frac{mg}{m^3}} \longrightarrow \text{OSHA is happy} \ ☺$$

@ $x = L$,

Let $x = L = 2000$ m

$$C(x) = C_{max} - [C_{max} - c(0)] \exp[-Ax]$$

$$\boxed{C(L) = 38.5 \ mg/m^3}$$

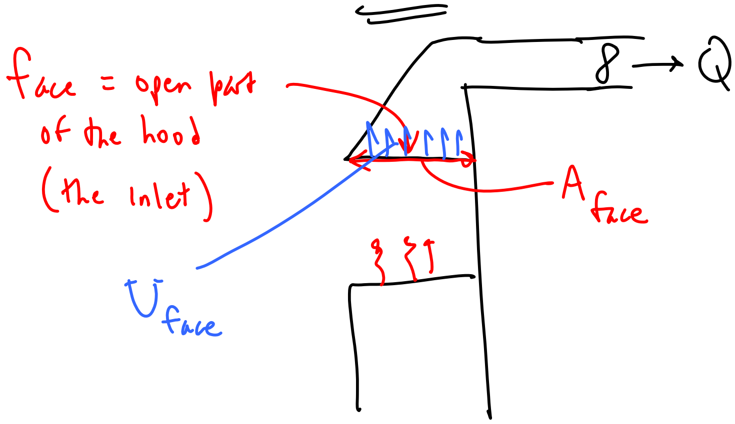
Almost @ C_{max}, but not quite

CH. 6 Local Ventilation

- General Ventilation → vent. in a room or building (large scale)

- Local ventilation → vent. locally within a room (small scale)

  ↓ i.e., hoods



We want to remove the contaminant before it enters the room air

- Hood design → different kinds of hoods for
  - particle capture
  - vapor capture

- Terminology

  A_{face} = face area = opening area of the hood

  U_{face} = avg. speed @ the face

$$\boxed{U_{face} = \frac{Q}{A_{face}}}$$

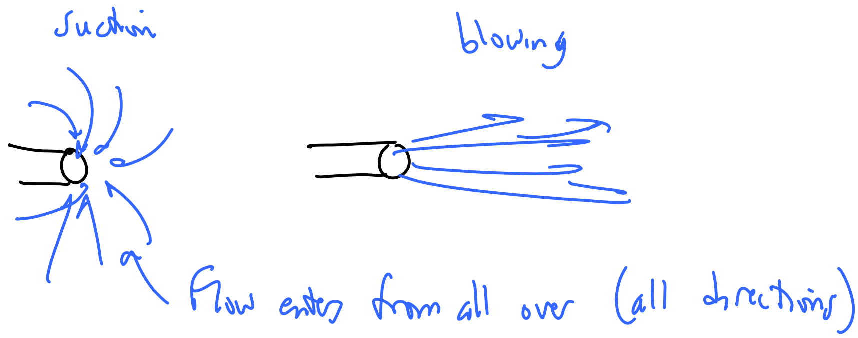
Q = volume flow rate through the hood

Fluid Mechanics → There is a fundamental problem with hood design

"The candle effect"

You can easily blow out a candle → directed momentum

You cannot suck out a candle



See figs eg. 6.8. in text → Air speed drops dramatically with distance from the face

Example: Rectangular inlet of aspect ratio 10:

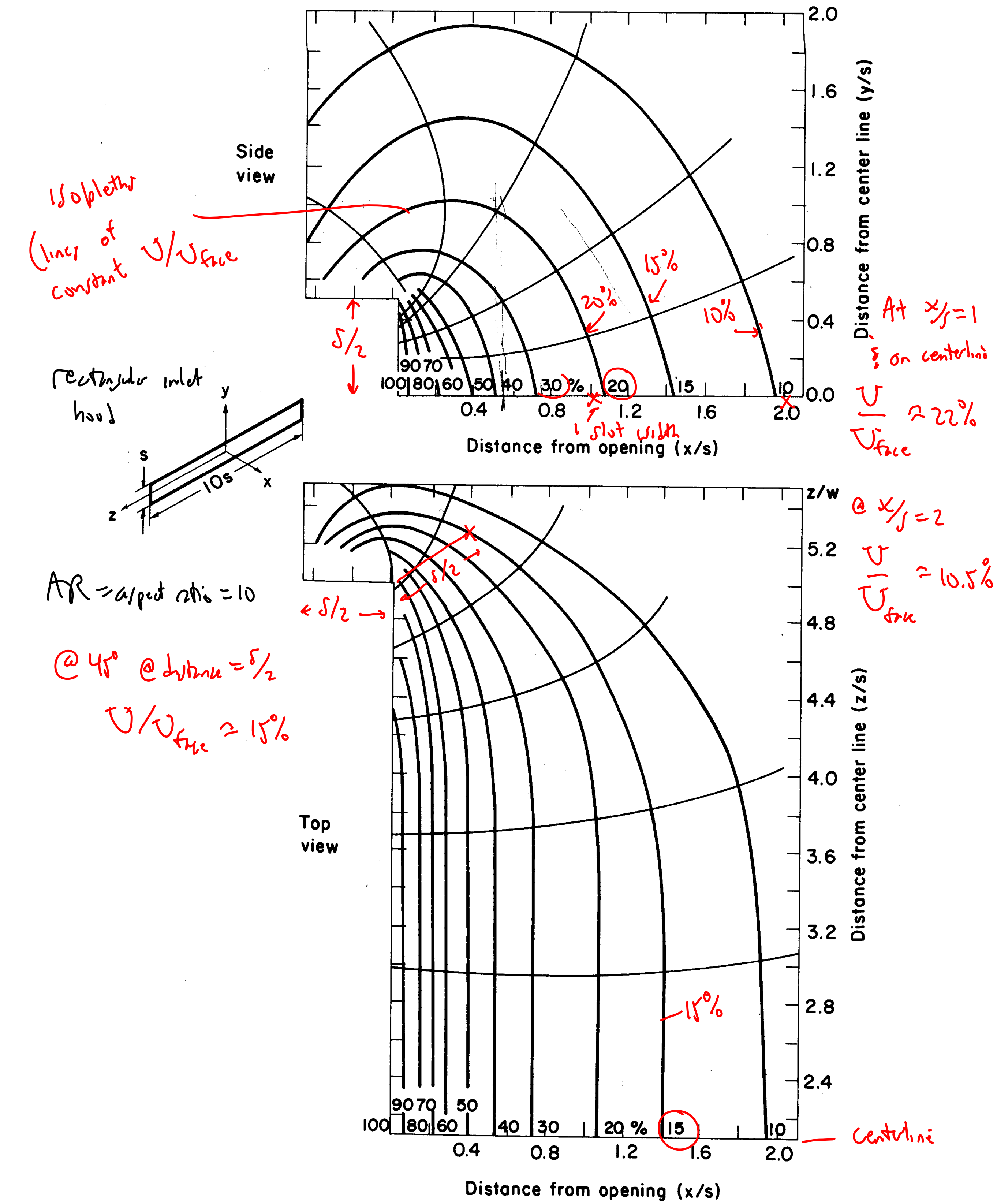


Figure 6.8 Velocity isopleths (curves of constant U/U_{face}, %) for an unflanged rectangular opening, aspect ratio 1:10 (adapted from Baturin, 1972).