Today, we will:

- Discuss **terminal settling speed in Stokes flow** in **Section 8.5**
- Discuss **gravimetric settling in rooms** in **Section 8.7**
- Discuss **gravimetric settling in ducts** in **Section 8.8**
- Discuss **inertial separation – particles in curved flow** in **Section 8.11**
- Do Candy Questions for Candy Friday

Terminal settling velocity in Stokes flow

if $Re \lesssim 0.1$, $C_D = \frac{24}{Re} = \frac{24\mu}{\rho V_t D_p}$

Can solve for V_t in closed form

$$V_t = \sqrt{\frac{4}{3}\frac{(\rho_p - \rho) D_p g C}{\rho C_D}}$$

$$\boxed{V_t = \frac{\rho_p - \rho}{\rho_p} \frac{\rho_p D_p^2}{18\mu} g C}$$

$$\boxed{\tau_p \equiv \frac{\rho_p D_p^2}{18\mu} = \text{particle relaxation time}}$$

$$\boxed{V_t = C g \tau_p \frac{\rho_p - \rho}{\rho_p}} = \text{terminal settling velocity for Stokes flow}$$

Significance of τ_p

$$\boxed{\tau_p C = \text{first-order time constant in our first-order ODEs}}$$

When $\Delta t \approx 7 \times$ time const, value is 99.9% of its final value

$$\boxed{\text{Here, it takes } \approx 7\tau_p C \text{ for a particle to reach 99.9\% of its final (terminal) settling speed}}$$

eg @ $D_p = 50\,\mu m$ → For air @ STP,

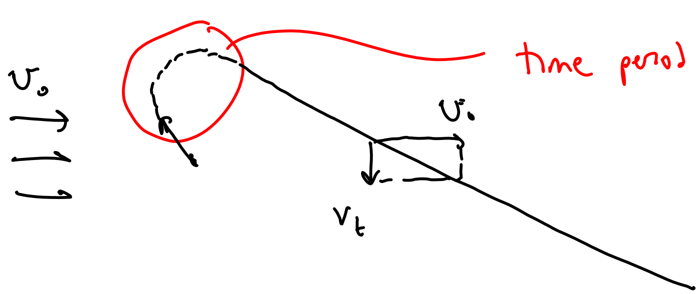
$$\left.\begin{aligned} \tau_p &= 7.58 \times 10^{-3}\ s \\ C &= 1.003 \end{aligned}\right\} 7\tau_p C = 5.3 \times 10^{-2}\ s$$

$$= \underline{\underline{0.053\ s}}$$

THIS IS VERY SHORT

Compared to time scales in a typical flow, $7\tau_p C$ is $\underline{\underline{\text{short}}}$

We can $\underline{\underline{\text{approximate}}}$ by assuming that particle "immediately" falls @ speed v_t



time period to reach steady linearly falling trajectory is negligibly small

APPLICATIONS → (rest of Ch. 8)

Sec. 8.7 Gravimetric settling in a room

(quiescent air — still air — in the room)

Two models:

- Laminar settling model
- Well-mixed settling model [some call it "turbulent"]

Analogous to displacement vs dilution ventilation

$\boxed{\text{See Fig 8.18 in text}}$ ☆

[consider a mono-particle — monodisperse aerosol]

↓

all particles are same diameter

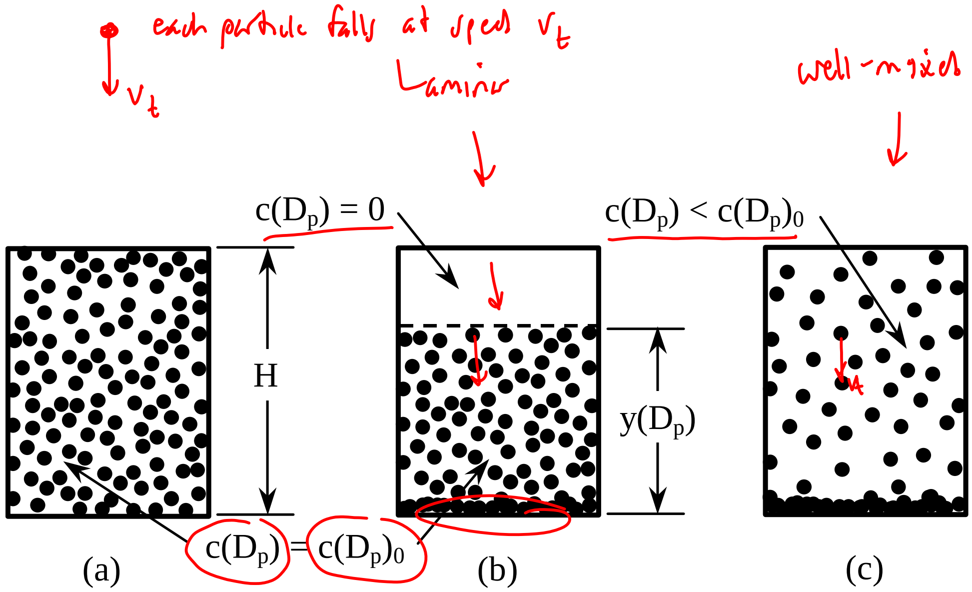


Figure 8.18 Gravimetric settling of a monodisperse aerosol in quiescent room air: (a) initial condition for both cases, t = 0, (b) laminar settling model for t > 0, (c) well-mixed model for t > 0.

Actual settling will most likely be in between these two

Sec 8.8 — GRAVIMETRIC SETTLING IN DUCTS

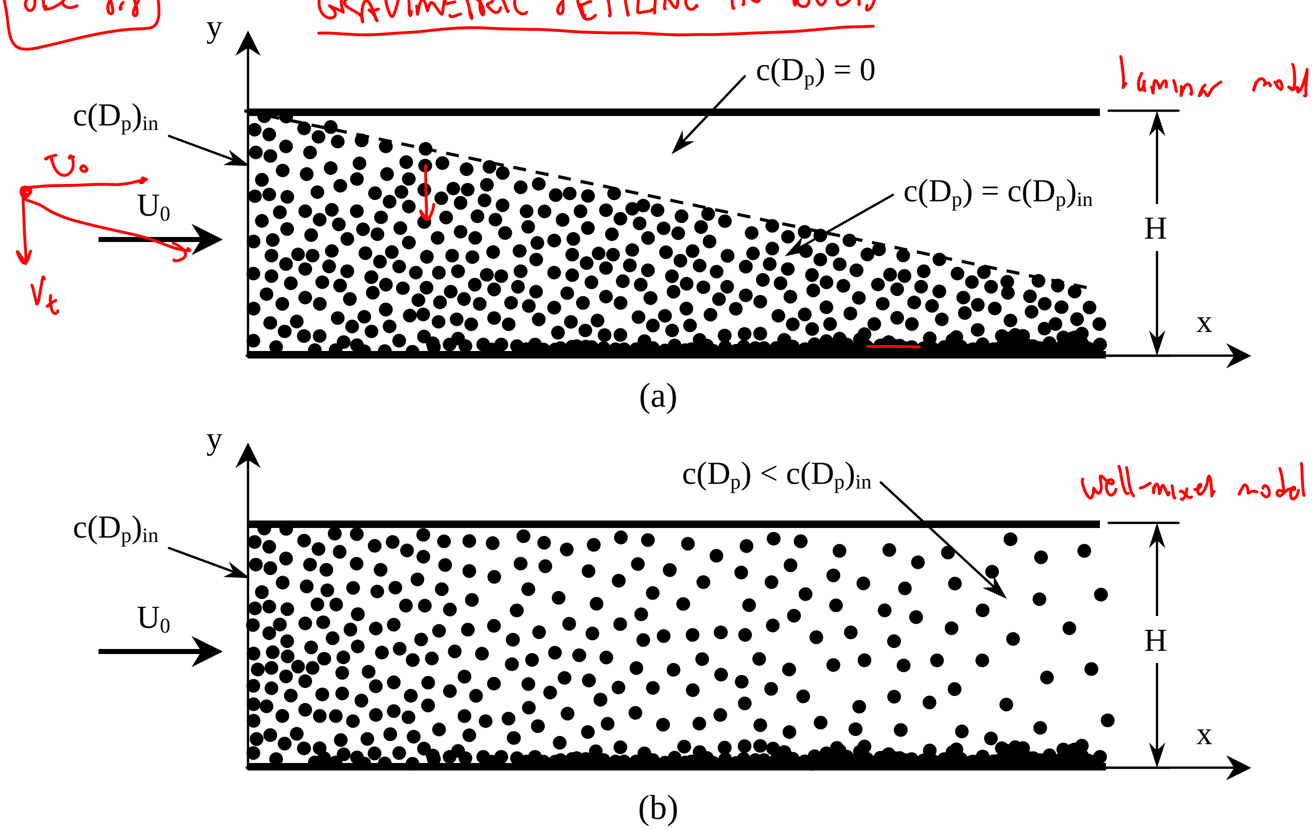
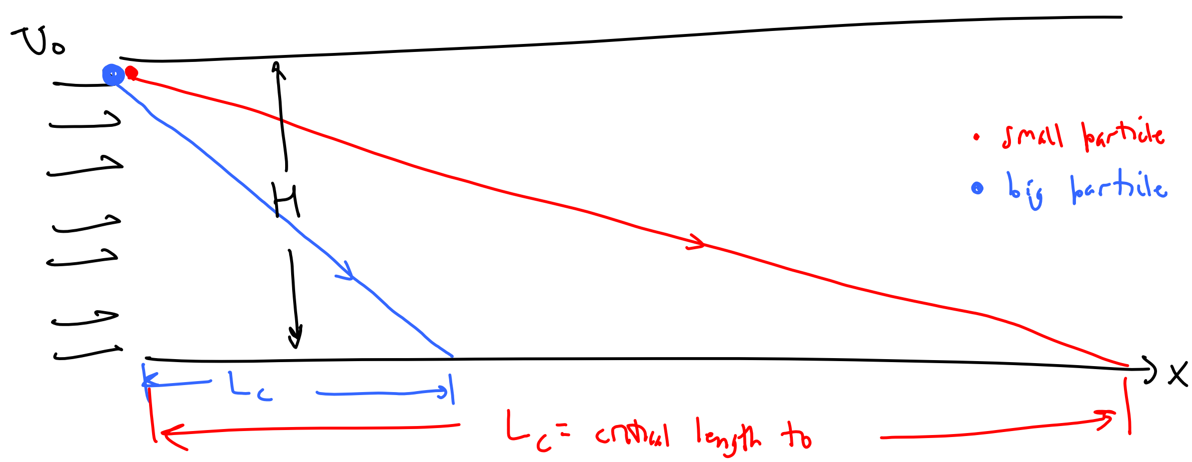


Figure 8.19 Gravimetric settling of a monodisperse aerosol in a horizontal duct with uniform air flow: (a) laminar conditions, (b) well-mixed conditions.

At different particle diameters, the settling speed v_t will be different, but U_0 will remain the same.

Laminar model



$\boxed{\text{But, } L_c \text{ depends on } D_p}$

$$\boxed{L_c = H\frac{U_0}{v_t}} \; ☆ = \text{fnc. of } D_p$$

We can define a grade efficiency for particles settling in a duct

$$\boxed{\eta(D_p) = 1 - \frac{\bar{c}(D_p)}{c(D_p)_{in}}}$$

$\bar{c}(D_p)$ = avg mass conc. at some location x

For laminar settling

$$\boxed{\eta(D_p) = \frac{x}{H}\frac{v_t}{U_0} = \frac{x}{L_c}} \; ☆$$

up to $x = L_c$, then $\eta(D_p) = 0$ from then on $(x > L_c)$

Well-mixed:

$$\boxed{\eta(D_p) = 1 - \exp\left(\frac{-x}{L_c}\right)} \; ☆$$

Table 8.8 Gravimetric settling of unit density spheres in ducts; duct velocity = 22.86 m/s (4500 FPM), air at STP.

we want a low efficiency

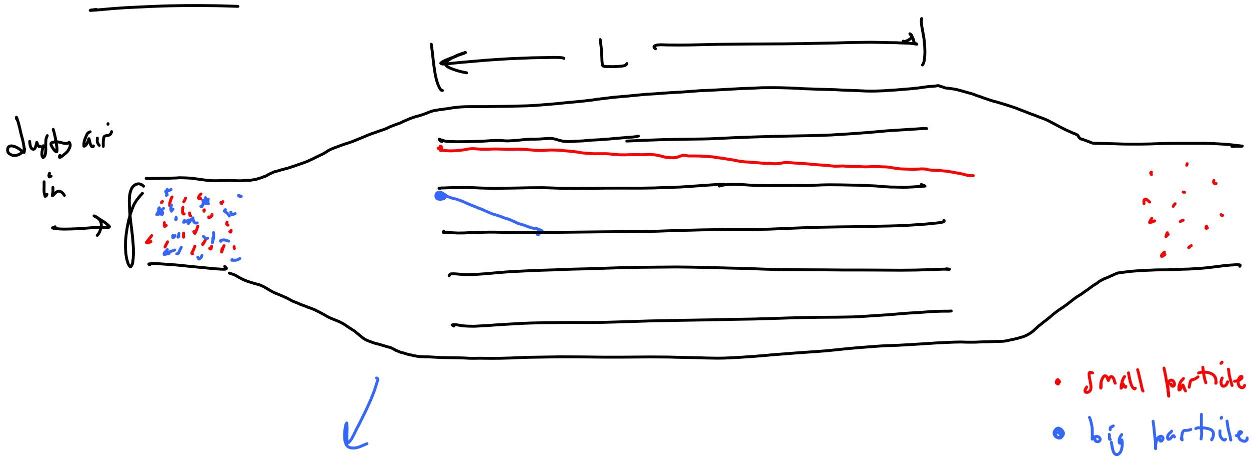
		efficiency = 1%		efficiency = 10%	
D_p (µm)	v_t (m/s)	$(x/H)_{lam}$	$(x/H)_{turb}$	$(x/H)_{lam}$	$(x/H)_{turb}$
10	3.1×10^{-3}	74.6	74.6	745.8	785.8
50	7.3×10^{-2}	3.12	3.12	31.2	32.9
100	2.5×10^{-1}	0.92	0.92	9.2	9.7
200	0.7	0.33	0.33	3.3	3.4
600	2.5	0.09	0.09	0.9	1.0
1,000	4.0	0.06	0.06	0.6	0.6

well mixed

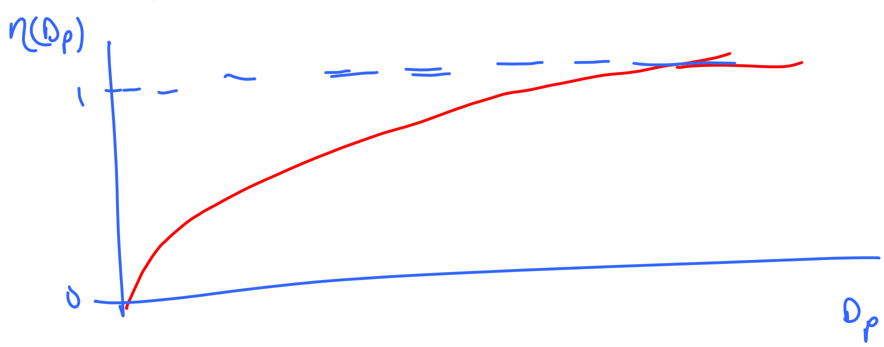
"Unit density" means $SG = 1.00 \rightarrow \rho = 1000 \frac{kg}{m^3}$

(density of water)

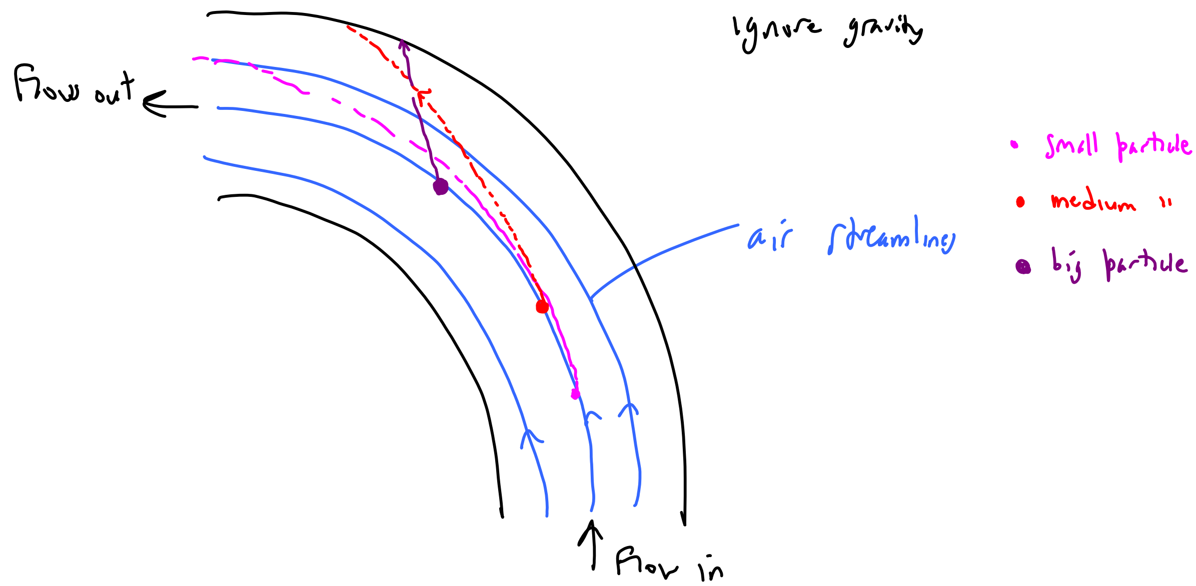
Elutriator → simple air cleaning device to remove big particles



Can generate grade effic. curve



SEC 8.11 → INERTIAL SEPARATION (particles in curved ducts)



See book eq's → How to calculate the trajectory of particles

ANALOGY BETWEEN GRAVIMETRIC SETTLING
& SETTLING IN CURVED DUCTS

replace $\boxed{g \text{ by } \frac{U^2}{r}}$!

Gravimetric settling → $\boxed{V_t = C \tau_p g}$ $\frac{\rho_p - \rho}{\rho_p}$ → assume heavy particles so that $\rho_p \gg \rho$

Inertial settling (centrifugal) → $\boxed{V_r = C \tau_p \frac{U_\theta^2}{r}}$

Section 8.11 – Inertial separation of particles in curved flow fields

laminar model

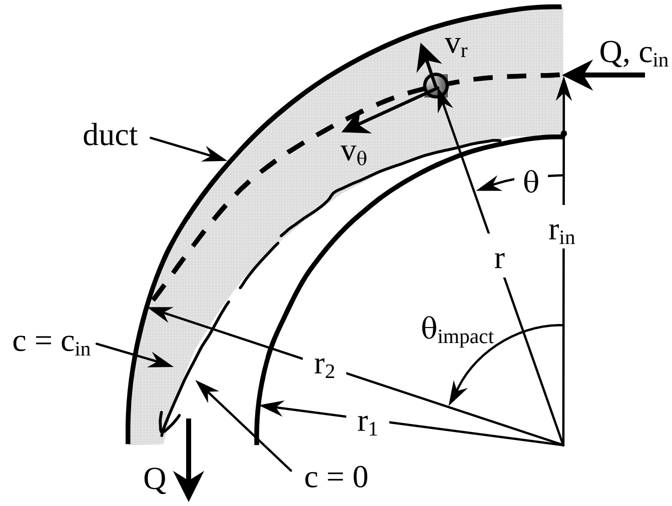


Figure 8.24 Particle trajectory (dashed line) and mass concentration in a curved duct of rectangular cross section for the laminar flow model; particle enters at $r = r_{in}$, $\theta = 0$, and impacts the outer wall at $r = r_2$, $\theta = \theta_{impact}$; particle shown at arbitrary time.

Well-mixed model

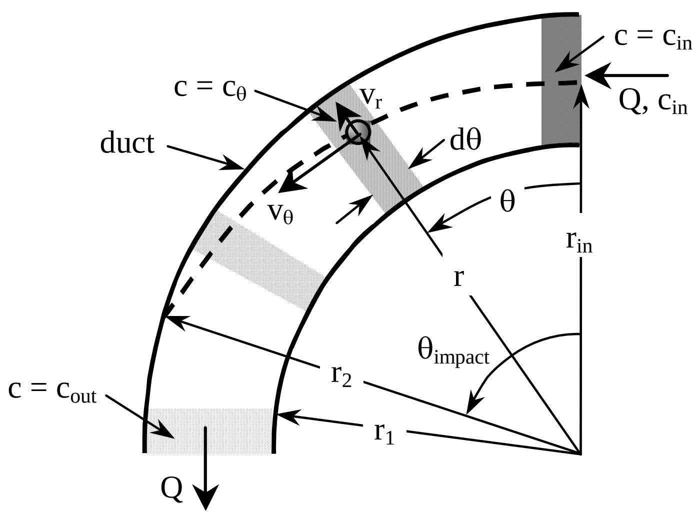


Figure 8.25 Particle trajectory (dashed line) and mass concentration in a curved duct of rectangular cross section for the well-mixed model; particle enters at $r = r_{in}$, $\theta = 0$, and impacts the outer wall at $r = r_2$, $\theta = \theta_{impact}$; particle shown at arbitrary time at location (r,θ). Four control volumes are shown with degree of shading indicating how mass concentration decreases with θ.

* AGAIN – ACTUAL AMOUNT OF PARTICLE REMOVAL WILL TYPICALLY LIE BETWEEN THE TWO – LAMINAR vs. WELL MIXED