

Equation Sheet for ME 420

For homework, in-class problems, exams, and future reference. Author: John M. Cimbala, Penn State University. Latest revision, 27 March 2025

General and conversions: $g = 9.807 \frac{\text{m}}{\text{s}^2}$, $\frac{0.3048 \text{ m}}{1 \text{ ft}}$, $\frac{1 \text{ mile}}{1609.3 \text{ m}}$, $\frac{1 \text{ kPa} \cdot \text{m}^2}{1 \text{ kN}}$, $\frac{1 \text{ kN} \cdot \text{m}}{1 \text{ kJ}}$, $\frac{1 \text{ kW} \cdot \text{s}}{1 \text{ kJ}}$, $\frac{1 \text{ Btu}}{1.055056 \text{ kJ}}$,
 $\frac{1 \text{ kg}}{2.205 \text{ lbm}}$, $\frac{1 \text{ ton}}{2000 \text{ lbm}}$, $\frac{1 \text{ tonne (metric ton)}}{1000 \text{ kg}}$, $\frac{1 \text{ g}}{10^6 \mu\text{g}}$, $\frac{1 \text{ m}}{10^6 \mu\text{m}}$, $\frac{1 \text{ m}}{10^9 \text{ nm}}$, $V_{\text{sphere}} = \frac{4}{3}\pi(R_p)^3 = \frac{1}{6}\pi(D_p)^3$.

Molecular weights and mols: $m = nM$, $M_{\text{air}} = 28.97 \text{ g/mol}$, $M_{\text{water}} = 18.02 \text{ g/mol}$, **Avagadro's number:** 6.02214×10^{23} .

Air at SATP (25°C): $P_{\text{SATP}} = 101.325 \text{ kPa} = 760 \text{ mm Hg}$, $T_{\text{SATP}} = 25^\circ \text{C} = 298.15 \text{ K}$, $\rho = 1.184 \text{ kg/m}^3$, $\lambda = 0.06704 \mu\text{m}$, $\mu = 1.849 \times 10^{-5} \text{ kg/(m s)}$.

Air at any T: $P = \rho R_{\text{air}} T$, $\lambda \approx \frac{\mu}{0.499} \sqrt{\frac{\pi}{8\rho P}}$, $\mu \approx \mu_s \left(\frac{T}{T_{s,0}} \right)^{3/2} \frac{T_{s,0} + T_s}{T + T_s}$, $T_{s,0} = 298.15 \text{ K}$, $T_s = 110.4 \text{ K}$, $\mu_s = 1.849 \times 10^{-5} \frac{\text{kg}}{\text{m} \cdot \text{s}}$.

Ideal gas: $PV = nR_u T = mRT$, $R = \frac{R_u}{M}$, $P = \rho RT$, $R_u = 8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$, $R_{\text{air}} = 0.2870 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 287.0 \frac{\text{J}}{\text{kg} \cdot \text{K}} = 287.0 \frac{\text{m}^2}{\text{s}^2 \cdot \text{K}}$.

Mass flow rate in a duct: $\dot{m} = \rho VA = \text{constant}$, where V is the magnitude of velocity averaged over cross-sectional area A .

Thermo review: $e = u + \frac{V^2}{2} + gz$, $v = \frac{1}{\rho}$, $h = u + Pv = u + \frac{P}{\rho}$, $c_v = \left(\frac{\partial u}{\partial T} \right)_v$, $c_p = \left(\frac{\partial h}{\partial T} \right)_p$, $Tds = du + PdV$, $Tds = dh - vdp$.

Ideal gas: $u = c_v T$, $h = c_p T$, $c_p - c_v = R$, $\gamma = \frac{c_p}{c_v}$, $c_v = \frac{R}{\gamma - 1}$, $c_p = \frac{R\gamma}{\gamma - 1}$, $s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = c_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$.

Air: $\gamma = 1.40$, $c_v = 717.5 \frac{\text{J}}{\text{kg} \cdot \text{K}} = 0.7175 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 717.5 \frac{\text{m}^2}{\text{s}^2 \cdot \text{K}}$, $c_p = 1004.5 \frac{\text{J}}{\text{kg} \cdot \text{K}} = 1.0045 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 1004.5 \frac{\text{m}^2}{\text{s}^2 \cdot \text{K}}$.

Isentropic flow of ideal gas: $\frac{P_2}{P_1} = \left(\frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma-1}}$, $\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1} \right)^{\frac{1}{\gamma-1}}$. For air ($\gamma = 1.40$), $\frac{P_2}{P_1} = \left(\frac{T_2}{T_1} \right)^{3.5}$, $\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1} \right)^{2.5}$.
Ma sometimes instead of M

Mach number M, speed of sound a, Mach angle μ: $M = \frac{V}{a}$, $a = \sqrt{\left(\frac{\partial P}{\partial \rho} \right)_s}$ [ideal gas, $a = \sqrt{\gamma RT}$], $\mu = \sin^{-1} \frac{1}{M}$.
c sometimes instead of a

Steady, adiabatic duct flow: $\dot{m}_1 = \dot{m}_2$, $h_0 = h + \frac{V^2}{2}$, $h_{0,1} = h_{0,2}$, $\frac{dA}{A} + \frac{dV}{V} + \frac{d\rho}{\rho} = 0$, $\frac{dV}{V} = \frac{1}{M^2 - 1} \frac{dA}{A}$.

Critical or sonic values when $M = 1$ (at throat or at duct exit): $a^* = \sqrt{\gamma RT^*}$, $M^* = V/a^*$ = characteristic Mach number.

Adiabatic, isentropic, 1-D duct flow: $\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2$, $\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{1}{\gamma-1}}$, $\frac{P_0}{P} = \left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{\gamma}{\gamma-1}}$, $\frac{a_0}{a} = \left(\frac{T_0}{T} \right)^{1/2}$,

$$\frac{T_0}{T^*} = \frac{\gamma+1}{2}, \quad \frac{\rho_0}{\rho^*} = \left(\frac{T_0}{T^*} \right)^{\frac{1}{\gamma-1}}, \quad \frac{P_0}{P^*} = \left(\frac{T_0}{T^*} \right)^{\frac{\gamma}{\gamma-1}}, \quad \frac{a_0}{a^*} = \left(\frac{T_0}{T^*} \right)^{1/2}, \quad \frac{A}{A^*} = \frac{1}{M} \left[\left(\frac{2}{\gamma+1} \right) \left(1 + \frac{\gamma-1}{2} M^2 \right) \right]^{\frac{\gamma+1}{2(\gamma-1)}}, \quad M^2 = \frac{2}{\frac{\gamma+1}{M^{*2}} - (\gamma-1)}.$$

Air ($\gamma=1.4$): $\frac{T_0}{T} = 1 + 0.2M^2$, $\frac{\rho_0}{\rho} = (1 + 0.2M^2)^{2.5} = \left(\frac{T_0}{T} \right)^{2.5}$, $\frac{P_0}{P} = (1 + 0.2M^2)^{3.5} = \left(\frac{T_0}{T} \right)^{3.5}$, $\frac{a_0}{a} = \left(\frac{T_0}{T} \right)^{1/2}$,

$$\frac{T_0}{T^*} = \frac{\gamma+1}{2} = 1.2, \quad \frac{\rho_0}{\rho^*} = 1.577, \quad \frac{P_0}{P^*} = 1.893, \quad \frac{a_0}{a^*} = 1.095, \quad \frac{A}{A^*} = \frac{1}{M} \frac{(1 + 0.2M^2)^3}{1.728}, \quad M^2 = \frac{2}{(2.4/M^{*2}) - 0.4}.$$

Mass flow rate: General case: $\dot{m} = P_0 A M \sqrt{\frac{\gamma}{RT_0}} \left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{-(\gamma+1)}{2(\gamma-1)}}$, Choked case: $\dot{m} = \dot{m}_{\max} = P_0 A^* \sqrt{\frac{\gamma}{RT_0}} \left(\frac{\gamma+1}{2} \right)^{\frac{-(\gamma+1)}{2(\gamma-1)}}$.

Normal shock equations (stationary, ideal gas): $h_{0,1} = h_{0,2}$, $T_{0,1} = T_{0,2}$, $\frac{T_2}{T_1} = \frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2}$, $M_2^* = \frac{1}{M_1^*}$. The pressure ratio

must satisfy both the [Fanno curve](#) $\frac{P_2}{P_1} = \frac{M_1}{M_2} \sqrt{\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2}}$ and the [Rayleigh curve](#) $\frac{P_2}{P_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}$. Equating these, solve

$$\text{for } M_2^2: \text{Method 1: } M_2^2 = \frac{-B - \sqrt{-(B + 2A)}}{1 + \gamma B} \quad \text{where } A = M_1^2 \frac{1 + \frac{\gamma-1}{2} M_1^2}{(1 + \gamma M_1^2)^2}, \quad B = 2\gamma A - 1. \quad \text{Method 2: } M_2^2 = \frac{1 + \frac{\gamma-1}{2} M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}}.$$

Either method yields M_2 after the shock in terms of γ and M_1 before the shock. Other useful normal shock relations:

$$\frac{P_2}{P_1} = \frac{2\gamma M_1^2 - \gamma + 1}{\gamma + 1}, \quad \frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{(\gamma + 1)M_1^2}{2 + (\gamma - 1)M_1^2}, \quad \frac{P_{02}}{P_{01}} = \frac{M_1}{M_2} \left[\frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_1^2} \right]^{\frac{\gamma+1}{2(\gamma-1)}} = \left[\frac{\frac{\gamma+1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_1^2} \right]^{\frac{\gamma}{\gamma-1}} \left[\frac{1 - \gamma + 2\gamma M_1^2}{\gamma + 1} \right]^{\frac{1}{\gamma-1}},$$

$$\frac{P_1}{P_{0,2}} = \left[\frac{1 - \gamma + 2\gamma M_1^2}{\gamma + 1} \right]^{\frac{1}{\gamma-1}} \left[\frac{\gamma + 1}{2} M_1^2 \right]^{\frac{1}{\gamma-1}}. \quad \text{Hugoniot: } u_2 - u_1 = \frac{P_1 + P_2}{2} (v_1 - v_2); \quad \text{Ideal gas Hugoniot: } \frac{P_2}{P_1} = \frac{\frac{\gamma+1}{\gamma-1} \rho_2 - 1}{\frac{\gamma+1}{\gamma-1} - \frac{\rho_2}{\rho_1}}.$$

$$\text{Normal shock in one-D duct: } \frac{A_2^*}{A_1^*} = \frac{M_2}{M_1} \left[\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} \right]^{\frac{\gamma+1}{2(\gamma-1)}} = \left[\frac{1 + \frac{\gamma-1}{2} M_1^2}{\frac{\gamma+1}{2} M_1^2} \right]^{\frac{\gamma}{\gamma-1}} \left[\frac{1 - \gamma + 2\gamma M_1^2}{\gamma + 1} \right]^{\frac{1}{\gamma-1}}.$$

$$\text{Air } (\gamma=1.4): \quad h_{0,1} = h_{0,2}, \quad T_{0,1} = T_{0,2}, \quad \frac{T_2}{T_1} = \frac{1 + 0.2M_1^2}{1 + 0.2M_2^2}, \quad \text{Fanno: } \frac{P_2}{P_1} = \frac{M_1}{M_2} \sqrt{\frac{1 + 0.2M_1^2}{1 + 0.2M_2^2}}, \quad \text{Rayleigh: } \frac{P_2}{P_1} = \frac{1 + 1.4M_1^2}{1 + 1.4M_2^2}.$$

$$M_2^2 = \frac{1 + 0.2M_1^2}{1.4M_1^2 - 0.2}, \quad \frac{P_2}{P_1} = \frac{2.8M_1^2 - 0.4}{2.4}, \quad \frac{\rho_2}{\rho_1} = \frac{2.4M_1^2}{2 + 0.4M_1^2}, \quad \frac{P_{02}}{P_{01}} = \frac{M_1}{M_2} \left[\frac{1 + 0.2M_2^2}{1 + 0.2M_1^2} \right]^3 = \left[\frac{1.2M_1^2}{1 + 0.2M_1^2} \right]^{3.5} \left[\frac{-0.4 + 2.8M_1^2}{2.4} \right]^{-2.5},$$

$$\frac{P_1}{P_{02}} = \left[\frac{-0.4 + 2.8M_1^2}{2.4} \right]^{2.5} \left[\frac{1.2M_1^2}{1 + 0.2M_1^2} \right]^{-3.5}. \quad \text{Hugoniot: } \frac{P_2}{P_1} = \frac{\frac{6\rho_2}{\rho_1} - 1}{6 - \frac{\rho_2}{\rho_1}}. \quad \text{One-D duct: } \frac{A_2^*}{A_1^*} = \left[\frac{1 + 0.2M_1^2}{1.2M_1^2} \right]^{3.5} \left[\frac{-0.4 + 2.8M_1^2}{2.4} \right]^{2.5}.$$

$$\text{Blast waves: } R \rho_{\text{atm}}^{1/5} E^{-1/5} t^{-2/5} = \text{constant}_1, \quad V_s = \frac{dR}{dt} = \text{constant}_2 \cdot t^{-3/5}.$$

$$\text{Pitot probes: Incompressible: } V = \sqrt{2 \frac{(P_0 - P)}{\rho}}, \quad \text{Subsonic compressible: } V = \sqrt{\frac{2\gamma RT_1}{\gamma - 1} \left[\left(\frac{P_{01}}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]}, \quad \text{Supersonic (the)}$$

$$\text{Rayleigh-Pitot formula: } \frac{P_1}{P_{02}} = \left(\frac{1 - \gamma + 2\gamma M_1^2}{\gamma + 1} \right)^{\frac{1}{\gamma-1}} \left(\frac{\gamma + 1}{2} M_1^2 \right)^{\frac{-\gamma}{\gamma-1}}. \quad \text{For air, } \frac{P_1}{P_{02}} = \left(\frac{-0.4 + 2.8M_1^2}{2.4} \right)^{2.5} \left(1.2M_1^2 \right)^{-3.5}.$$

Moving normal shocks: Static property ratios $\left(\frac{T_2}{T_1}, \frac{P_2}{P_1}, \frac{\rho_2}{\rho_1}, \text{etc.} \right)$ are same as stationary shock, but stagnation property

$$\text{ratios } \left(\frac{T_{02}}{T_{01}}, \frac{P_{02}}{P_{01}}, \frac{\rho_{02}}{\rho_{01}}, \text{etc.} \right) \text{ are not. } M_1 = \frac{V_1}{a_1} = M_s = \frac{V_s}{a_1}, \quad h_{0,as} = h_2 + \frac{V_{as}}{2}, \quad \frac{V_1}{V_2} = \frac{\rho_2}{\rho_1} = \frac{V_s}{V_s - V_{as}}, \quad V_{as} = \left(1 - \frac{V_2}{V_1} \right) V_s.$$

Shock tubes: Here, 4 is the initially high-pressure region and 1 is the initially low-pressure region (ideal gas only).

$$\text{Expansion fan: } \frac{a_3}{a_4} = 1 - \frac{\gamma_4 - 1}{2} \frac{V_3}{a_4}, \quad \frac{T_3}{T_4} = \left(\frac{a_3}{a_4} \right)^2 = \left(1 - \frac{\gamma_4 - 1}{2} \frac{V_3}{a_4} \right)^2, \text{ similarly, } \frac{P_3}{P_4} = \left(\frac{a_3}{a_4} \right)^{\frac{2\gamma_4}{\gamma_4-1}} \text{ and } \frac{\rho_3}{\rho_4} = \left(\frac{a_3}{a_4} \right)^{\frac{2}{\gamma_4-1}}.$$

$$\text{Shock tube: } V_2 = V_3 = V_{CS} = V_{as} = \frac{2a_1}{\gamma_1+1} \left(M_s - \frac{1}{M_s} \right), \quad P_2 = P_3, \quad \frac{P_4}{P_1} = \frac{2\gamma_1 M_s^2 - (\gamma_1 - 1)}{\gamma_1 + 1} \left[1 - \frac{\gamma_4 - 1}{\gamma_1 + 1} \frac{a_1}{a_4} \left(M_s - \frac{1}{M_s} \right) \right]^{\frac{-2\gamma_4}{\gamma_4-1}}.$$

$$\text{Reflecting shock (2=before, 5=after): } \frac{V_{\text{before}}}{V_{\text{after}}} = \frac{V_{sR} + V_{CS}}{V_{sR}} = \frac{(\gamma_1 + 1) M_{sR}^2}{2 + (\gamma_1 - 1) M_{sR}^2}, \quad M_{sR} = \frac{V_{\text{before}}}{a_{\text{before}}} = \frac{V_{sR} + V_{CS}}{a_{\text{before}}} = \frac{V_{sR} + V_{CS}}{\sqrt{\gamma_1 R_1 T_{\text{before}}}}.$$

$$\text{Rayleigh flow (Ideal gas): Cons. laws: } \rho_1 V_1 = \rho_2 V_2, \quad P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2, \quad q = \frac{\dot{Q}}{\dot{m}} = c_p (T_{02} - T_{01}) = c_p (T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}.$$

$$\frac{P_2}{P_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}, \quad \frac{P}{P^*} = \frac{1 + \gamma}{1 + \gamma M^2}, \quad \frac{T_2}{T_1} = \left[\frac{M_2}{M_1} \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \right]^2, \quad \frac{T}{T^*} = \left[\frac{(1 + \gamma) M}{1 + \gamma M^2} \right]^2, \quad \frac{\rho^*}{\rho} = \frac{V}{V^*} = \frac{(1 + \gamma) M^2}{1 + \gamma M^2},$$

$$\frac{T_0}{T_0^*} = \frac{[2 + (\gamma - 1) M^2] (1 + \gamma) M^2}{[1 + \gamma M^2]^2}, \quad \frac{P_0}{P_0^*} = \left[\frac{2 + (\gamma - 1) M^2}{1 + \gamma} \right]^{\frac{\gamma}{\gamma-1}} \frac{(1 + \gamma)}{1 + \gamma M^2}, \quad q_{\max} = \frac{\dot{Q}_{\max}}{\dot{m}} = c_p (T_0^* - T_{01}).$$

$$\text{Air } (\gamma=1.4): \quad \frac{P_2}{P_1} = \frac{1 + 1.4 M_1^2}{1 + 1.4 M_2^2}, \quad \frac{P}{P^*} = \frac{2.4}{1 + 1.4 M^2}, \quad \frac{T_2}{T_1} = \left[\frac{M_2}{M_1} \frac{1 + 1.4 M_1^2}{1 + 1.4 M_2^2} \right]^2, \quad \frac{T}{T^*} = \left[\frac{2.4 M}{1 + 1.4 M^2} \right]^2, \quad \frac{\rho^*}{\rho} = \frac{V}{V^*} = \frac{(1 + 1.4) M^2}{1 + 1.4 M^2},$$

Exam 2 material ends here.

$$\frac{T_0}{T_0^*} = \frac{[2 + 0.4 M^2] (2.4) M^2}{[1 + 1.4 M^2]^2}, \quad \frac{P_0}{P_0^*} = \left[\frac{2 + 0.4 M^2}{2.4} \right]^{3.5} \frac{2.4}{1 + 1.4 M^2}.$$

$$\text{Fanno flow (Ideal gas): Cons. laws: } \rho_1 V_1 = \rho_2 V_2, \quad P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2 + \frac{F_{\text{friction}}}{A}, \quad T_{01} = T_{02} \quad \text{or} \quad c_p T_1 + \frac{V_1^2}{2} = c_p T_2 + \frac{V_2^2}{2}.$$

$$f = \frac{8\tau_w}{\rho V^2} = \text{Darcy friction factor from the Moody chart or the Churchill equation, } f = 8 \left[\left(\frac{8}{\text{Re}} \right)^{12} + (A + B)^{-1.5} \right]^{1/12}, \text{ where}$$

$$\text{Re} = \frac{\rho V D_h}{\mu} = \frac{V D_h}{\nu}, \quad A = \left\{ -2.457 \cdot \ln \left[\left(\frac{7}{\text{Re}} \right)^{0.9} + 0.27 \frac{\varepsilon}{D_h} \right] \right\}^{16}, \quad B = \left(\frac{37530}{\text{Re}} \right)^{16}, \quad D_h = \frac{4A_c}{p} = \text{hydraulic diameter, } A_c \text{ is duct}$$

cross-sectional area, p is the perimeter. For solutions it is typical to use ratios at the **critical reference state**:

$$\frac{fL^*}{D_h} = \frac{1 - M^2}{\gamma M^2} + \frac{\gamma + 1}{2\gamma} \ln \left(\frac{(\gamma + 1) M^2}{2 + (\gamma - 1) M^2} \right), \quad \frac{fL}{D_h} = \left(\frac{fL^*}{D_h} \right)_1 - \left(\frac{fL^*}{D_h} \right)_2, \quad \frac{T}{T^*} = \frac{\gamma + 1}{2 + (\gamma - 1) M^2}, \quad \frac{\rho^*}{\rho} = \frac{V}{V^*} = M \sqrt{\frac{\gamma + 1}{2 + (\gamma - 1) M^2}},$$

$$\frac{P}{P^*} = \frac{1}{M} \sqrt{\frac{\gamma + 1}{2 + (\gamma - 1) M^2}}, \quad \frac{P_0}{P_0^*} = \frac{\rho_0}{\rho_0^*} = \frac{1}{M} \left[\frac{2 + (\gamma - 1) M^2}{\gamma + 1} \right]^{\frac{\gamma+1}{2(\gamma-1)}}. \quad \text{For air } (\gamma=1.4): \quad \frac{fL^*}{D_h} = \frac{1 - M^2}{1.4 M^2} + \frac{2.4}{2.8} \ln \left(\frac{2.4 M^2}{2 + 0.4 M^2} \right),$$

$$\frac{T}{T^*} = \frac{2.4}{2 + 0.4 M^2}, \quad \frac{\rho^*}{\rho} = \frac{V}{V^*} = M \sqrt{\frac{2.4}{2 + 0.4 M^2}}, \quad \frac{P}{P^*} = \frac{1}{M} \sqrt{\frac{2.4}{2 + 0.4 M^2}}, \quad \frac{P_0}{P_0^*} = \frac{\rho_0}{\rho_0^*} = \frac{1}{M} \left[\frac{2 + 0.4 M^2}{2.4} \right]^3.$$

$$\text{Oblique shocks (Ideal gas): } \theta - \beta - M \text{ equation: } \tan \theta = \frac{2 \cot \beta (M_1^2 \sin^2 \beta - 1)}{M_1^2 [\gamma + \cos(2\beta)] + 2}, \quad M_{1,n} = M_1 \sin \beta, \quad M_{2,n} = M_2 \sin(\beta - \theta).$$

All else identical to normal shocks except replace M_1 with $M_{1,n}$ and replace M_2 with $M_{2,n}$. Repeat for reflected shocks.

$$\text{Prandtl-Meyer expansion waves (Ideal gas): P-M function: } v(M) = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1} \left[\sqrt{\frac{\gamma - 1}{\gamma + 1}} (M^2 - 1) \right] - \tan^{-1} \left[\sqrt{(M^2 - 1)} \right],$$

$$\theta_2 = v(M_2) - v(M_1), \text{ then use isentropic relations. Air } (\gamma=1.4): \quad v(M) = \sqrt{6} \tan^{-1} \left[\sqrt{\frac{1}{6}} (M^2 - 1) \right] - \tan^{-1} \left[\sqrt{(M^2 - 1)} \right].$$