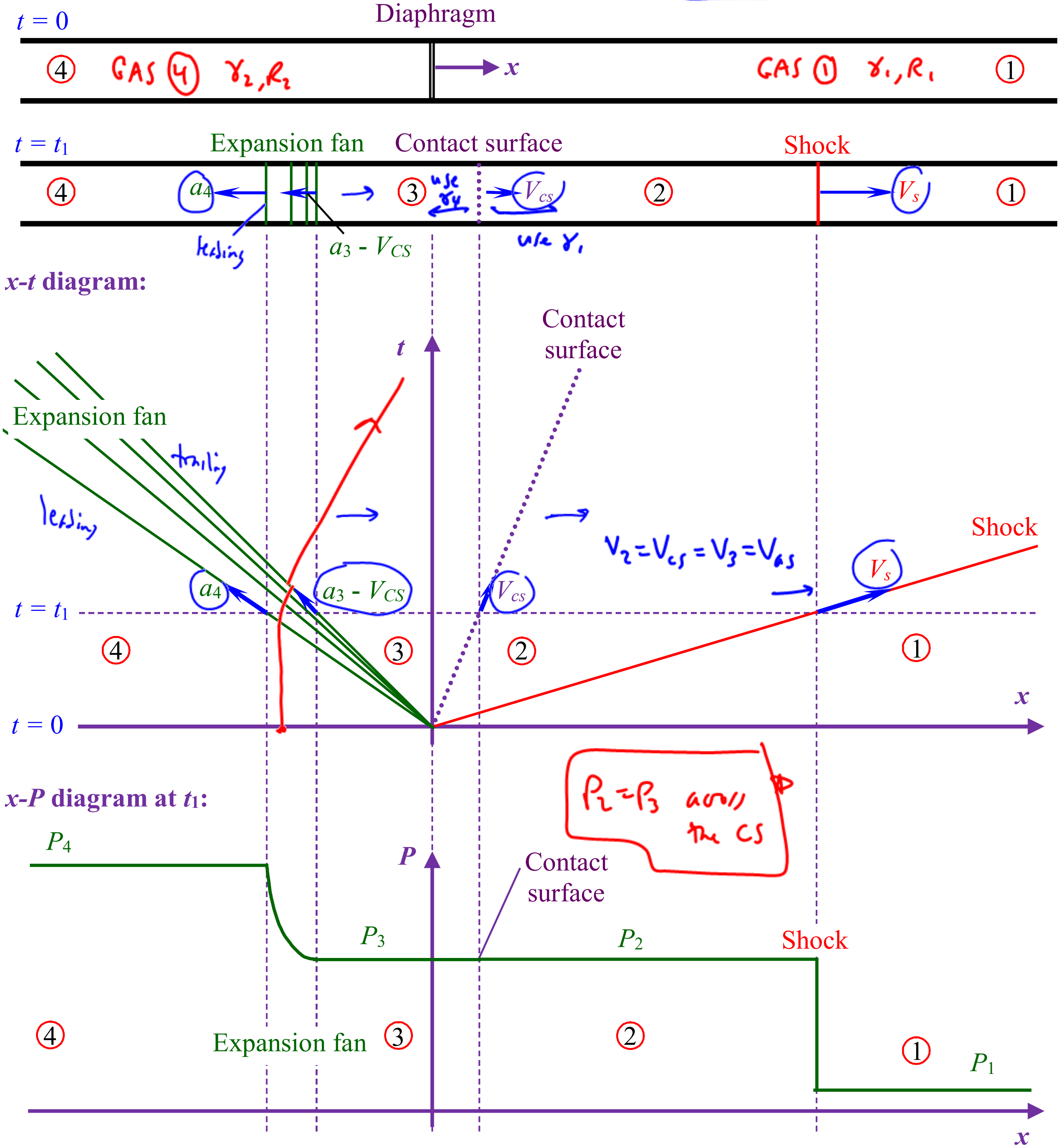
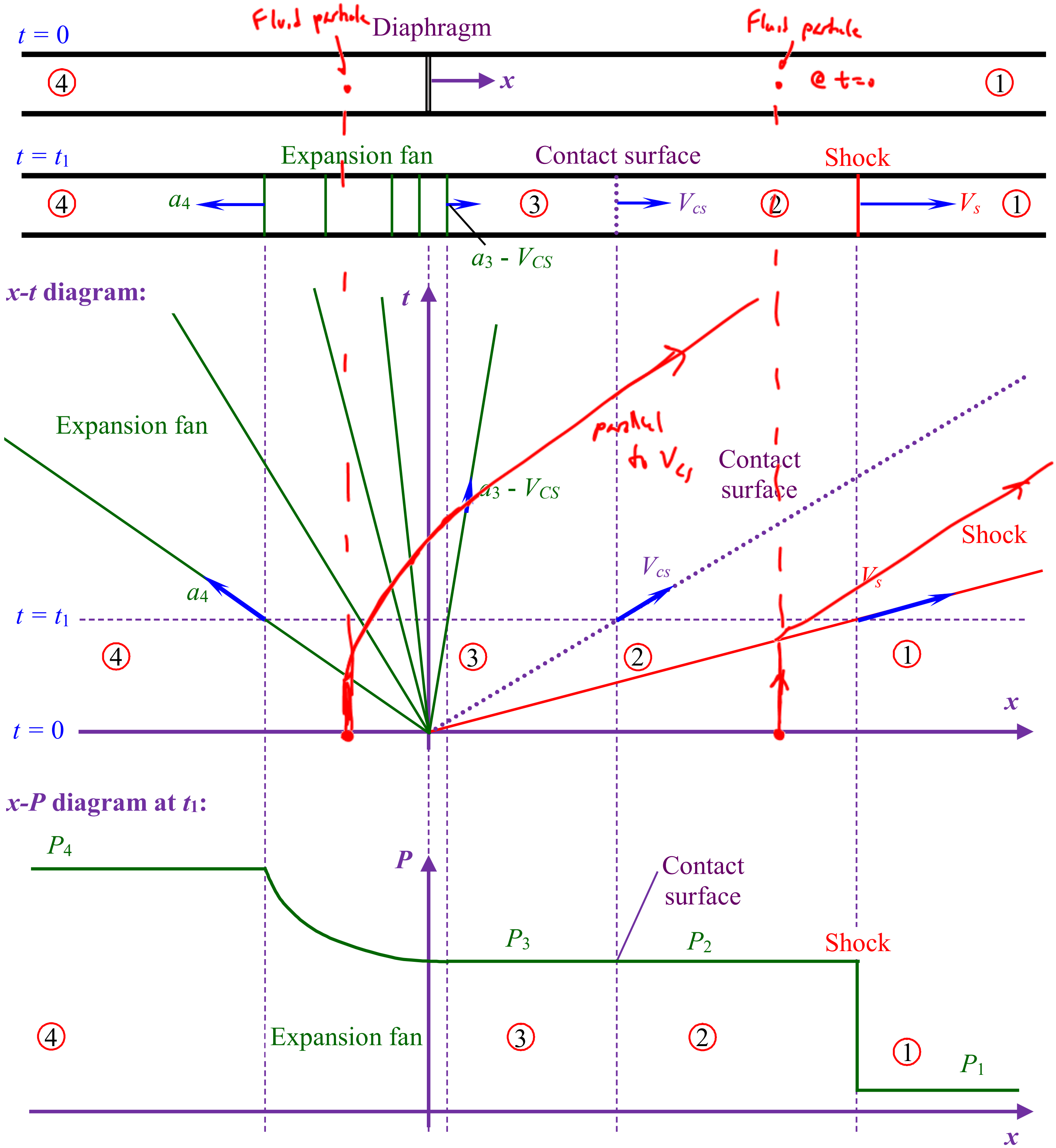
Today, we will:

- Discuss the equations for analyzing a shock tube
- Do an example problems – shock tube
- ~~If time, begin to discuss reflecting shocks in shock tubes~~
- Do Candy Questions for Candy Friday

Recall, our x-t and x-P diagrams for a shock tube with **subsonic V_{cs}** (compared to a_3):



Note: **If V_{cs} is supersonic** with respect to region 3 (> a_3), then a_3 - V_{cs} < 0 and the trailing wave of the expansion fan actually moves to the *right* instead of the left.

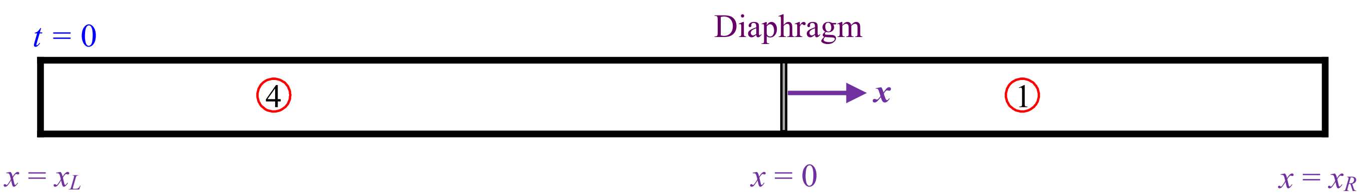


☆ QUANTITATIVE ANAL (ideal gas, adiabatic, isentropic except across shock)

Example: Shock tube (we will learn by example, including shock tube equations)

Given: A shock tube is set up with the following properties and two different gases.

- Right (low pressure) side:
  Air: P_1 = 100 kPa, T_1 = 298.15 K, γ_1 = 1.40, R_1 = 287. J/(kg K).
- Left (high pressure) side:
  Helium: P_4 = 800 kPa, T_4 = 298.15 K, γ_4 = 1.663, R_4 = 2077.15 J/(kg K).
- x_L = -15 m, x_R = 10 m.



At $t = 0$ the diaphragm ruptures.

To do:

(a) Predict properties and velocities on the left and right sides of the shock tube after rupture (P_2, T_2, T_3, V_{CS}, a_3, …).

(b) Plot the x-t diagram up to the time when the shock hits the right wall ($x = x_R$).

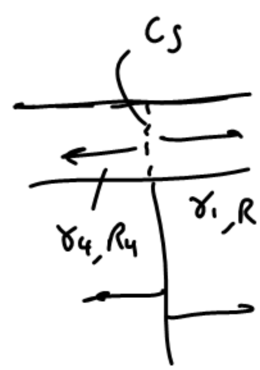
Solution:

Assumptions and Approximations:

1. Both gases are ideal gases.
2. The flow is approximated as isentropic except across the shock.
3. The diaphragm instantly disappears at $t = 0$.
4. There is no mixing of the two gases across the contact surface.
5. The duct is well insulated so that the flow is approximated as adiabatic.
6. The flow at any cross-section of the duct is approximated as one-dimensional.

(a) Calc $a_1 = \sqrt{\gamma_1 R_1 T_1} = \boxed{346.12 \text{ m/s} = a_1}$

$a_4 = \sqrt{\gamma_4 R_4 T_4} = \boxed{1014.84 \text{ m/s} = a_4}$



Ideal gas $\rho_1 = \dfrac{P_1}{R_1 T_1} = \boxed{1.1687 \ \frac{\text{kg}}{\text{m}^3} = \rho_1}$

$\rho_4 = \dfrac{P_4}{R_4 T_4} = \boxed{1.2918 \ \frac{\text{kg}}{\text{m}^3} = \rho_4}$

• **KEY** $\boxed{P_2 = P_3}$ across the CS

$\therefore$ $$\boxed{V_2 = V_3 = V_{cs} = V_{as}}$$

Get M_s (moving shock)

Get $$\frac{P_4}{P_1} = \frac{2\gamma_1 M_s^2 - (\gamma_1 - 1)}{(\gamma_1 + 1)}\left[1 - \frac{\gamma_4 - 1}{\gamma_1 + 1}\frac{a_1}{a_4}\left(M_s - \frac{1}{M_s}\right)\right]^{\frac{-2\gamma_4}{\gamma_4 - 1}}$$ ☆

Implicit in M_s. Here we know all except M_s

Must solve with software False Position Method, etc. ...

$$\boxed{M_s = 1.8926}$$

$[M_s = M_1]$

• Across the shock, can use all our old shock eq's for static properties

for a normal shock → $$\boxed{\frac{P_2}{P_1} = \frac{2\gamma_1 M_s^2 - (\gamma_1 - 1)}{\gamma_1 + 1}} = \underline{\underline{4.0121}}$$

$$\therefore \; P_2 = \frac{P_2}{P_1} P_1 = (4.0121)(100 \text{ kPa}) = \boxed{401.21 \text{ kPa} = P_2} \; \boxed{= P_3}$$

Region ②

$$\boxed{V_2 = V_{as} = \frac{2}{\gamma_1+1} a_1 \left(M_s - \frac{1}{M_s}\right)} \rightarrow \boxed{V_2 = 393.46 \text{ m/s}}$$

$$\boxed{V_3 = V_2 = V_{cs} = V_{as} = 393.46 \text{ m/s}}$$

Recall,

$$\boxed{\frac{a_3}{a_4} = 1 - \frac{\gamma_4 - 1}{2}\frac{V_3}{a_4}} = \boxed{0.87147 = \frac{a_3}{a_4}}$$

$$\boxed{a_3 = \frac{a_3}{a_4} a_4 = 884.4 \text{ m/s}}$$

[ALL THIS IS WITH GAS 4]

$$\boxed{\frac{T_3}{T_4} = \left(\frac{a_3}{a_4}\right)^2} \rightarrow \boxed{\frac{P_3}{P_4} = \left(\frac{a_3}{a_4}\right)^{\frac{2\gamma_4}{\gamma_4-1}}} \rightarrow \boxed{\frac{\rho_3}{\rho_4} = \left(\frac{a_3}{a_4}\right)^{\frac{2}{\gamma_4-1}}}$$

$$\boxed{\frac{P_3}{P_4} = \left(1 - \frac{\gamma_4-1}{2}\frac{V_{cs}}{a_4}\right)^{\frac{2\gamma_4}{\gamma_4-1}}} \star = \underline{\underline{0.50151}}$$

OR

$$\frac{P_3}{P_4} = \frac{P_3}{P_1}\frac{P_1}{P_4} = \left(\frac{P_2}{P_1}\right)\left(\frac{P_1}{P_4}\right) = \underline{\underline{0.50151}}$$

calculate across shock

given

then $P_3 = \frac{P_3}{P_4} P_4 = \boxed{401.21 \text{ kPa} = P_3 = P_2}$ ✓

Also $\quad \frac{\rho_3}{\rho_4} = \left(\frac{a_3}{a_4}\right)^{\frac{2}{\gamma_4 - 1}} = 0.66035$

$$\rho_3 = \frac{\rho_3}{\rho_4}\rho_4 = \boxed{0.85302 \ \text{kg/m}^3}$$

$$T_3 = \frac{P_3}{R_4 \rho_3} \longrightarrow \boxed{T_3 = 226.43 \ \text{K}}$$

$$a_3 = \sqrt{\gamma_4 R_4 T_3} = \boxed{884.41 \ \text{m/s} = a_3}$$

☺ ✓

<u>OR</u> $\quad a_3 = \sqrt{\gamma_4 \frac{P_3}{\rho_3}} = \boxed{884.41 \ \frac{m}{s} = a_3}$ ☺ ✓

<u>PLOT ON x-t diagram</u>

$$\boxed{V_s = M_s a_1} = \boxed{V_s = 655.03 \ \text{m/s}} = \frac{dx}{dt} \text{ on x-t diagram}$$

Plot M_s line

Plot V_{cs} line $\rightarrow$ $[V_s = 393.462 \ \text{m/s}]$

leading wave of exp. fan $\rightarrow$ plot a_4 line

trailing " ---- $\rightarrow$ plot $\underline{a_3 - V_{cs}}$ line

[see plot]

<u>Shock</u> $\quad x = V_s t \rightarrow$ hits right wall when $x = x_R = 10$ m

$$t_R = \frac{x_R}{V_{cs}} = \frac{10 \ \text{m}}{655.03 \ \text{m/s}} = \underline{0.015267 \ \text{s}} \quad (15.2 \ \text{ms})$$

Results plotted in Excel for accuracy: