Equation Sheet for ME 433

Print out for homework, quizzes, exams, and future reference.

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General and conversions: $g = 9.807\frac{\text{m}}{\text{s}^2}$, $\frac{0.3048\text{ m}}{1\text{ ft}}$, $\frac{1\text{ mile}}{1.6093\text{ m}}$, $\frac{1\text{ kPa}\cdot\text{m}^2}{1\text{ kN}}$, $\frac{1\text{ kN}\cdot\text{m}}{1\text{ kJ}}$, $\frac{1\text{ kW}\cdot\text{s}}{1\text{ kJ}}$, $\frac{1\text{ Btu}}{1.055056\text{ kJ}}$, $\frac{1\text{ kg}}{2.205\text{ lbm}}$, $\frac{1\text{ ton}}{2000\text{ lbm}}$, $\frac{1\text{ tonne (metric ton)}}{1000\text{ kg}}$, $\frac{1\text{ g}}{10^6\ \mu\text{g}}$, $\frac{1\text{ m}}{10^6\ \mu\text{m}}$, $\frac{1\text{ m}}{10^9\text{ nm}}$, $V_{\text{sphere}} = \frac{4}{3}\pi\left(R_p\right)^3 = \frac{1}{6}\pi\left(D_p\right)^3$.

Molecular weights and mols: $m = nM$, $M_{\text{air}} = 28.97\text{ g/mol}$, $M_{\text{water}} = 18.02\text{ g/mol}$, Avagadro's number: 6.0225×10^{23}.

Air at SATP: $P_{\text{SATP}} = 101.325\text{ kPa},\ T_{\text{SATP}} = 298.15\text{ K}$, $\rho = 1.184\text{ kg/m}^3$, $\mu = 1.849\times10^{-5}\text{ kg/(m s)}$, $\lambda = 0.06704\ \mu\text{m}$.

Air at *P* & *T*: $\rho = \frac{P}{R_{\text{air}}T}$, $\mu \approx \mu_s\left(\frac{T}{T_{s,0}}\right)^{3/2}\frac{T_{s,0}+T_s}{T+T_s}$, $T_{s,0} = 298.15$ K, $T_s = 110.4$ K, $\mu_s = 1.849\times10^{-5}\frac{\text{kg}}{\text{m}\cdot\text{s}}$, $\lambda = \frac{\mu}{0.499}\sqrt{\frac{\pi}{8\rho P}}$

Ideal gas: $PV = nR_uT$, $R = R_u/M$, $PV = mRT$, $P = \rho RT$, $R_u = 8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}$, $R_{\text{air}} = 0.287\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 287.0\frac{\text{J}}{\text{kg}\cdot\text{K}}$.

Volume and mass flow rate: $Q = \dot{V} = UA_c$, $\dot{m} = \rho Q = \rho\dot{V}$, $Q_{\text{standard}} = Q_{\text{actual}}\frac{P}{P_{\text{SATP}}}\frac{T_{\text{SATP}}}{T}$, $P_{\text{SATP}} = 101.325\text{ kPa} = 760\text{ mm Hg}$, $T_{\text{SATP}} = 25^\circ\text{C} = 298.15\text{ K}$.

Ideal gas mixture: $m_t = \sum_{j=1}^{J} m_j$, $n_t = \sum_{j=1}^{J} n_j$, $P = \sum_{j=1}^{J} P_j$, $V = \sum_{j=1}^{J} V_j$, $f_j = \frac{m_j}{m_t} = y_j\frac{M_j}{M_t}$, $y_j = \frac{n_j}{n_t} = \frac{P_j}{P} = \frac{V_j}{V}$, $M_j = \frac{m_j}{n_j}$,

$PV = n_tR_uT$, $P_jV = n_jR_uT$, $PV_j = n_jR_uT$, $M_t = \sum_{j=1}^{J}\left(y_jM_j\right)$, $c_j = \frac{m_j}{V}$, $c_{\text{molar},j} = \frac{n_j}{V} = \frac{c_j}{M_j}$, $c_j = y_j\frac{M_j}{R_u}\frac{P}{T}$, $\dot{m}_j = c_jQ$.

Relative humidity & vapor pressure: $RH = \Phi = \frac{P_{H_2O}}{P_{v,H_2O}}\times100\% = \frac{P_{H_2O}}{P_{\text{sat},H_2O}}\times100\%$, $y_{H_2O} = \frac{P_{H_2O}}{P_{\text{atm}}}$, $P_v = P_{\text{sat}}$ for any VOC.

First-order ODE: $\frac{dy}{dt} = B - Ay$. For a step function change and constant A and B, $t_{1/2} = \frac{-\ln(1/2)}{A} = -\ln(1/2)\tau$, $y_{ss} = \frac{B}{A}$, $\tau = 1/A$, and the solution at any time t is $y(t) = y_{ss} - \left[y_{ss} - y(0)\right]\exp(-At)$.

Emission factors (EPA AP-42): $EF = \frac{\dot{m}_{\text{pollutant}}\ \left(\text{or } m_{\text{pollutant}}\right)}{\text{some appropriate denominator}}$, $\dot{m}_{\text{d (discharged)}} = (1-\eta)\dot{m}_{\text{g (generated)}}$ or $m_d = (1-\eta)m_g$ where η = APCS removal efficiency.

Combustion of hydrocarbons: $C_xH_y + a\left(O_2 + 3.76N_2\right) \rightarrow bCO_2 + cH_2O + dN_2$, $a = a_{\text{stoich}}$ if *stoichiometric* combustion,

Dry air = 21% O_2, 79% N_2, $\text{Equivalence ratio} = \Phi = \frac{(\text{F/A})_n}{(\text{F/A})_{n,\text{ stoich}}} = \frac{1/a}{1/a_{\text{stoich}}} = \frac{a_{\text{stoich}}}{a}$ where a = actual molar coefficient.

Flux chamber: $\frac{dm_j}{dt} = V\frac{dc_j}{dt} = c_{j,a}Q_a + S_j - c_jQ_a$, $\dot{m}_{j,\text{ generated}} = S_j = \left(c_{j,ss} - c_{j,a}\right)Q_a$, $c_{j,ss} = c_{j,a} + \frac{S_j}{Q_a}$, $\tau = \frac{V}{Q_a}$,

$t_{1/2} = -\ln(1/2)\frac{V}{Q_a}$, and the solution at any time t is $c_j(t) = c_{j,ss} - \left[c_{j,ss} - c_j(0)\right]\exp\left(-\frac{Q_a}{V}t\right)$.

Tank filling: $m_{j,\text{ displaced}} = f\frac{M_jP_{v,j}}{R_uT}V_{\text{liquid in}}$, $\dot{m}_{j,\text{ displaced}} = f\frac{M_jP_{v,j}}{R_uT}Q_{\text{liquid in}}$, filling factor $f = \frac{P_j}{P_{v,j}}$, Loading factor $L_r = \frac{Q_{\text{liquid in}}}{V}$.

When a liquid puddle of species k sits at the bottom of a tank being filled with species j, emissions come from *both* j & k,

$\dot{m}_{\text{total}} = \dot{m}_{j,\text{ displaced}} + \dot{m}_{k,\text{ displaced}} = f_j\frac{M_jP_{v,j}}{R_uT}Q_{\text{liquid in}} + f_k\frac{M_kP_{v,k}}{R_uT}Q_{\text{liquid in}}$, where $f_j = \frac{P_j}{P_{v,j}}$ & $f_k = \frac{P_k}{P_{v,k}} = 1$ since k is saturated.

Gradient diffusion of *A*: $J_A = -b\frac{da}{dz}$, where $a = \frac{A}{V}$ and A = mass, energy, momentum, … For mass, $M_jJ_j = -D_{aj}\frac{dc_j}{dz}$.

Gaussian plume model: [h_s is actual stack height, δh is additional plume elevation due to buoyancy of the plume]

- Absorbing ground: $c_j = \dfrac{\dot{m}_{j,s}}{2\pi U \sigma_y \sigma_z} \exp\left\{-\dfrac{1}{2}\left[\left(\dfrac{y}{\sigma_y}\right)^2 + \left(\dfrac{z-H}{\sigma_z}\right)^2\right]\right\}$, where $H = h_s + \delta h$ = effective stack height.
- Reflecting ground: $c_j = \dfrac{\dot{m}_{j,s}}{2\pi U \sigma_y \sigma_z}\left[\exp\left\{-\dfrac{1}{2}\left[\left(\dfrac{y}{\sigma_y}\right)^2 + \left(\dfrac{z-H}{\sigma_z}\right)^2\right]\right\} + \exp\left\{-\dfrac{1}{2}\left[\left(\dfrac{y}{\sigma_y}\right)^2 + \left(\dfrac{z+H}{\sigma_z}\right)^2\right]\right\}\right]$.
- Absorbing ground w/ inversion: $c_j = \dfrac{\dot{m}_{j,s}}{2\pi U \sigma_y \sigma_z} \exp\left[-\dfrac{1}{2}\left(\dfrac{y}{\sigma_y}\right)^2\right]\left\{\exp\left[-\dfrac{1}{2}\left(\dfrac{z-H}{\sigma_z}\right)^2\right] + \exp\left[-\dfrac{1}{2}\left(\dfrac{z-(2H_T-H)}{\sigma_z}\right)^2\right]\right\}$,
  where H = effective stack height and H_T is the elevation of the reflecting part of the inversion.
- Reflecting ground w/ elevated inversion (fumigating plume):
  $$c_j = \frac{\dot{m}_{j,s}}{2\pi U \sigma_y \sigma_z} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] \sum_{i=-\infty}^{\infty}\left\{\exp\left[-\frac{1}{2}\left(\frac{z-H-2iH_T}{\sigma_z}\right)^2\right] + \exp\left[-\frac{1}{2}\left(\frac{z+H-2iH_T}{\sigma_z}\right)^2\right]\right\}$$
- Fumigating ***approximation***, reflecting ground, elevated inversion, far downwind: $c_{j,F} = \dfrac{\dot{m}_{j,s}}{\sqrt{2\pi} U \sigma_y H_T} \exp\left[-\dfrac{1}{2}\left(\dfrac{y}{\sigma_y}\right)^2\right]$.
- For **all** the above, $\sigma_y = ax^b$, $\sigma_z = cx^d + f$. *Note*: x in units of km and σ_y and σ_z in units of m. Use **Tables 1** & **2** for *a-f*.

Table 1. Stability Classifications for Calculation of Dispersion Coefficients, Gaussian Plume Model
(adapted from Martin, 1976)

Stability increases as classification letter increases: A very unstable … C & D near neutral … F very stable

Wind Speed (m/s)[1]	Daytime Incoming Solar Radiation			Nighttime Cloudiness[5]		Day or Night
	Strong[2]	Moderate[3]	Slight[4]	Cloudy (>1/2)	Clear (<1/2)	Overcast
< 2	A	A-B[6]	B	E	F	D
2-3	A-B	B	C	E	F	D
3-5	B	B-C	C	D	E	D
5-6	C	C-D	D	D	D	D
> 6	C	D	D	D	D	D

[1] Wind speed is measured at 10 m above the ground.
[2] Strong = clear summer day with sun higher than 60° above the horizon.
[3] Moderate = summer day with a few broken clouds, or a clear day with sun 35-60° above the horizon.
[4] Slight = fall afternoon, or a cloudy summer day, or a clear summer day with sun 15-35° above the horizon.
[5] Nighttime cloudiness is defined as the fraction of sky covered by clouds.
[6] For *two* stability classification letters like A-B, B-C, or C-D, ***average*** the two values obtained from Table 2.

Table 2. Curve-Fit Constants for Calculation of Dispersion Coefficients, Gaussian Plume Model
(adapted from Martin, 1976)

	All x		x < 1 km			x > 1 km		
Class	***a***	***b***	***c***	***d***	***f***	***c***	***d***	***f***
A	213	0.894	440.8	1.941	9.27	459.7	2.094	-9.6
B	156	0.894	106.6	1.149	3.3	108.2	1.098	2.0
C	104	0.894	61.0	0.911	0	61.0	0.911	0
D	68.0	0.894	33.2	0.725	-1.7	44.5	0.516	-13.0
E	50.5	0.894	22.8	0.678	-1.3	55.4	0.305	-34.0
F	34.0	0.894	14.35	0.740	-0.35	62.6	0.180	-48.6

Gaussian puff diffusion model:

- Absorbing ground: $c_j(x,y,z,t)=\frac{m_j}{\pi\sqrt{2\pi}\sigma_{xi}\sigma_{yi}\sigma_{zi}}\exp\left\{-\frac{1}{2}\left[\left(\frac{x-Ut}{\sigma_{xi}}\right)^2+\left(\frac{y}{\sigma_{yi}}\right)^2+\left(\frac{z-H}{\sigma_{zi}}\right)^2\right]\right\}$.
- Ground level dose, absorbing ground: $D_j(x,y,0)=\frac{m_j}{\pi U\sigma_{yi}\sigma_{zi}}\exp\left\{-\frac{1}{2}\left[\left(\frac{y}{\sigma_{yi}}\right)^2+\left(\frac{H}{\sigma_{zi}}\right)^2\right]\right\}$, (double for reflecting).
- For all the above, $\sigma_{xi}=\sigma_{yi}=ax^b$, $\sigma_{zi}=cx^d$. *Note*: x, σ_{yi}, and σ_{zi} are *all* in units of m. Use **Table 3** for *a*-*d*.

Table 3. Curve-Fit Constants for Instantaneous Dispersion Coefficients, Gaussian Puff Model
(adapted from Slade, 1968 as found in Heinsohn and Kabel, 1999)

Stability condition	*a*	*b*	*c*	*d*
Unstable	0.14	0.92	0.53	0.73
Neutral	0.06	0.92	0.15	0.70
Very Stable	0.02	0.89	0.05	0.61

Particles: $c_{\text{number},j}=\frac{c_j}{m_{p,\text{mean}}}$, $m_{p,\text{mean}}=\rho_p\frac{1}{6}\pi\left(D_{p,am}(\text{mass})\right)^3$, $\vec{F}_{\text{gravity}}=(\rho_p-\rho)\frac{\pi}{6}D_p^{\,3}\vec{g}$, $\vec{F}_{\text{drag}}=-\frac{\rho}{8}\frac{C_D}{C}\pi D_p^{\,2}\vec{v}_r\left|\vec{v}_r\right|$,

where v_r = **relative particle velocity**, $\vec{v}_r=\vec{v}-\vec{U}$, where $\vec{v}$ is the particle velocity and $\vec{U}$ is the air velocity.

Kn is the **Knudsen number**, λ is the **mean free path** of air molecules, and C is the **Cunningham correction factor**,

$\text{Kn}=\frac{\lambda}{D_p}$, $\lambda=\frac{\mu}{0.499}\sqrt{\frac{\pi}{8\rho P}}$, $C=1+\text{Kn}\left[2.514+0.80\exp\left(-\frac{0.55}{\text{Kn}}\right)\right]$, and $C_D=C_D(\text{Re})$, where $\text{Re}=\frac{\rho\left|\vec{v}_r\right|D_p}{\mu}$,

Stokes: $C_D=\frac{24}{\text{Re}}$ for Re < 0.1, **Morrison**: $C_D\approx\frac{24}{\text{Re}}+\frac{2.6\left(\frac{\text{Re}}{5.0}\right)}{1+\left(\frac{\text{Re}}{5.0}\right)^{1.52}}+\frac{0.411\left(\frac{\text{Re}}{2.63\times10^5}\right)^{-7.94}}{1+\left(\frac{\text{Re}}{2.63\times10^5}\right)^{-8.00}}+\frac{0.25\left(\frac{\text{Re}}{10^6}\right)}{1+\left(\frac{\text{Re}}{10^6}\right)}$ for $\text{Re}<10^6$.

Terminal settling speed: $V_t=\sqrt{\frac{4}{3}\frac{\rho_p-\rho}{\rho}gD_p\frac{C}{C_D}}$, $\text{Re}=\frac{\rho V_t D_p}{\mu}$. **Stokes flow *approx.*** $(\text{Re}<0.1)$, $V_t=\frac{\rho_p-\rho}{18}D_p^{\,2}g\frac{C}{\mu}$.

Equation of particle motion: $\frac{d\vec{v}}{dt}=\frac{\rho_p-\rho}{\rho_p}\vec{g}-\frac{3}{4}\frac{\rho}{\rho_p}\frac{C_D}{C}\frac{1}{D_p}\vec{v}_r\left|\vec{v}_r\right|$.

Grade efficiency for particulate APCS: V_t (or v_r) = fnc(D_p), so η = fnc(D_p) & **Grade efficiency**: $\eta(D_p)=1-\frac{c}{c(\text{in})}$.

Settling in box, room, container of height *H*: $t_c=H/V_t$ = critical time; **laminar and well-mixed are two extremes**:

Laminar: $\frac{c_{\text{avg}}}{c_0}=1-\frac{t}{t_c}$, $\eta(D_p)=\frac{t}{t_c}$ if $t\le t_c$; $\frac{c_{\text{avg}}}{c_0}=0$, $\eta(D_p)=1$ if $t>t_c$ Well-mixed: $\frac{c}{c_0}=\exp\left(-\frac{t}{t_c}\right)$, $\eta(D_p)=1-\exp\left(-\frac{t}{t_c}\right)$

Settling in duct: $L_c=\frac{HU}{V_t}$, Laminar: $\eta(D_p)=\frac{L}{L_c}$ if $L\le L_c$; $\eta(D_p)=1$ if $L>L_c$, Well-mixed: $\eta(D_p)=1-\exp\left(-\frac{L}{L_c}\right)$.

Non-spherical: Aero: $V_t=\sqrt{\frac{4}{3}\frac{\rho_0-\rho}{\rho}gD_{ae}\frac{C}{C_D}}$, $\rho_0=1000\frac{\text{kg}}{\text{m}^3}$, Spherical: $V_t=\sqrt{\frac{4}{3}\frac{\rho_p-\rho}{\rho}gD_{se}\frac{C}{C_D}}$, Volume: $V_p=\frac{\pi D_{ve}^{\,3}}{6}$.

Gaussian Plume with Particle Settling: $c_j=\frac{\dot{m}_{j,s}}{2\pi U\sigma_y\sigma_z}\exp\left\{-\frac{1}{2}\left[\left(\frac{y}{\sigma_y}\right)^2+\left(\frac{z-\left[H_0-(V_t x/U)\right]}{\sigma_z}\right)^2\right]\right\}$, $H_0=H$ at $x=0$.

Inertial separation devices:

Terminal radial speed, inertial separation: $v_r = \sqrt{\frac{4}{3}\frac{\rho_p - \rho}{\rho}\frac{U_\theta^2}{r_m}D_p\frac{C}{C_D}}$, $\mathrm{Re} = \frac{\rho v_r D_p}{\mu}$, where $\frac{U_\theta^2}{r_m}$ replaces g in the equations, r_m = mean radius, $x = r_m\theta$, $L_c = \frac{WU_\theta}{v_r}$, $\theta_c = \frac{L_c}{r_m}$. For **Stokes flow *approx***. ($\mathrm{Re} < 0.1$), $v_r = \frac{\rho_p - \rho}{18}D_p^2\frac{U_\theta^2}{r_m}\frac{C}{\mu}$.

Laminar settling: $\eta(D_p) = \frac{L}{L_c}$ if $L < L_c$; $\eta(D_p) = 1$ if $L > L_c$. Well-mixed settling: $\eta(D_p) = 1 - \exp\left(-\frac{L}{L_c}\right)$.

Standard Lapple cyclone: $\eta(D_p) = \frac{1}{1 + \left(D_{p,\mathrm{cut}}/D_p\right)^2}$, where $D_{p,\mathrm{cut}} = \sqrt{\frac{3\mu D_2^3}{128\pi Q(\rho_p - \rho)}}$, D_2 = overall cyclone diameter.

Pressure drop and required power: $\Delta P = 40.96\rho\left(\frac{Q}{WH}\right)^2 = 2621.44\rho\frac{Q^2}{D_2^4}$, $\dot{W}_{\mathrm{blower}} = \frac{Q\Delta P}{\eta_{\mathrm{blower}}}$, where $W = \frac{D_2}{4}$ & $H = \frac{D_2}{2}$.

Air cleaners in series and parallel: For m cleaners, j is cleaner number in series or parallel.

Parallel: Gases: $\eta_{\mathrm{overall}} = 1 - \sum_{j=1}^{m} f_j\left[1 - \eta_j\right]$ Particles: $\eta(D_p)_{\mathrm{overall}} = 1 - \sum_{j=1}^{m} f_j\left[1 - \eta(D_p)_j\right]$, where $f_j = \frac{Q_j}{Q_{\mathrm{total}}}$.

Series: Gases: $\eta_{\mathrm{overall}} = 1 - \prod_{j=1}^{m}\left[1 - \eta_j\right]$ Particles: $\eta(D_p)_{\mathrm{overall}} = 1 - \prod_{j=1}^{m}\left[1 - \eta(D_p)_j\right]$, and Q is the same through each cleaner.

Rain, Spray Chambers, and Wet Scrubbers as Air Pollution Control Systems:

Single-drop collection grade efficiency: $\eta_d(D_p) = \left(\frac{r_1}{R_c}\right)^2 = \left(\frac{Stk}{Stk + 0.35}\right)^2$, where $Stk = \frac{(\rho_p - \rho)D_p^2(U_0 - V_{t,p})}{18\mu D_c}$.

Overall collection grade efficiency: $\eta(D_p) = 1 - \exp\left(-L/L_c\right)$, where $L_c = \frac{2}{3}\frac{Q_a}{Q_s}\frac{V_c}{V_{t,c}}\frac{D_c}{\eta_d(D_p)}$, $V_c = V_{t,c} - U_a$ for a ***spray chamber***. L_c must be estimated or calculated for a ***wet scrubber*** – depends on size and shape of the ***packing material***.

Air filters: (ε = porosity, U_0 = air speed, L = filter thickness, $\eta_f(D_p)$ = single-fiber collection efficiency)

$Stk = \frac{(\rho_p - \rho)D_p^2(U_0/\varepsilon)}{18\mu D_f}$, $\eta_f(D_p) = \left(\frac{Stk}{Stk + 0.425}\right)^2$, $L_c = \frac{\pi}{4}\frac{\varepsilon}{1 - \varepsilon}\frac{D_f}{\eta_f(D_p)}$, $\eta(D_p) = 1 - \exp\left(-L/L_c\right)$.

Electrostatic Precipitators: (ESPs)

$\eta(D_p) = 1 - \exp\left(-L/L_c\right)$, where L_c is dependent on voltages, particle composition, air speed, gap widths, and many other parameters. On quizzes and exams, L_c would either be given, or would be the variable to be calculated.

Polydisperse Aerosol Particle Statistics: j is bin number.

- j = class (bin) number with range $D_{p,\mathrm{min},j} < D_p \le D_{p,\mathrm{max},j}$, width $\Delta D_{p,j}$, and mid value $D_{p,j}$ for $j = 1$ to J.
- n_j = number of particles in bin j, and n_t = total number of particles in the sample, $n_t = \Sigma n_j$.
- $f(D_{p,j})$ = fraction of particles per bin width = $n_j/(\Delta D_{p,j}\, n_t)$.

Probability of particle in bin j: $\mathrm{Prob} = f(D_{p,j})\cdot\Delta D_{p,j} = \frac{n_j}{n_t}$, **Cumulative number distribution**: $F(a) = \int_0^a f(D_{p,j})\,dD_p$.

Median diameter: $F(D_{p,50}) = 0.50$. For number distribution, use $D_{p,50}$ (number); for mass use $D_{p,50}$ (mass).

Arithmetic mean diameter: $D_{p,am} = \int_0^\infty D_p f(D_p)\,dD_p = \frac{1}{n_t}\sum_{j=1}^{J}(n_j D_{p,j})$.

Geometric mean diameter: $D_{p,gm} = \left(D_{p,1}^{n_1} D_{p,2}^{n_2} \ldots D_{p,j}^{n_j} \ldots D_{p,J}^{n_J}\right)^{\frac{1}{n_t}} = \exp\left[\frac{1}{n_t}\sum_{j=1}^{J}\left(n_j \ln(D_{p,j})\right)\right]$, $D_{p,gm} = D_{p,50} = D_{p,\mathrm{median}}$.

Geometric standard deviation: $\sigma_g = e^{\ln(\sigma_g)}$, $\ln(\sigma_g) = \sqrt{\dfrac{\sum_{j=1}^{J}\left\{n_j\left[\ln(D_{p,j}) - \ln(D_p)_{,am}\right]^2\right\}}{n_t - 1}}$, $\ln(D_p)_{,am} = \dfrac{\sum_{j=1}^{J}\left[n_j \ln(D_{p,j})\right]}{n_t}$.

Or, $\sigma_g = \dfrac{D_{p,50}}{D_{p,15.9}} = \dfrac{D_{p,84.1}}{D_{p,50}} = \sqrt{\dfrac{D_{p,84.1}}{D_{p,15.9}}}$, and σ_g is the same whether based on the number or the mass distribution. So, we can use *either* the number or mass values of $D_{p,50}$, $D_{p,15.9}$, and $D_{p,84.1}$, i.e., $\sigma_g = \dfrac{D_{p,50}(\text{number})}{D_{p,15.9}(\text{number})} = \dfrac{D_{p,50}(\text{mass})}{D_{p,15.9}(\text{mass})}$, etc., where $D_{p,gm}(\text{number}) = D_{p,50}(\text{number})$, and $D_{p,gm}(\text{mass}) = D_{p,50}(\text{mass})$.

Conversion from number distribution to mass distribution: $m_j = n_j \rho_p \dfrac{\pi}{6}(D_{p,j})^3$. **Mass fraction**: $g(D_{p,j}) = \dfrac{m_j}{m_t}$, but we plot histograms as $\dfrac{g(D_{p,j})}{\Delta D_{p,j}}$ for non-equal bin widths. **Cumulative mass distribution**: $G(a) = \int_0^a g(D_{p,j})\,dD_p$.

Since $\ln(D_{p,50}\ (\text{mass})) = \ln(D_{p,50}\ (\text{number})) + 3\left[\ln(\sigma_g)\right]^2$, $\sigma_g = \exp\left\{\sqrt{\dfrac{\left[\ln(D_{p,50}\ (\text{mass})) - \ln(D_{p,50}\ (\text{number}))\right]}{3}}\right\}$.

Overall particle removal efficiency for a single particle cleaner (grouped data) : j is bin number.

$\eta_{\text{overall}} = \sum_{j=1}^{J}\left[\eta(D_{p,j})\dfrac{m_j}{m_t}\right] = \sum_{j=1}^{J}\left[\eta(D_{p,j})\dfrac{c_j}{c_{\text{overall}}}\right]$ $\left(\dfrac{m_j}{m_t}\right)_{\text{out}} = \left(\dfrac{m_j}{m_t}\right)_{\text{in}} \dfrac{1-\eta(D_{p,j})}{1-\eta_{\text{overall}}}$ or $g(D_{p,j})_{\text{out}} = g(D_{p,j})_{\text{in}} \dfrac{1-\eta(D_{p,j})}{1-\eta_{\text{overall}}}$.