Avagadro’s number: \( N_A = 6.0225 \times 10^{23} \text{ mol}^{-1} \).

For a step function change and constant \( \mu \): \( \mu = \mu_s \), where \( \mu_s \) and the solution at any time \( t_s \), loading factor \( \lambda \).

**Gradient diffusion of A:** \( J_A = - \frac{a}{V} \frac{da}{dz} \), where \( a = \frac{A}{V} \) and \( A = \text{mass, energy, momentum, …} \). For mass, \( M J = -D_a \frac{dc}{dz} \).

**General and conversions:**
- \( g = 9.807 \text{ m/s}^2 \)
- \( 1 \text{ mile} = 1.6093 \text{ km} \)
- \( 1 \text{ kPa} \cdot \text{m}^2 = 1 \text{ kN} \cdot \text{m} \)
- \( 1 \text{ kW} \cdot \text{s} = 1 \text{ kJ} \)
- \( 1 \text{ Btu} = 1.055056 \text{ kJ} \)

**Molecular weights and mols:**
- \( m = n M \)
- \( M_{\text{air}} = 28.97 \text{ g/mol} \)
- \( M_{\text{water}} = 18.02 \text{ g/mol} \)

**Air at SATP:**
- \( P_{\text{SATP}} = 101.325 \text{ kPa} \)
- \( T_{\text{SATP}} = 298.15 \text{ K} \)

**Air at P & T:**
- \( \rho = \frac{P}{R_{\text{air}} T} \)
- \( \mu \approx \mu_s \left( \frac{T}{T_{s,0}} \right)^{3/2} \frac{T_{s,0} + T_s}{T + T_s} \), where \( T_{s,0} = 298.15 \text{ K} \), \( T_s = 110.4 \text{ K} \), \( \mu_s = 1.849 \times 10^{-5} \text{ kg/(m s)} \), \( \lambda = 0.06704 \mu \text{m} \)

**Ideal gas:**
- \( PV = nRT \)
- \( R = R_u / M \)
- \( PV = mRT \)

**Volume and mass flow rate:**
- \( Q = \dot{V} = UA \)
- \( m = \rho Q = \rho \dot{V} \)

**Ideal gas mixture:**
- \( m_j = \sum_{j=1}^{J} m_j \)
- \( n_j = \sum_{j=1}^{J} n_j \)
- \( P = \sum_{j=1}^{J} \frac{m_j}{M_j} \)

**Relative humidity & vapor pressure:**
- \( R_{\text{HH}} = \frac{P_{\text{v,HH}}}{P_{\text{sat,HH}}} \times 100\% \)
- \( R_{\text{v,HH}} = \frac{P_{\text{v,HH}}}{P_{\text{sat,HH}}} \times 100\% \)

**First-order ODE:**
- \( \frac{dy}{dt} = B - Ay \)
- For a step function change and constant \( A \) and \( B \), \( t_{1/2} = -\ln(1/2) \frac{B}{A} \), and the solution at any time \( t \) is \( y(t) = y_s - \left[ y_s - y(0) \right] \exp(-A t) \).

**Emission factors (EPA AP-42):**
- \( E_F = \frac{m_{\text{pollutant}} (\text{or } m_{\text{pollutant}})}{\text{some appropriate denominator}} \)

**Combustion of hydrocarbons:**
- \( C_x H_y + a(O_2 + 3.76 N_2) \rightarrow bCO_2 + cH_2O + dN_2 \)
- \( a = \text{stoichiometric} \)

**Dry air = 21% O_2, 79% N_2:**
- Equivalence ratio \( \Phi = \frac{(F/A)_a}{(F/A)_{a,\text{stoich}}} = \frac{1}{a} = \frac{a_{\text{stoich}}}{a} \)

**Flux chamber:**
- \( \frac{dm_j}{dt} = \dot{V} \frac{dc_j}{dt} = c_{j,a} Q_a + S_j - c_q Q_a \)
- \( m_j, \text{generated} = S_j = \left( c_{j,ss} - c_{j,a} \right) Q_a \)
- \( c_{j,ss} = c_{j,a} + \frac{S_j}{Q_a} \)

**Tank filling:**
- \( m_{j,\text{displaced}} = f_j M_j P_{v,j} \dot{V} \)
- \( m_{j,\text{generated}} = \dot{m}_j \frac{M_j P_{v,j} \dot{V}}{R_{g,j} T_{\text{liquid}}} \)
- \( f_j = 1 \) \( \text{and} \) \( f_k = 1 \) \( \text{since} \) \( k \) \( \text{is saturated} \).
Gaussian plume model: \[ h_s \text{ is actual stack height}, \quad \delta h \text{ is additional plume elevation due to buoyancy of the plume} \]

- Absorbing ground:
  \[ c_j = \frac{\dot{m}_{j,s}}{2\pi U \sigma_y \sigma_z} \exp\left\{ -\frac{1}{2}\left( \frac{y}{\sigma_y} \right)^2 + \left( \frac{z-H}{\sigma_z} \right)^2 \right\}, \text{ where } H = h_s + \delta h = \text{effective stack height.} \]

- Reflecting ground:
  \[ c_j = \frac{\dot{m}_{j,s}}{2\pi U \sigma_y \sigma_z} \exp\left\{ -\frac{1}{2}\left( \frac{y}{\sigma_y} \right)^2 + \left( \frac{z-H}{\sigma_z} \right)^2 \right\} + \exp\left\{ -\frac{1}{2}\left( \frac{y}{\sigma_y} \right)^2 + \left( \frac{z+H}{\sigma_z} \right)^2 \right\} \]

- Absorbing ground w/ inversion:
  \[ c_j = \frac{\dot{m}_{j,s}}{2\pi U \sigma_y \sigma_z} \exp\left\{ -\frac{1}{2}\left( \frac{y}{\sigma_y} \right)^2 \right\} \left\{ \exp\left[ -\frac{1}{2}\left( \frac{z-H}{\sigma_z} \right)^2 \right] + \exp\left[ -\frac{1}{2}\left( \frac{z-(2H_y-H)^2}{\sigma_z} \right)^2 \right] \right\} \]

where \( H \) = effective stack height and \( H_T \) is the elevation of the reflecting part of the inversion.

- Reflecting ground w/ elevated inversion (fumigating plume):
  \[ c_j = \frac{\dot{m}_{j,s}}{2\pi U \sigma_y \sigma_z} \left\{ \exp\left[ -\frac{1}{2}\left( \frac{z}{\sigma_z} \right)^2 \right] + \exp\left[ -\frac{1}{2}\left( \frac{z+(2H_y-H)^2}{\sigma_z} \right)^2 \right] \right\} \sum_{j=x} \]

- Fumigating approximation, reflecting ground, elevated inversion, far downwind:
  \[ c_{j,F} = \frac{\dot{m}_{j,s}}{\sqrt{2\pi U \sigma_y \sigma_z}} \exp\left[ -\frac{1}{2}\left( \frac{y}{\sigma_y} \right)^2 \right] \]

- For all the above, \( \sigma_y = ax^y, \sigma_z = cx^d + f \). Note: \( x \) in units of km and \( \sigma_y \) and \( \sigma_z \) in units of m. Use Tables 1 & 2 for \( a-f \).

### Table 1. Stability Classifications for Calculation of Dispersion Coefficients, Gaussian Plume Model
(adapted from Martin, 1976)

<table>
<thead>
<tr>
<th>Wind Speed (m/s)</th>
<th>Daytime Incoming Solar Radiation</th>
<th>Nighttime Cloudiness</th>
<th>Day or Night</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 2</td>
<td>Strong^1</td>
<td>Cloudy (1/2)</td>
<td>Overcast</td>
</tr>
<tr>
<td>2-3</td>
<td>A-B^2</td>
<td>Clear (1/2)</td>
<td>Overcast</td>
</tr>
<tr>
<td>3-5</td>
<td>B</td>
<td>E</td>
<td>D</td>
</tr>
<tr>
<td>5-6</td>
<td>C</td>
<td>F</td>
<td>D</td>
</tr>
<tr>
<td>&gt; 6</td>
<td>C</td>
<td>D</td>
<td>D</td>
</tr>
</tbody>
</table>

^1 Wind speed is measured at 10 m above the ground.

^2 Strong = clear summer day with sun higher than 60° above the horizon.

^3 Moderate = summer day with a few broken clouds, or a clear day with sun 35–60° above the horizon.

^4 Slight = fall afternoon, or a cloudy summer day, or a clear summer day with sun 15–35° above the horizon.

^5 Nighttime cloudiness is defined as the fraction of sky covered by clouds.

^6 For two stability classification letters like A-B, B-C, or C-D, average the two values obtained from Table 2.

### Table 2. Curve-Fit Constants for Calculation of Dispersion Coefficients, Gaussian Plume Model
(adapted from Martin, 1976)

<table>
<thead>
<tr>
<th>Class</th>
<th>( x &lt; 1 \text{ km} )</th>
<th>( x &gt; 1 \text{ km} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( c )</td>
<td>( d )</td>
</tr>
<tr>
<td>A</td>
<td>213.0</td>
<td>1.941</td>
</tr>
<tr>
<td>B</td>
<td>156.0</td>
<td>1.149</td>
</tr>
<tr>
<td>C</td>
<td>104.0</td>
<td>0.911</td>
</tr>
<tr>
<td>D</td>
<td>68.0</td>
<td>0.725</td>
</tr>
<tr>
<td>E</td>
<td>50.5</td>
<td>0.678</td>
</tr>
<tr>
<td>F</td>
<td>34.0</td>
<td>0.740</td>
</tr>
</tbody>
</table>
Gaussian puff diffusion model for absorbing and reflecting ground:

- **Absorbing:**
  \[ c_j(x, y, z, t) = \frac{m_j}{\pi \sqrt{2\pi \sigma_{xj} \sigma_{zj}}} \exp \left\{ -\frac{1}{2} \left( \frac{x - Ut}{\sigma_{xj}} \right)^2 + \left( \frac{y}{\sigma_{yj}} \right)^2 + \left( \frac{z - H}{\sigma_{zj}} \right)^2 \right\} \]
  but we care about dose not concentration since this is unsteady. After integration we get:
  \[ D_j(x, y, z) = \frac{m_j}{\pi U \sigma_{yj} \sigma_{zj}} \exp \left\{ -\frac{1}{2} \left( \frac{y}{\sigma_{yj}} \right)^2 + \left( \frac{z - H}{\sigma_{zj}} \right)^2 \right\} \]

- **Reflecting:**
  \[ c_j(x, y, z, t) = \frac{m_j}{\pi U \sigma_{yj} \sigma_{zj}} \exp \left\{ -\frac{1}{2} \left( \frac{y}{\sigma_{yj}} \right)^2 \right\} \exp \left\{ -\frac{1}{2} \left( \frac{x - Ut}{\sigma_{xj}} \right)^2 \right\} \exp \left\{ -\frac{1}{2} \left( \frac{z - H}{\sigma_{zj}} \right)^2 + \exp \left\{ -\frac{1}{2} \left( \frac{z + H}{\sigma_{zj}} \right)^2 \right\} \]
  \[ D_j(x, y, z) = \frac{m_j}{\pi U \sigma_{yj} \sigma_{zj}} \exp \left\{ -\frac{1}{2} \left( \frac{y}{\sigma_{yj}} \right)^2 \right\} \exp \left\{ -\frac{1}{2} \left( \frac{z - H}{\sigma_{zj}} \right)^2 + \exp \left\{ -\frac{1}{2} \left( \frac{z + H}{\sigma_{zj}} \right)^2 \right\} \]

- For all the above, \( \sigma_{xj} = \sigma_{yj} = ax^b \) and \( \sigma_{zj} = cx^d \). Note: \( x, \sigma_{yj}, \) and \( \sigma_{zj} \) are all in units of m. Use Table 3 for \( a-d \).

### Table 3. Curve-Fit Constants for Instantaneous Dispersion Coefficients, Gaussian Puff Model
(adapted from Slade, 1968 as found in Heinsohn and Kabel, 1999)

<table>
<thead>
<tr>
<th>Stability condition</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unstable</td>
<td>0.14</td>
<td>0.92</td>
<td>0.53</td>
<td>0.73</td>
</tr>
<tr>
<td>Neutral</td>
<td>0.06</td>
<td>0.92</td>
<td>0.15</td>
<td>0.70</td>
</tr>
<tr>
<td>Very Stable</td>
<td>0.02</td>
<td>0.89</td>
<td>0.05</td>
<td>0.61</td>
</tr>
</tbody>
</table>

**Particles:**
\[ c_{number,j} = \frac{c_j}{m_{p,mean}} \]  
\[ m_{p,mean} = \rho_p \frac{1}{6} \pi \left( D_{p,am} \text{ (mass)} \right) \]  
\[ \overline{F}_{gravity} = \left( \rho_p - \rho \right) \frac{\pi}{6} \rho V g \]  
\[ \overline{F}_{drag} = -\frac{\rho C_D}{8} \frac{\pi D_p^2 \rho V |\vec{V}_p|}{\rho} \]

where \( \vec{V}_p = \vec{v} - \vec{U} \), where \( \vec{v} \) is the particle velocity and \( \vec{U} \) is the air velocity.

Kn is the **Knudsen number**, \( \lambda \) is the mean free path of air molecules, and \( C \) is the Cunningham correction factor,
\[ Kn = \frac{\lambda}{D_p} \]  
\[ \lambda = \frac{\mu}{0.499} \sqrt{\frac{\pi}{8 \rho P}} \]  
\[ C = 1 + Kn \left[ 2.514 + 0.80 \exp \left( -\frac{0.55}{Kn} \right) \right] \]  
and \( C_D = C_D(Re) \), where \( Re = \frac{\rho |\vec{v}| D_p}{\mu} \).

Stokes: \( C_D = \frac{24}{Re} \) for \( Re < 0.1 \)  
Morrison: \( C_D \approx \frac{24}{Re} + \frac{2.6 \left( \frac{Re}{5.0} \right)^{0.85}}{1 + \left( \frac{Re}{5.0} \right)^{0.85}} + \frac{0.411 \left( \frac{Re}{2.63 \times 10^5} \right)^{7.94}}{1 + \left( \frac{Re}{2.63 \times 10^5} \right)^{8.00}} + \frac{0.25 \left( \frac{Re}{10^6} \right)^{7.94}}{1 + \left( \frac{Re}{10^6} \right)^{8.00}} \) for \( Re < 10^8 \).

Equation of particle motion:
\[ \frac{d\vec{v}_p}{dt} = \frac{\rho_p - \rho g}{\rho_p} - \frac{3 \rho C_D}{4 \rho_p C_D} \left( \frac{1}{D_p \rho V g} \right) |\vec{v}| \vec{v} \]

Terminal settling speed:
\[ V_t = \frac{4 \rho_p - \rho}{3 \rho} g D_p C_D \]  
Stokes flow approx. (\( Re < 0.1 \)), \( V_t = \frac{\rho_p - \rho}{18} D_p^2 g C_D \mu \)

Non-spherical: Aero: \( V_t = \frac{4 \rho_p - \rho}{3 \rho} g D_{ae} C_{D_{ae}} \rho_0 = 1000 \text{ kg/m}^3 \)  
Spherical: \( V_t = \frac{4 \rho_p - \rho}{3 \rho} g D_{se} C_{D_{se}} \rho_0 \)  
Volume: \( V_t = \frac{\pi D_{se}^3}{6} \)

Gaussian plume with particle settling (for particles the ground is always absorbing): Set \( H_0 = H \) at \( x = 0 \), and then
\[ c_j = \frac{m_j}{2\pi U \sigma_{yj} \sigma_{zj}} \exp \left\{ -\frac{1}{2} \left[ \frac{\left( \frac{y}{\sigma_{yj}} \right)^2 + \left( \frac{z - [H_0 - (V x/U)]}{\sigma_{zj}} \right)^2}{\sigma_j} \right] \right\} \]

Gaussian puff with particle settling (for particles the ground is always absorbing): Set \( H_0 = H \) at \( x = 0 \), and then
\[ D_j(x, y, z) = \frac{m_j}{\pi U \sigma_{yj} \sigma_{zj}} \exp \left\{ -\frac{1}{2} \left[ \frac{\left( \frac{y}{\sigma_{yj}} \right)^2 + \left( \frac{z - [H_0 - (V x/U)]}{\sigma_{zj}} \right)^2}{\sigma_j} \right] \right\} \]
Particle settling in containers, rooms, ducts, etc.:

Grade efficiency for particulate APCS: \( V_i \) (or \( v_c \)) = \( \text{fnc}(D_p) \), so \( \eta = \text{fnc}(D_p) \) & Grade efficiency: \[ \eta(D_p) = 1 - \frac{c}{c(\text{in})} \]

Settling in box, room, container of height \( H \): \( t_s = \frac{H}{V_i} \) = critical time; laminar and well-mixed are two extremes:

Laminar: \[ \frac{c_{\text{avg}}}{c_0} = 1 - \frac{t}{t_c}, \eta(D_p) = -\frac{L}{L_c} \text{ if } t \leq t_c; \eta(D_p) = 0, \text{ if } t > t_c \]
Well-mixed: \[ \frac{c_{\text{avg}}}{c_0} = \exp \left( -\frac{t}{t_c} \right), \eta(D_p) = 1 - \exp \left( -\frac{L}{L_c} \right) \]

Settling in duct: \( L_c = \frac{HU}{V_i} \), Laminar: \( \eta(D_p) = \frac{L_c}{L} \text{ if } L \leq L_c; \eta(D_p) = 1 \text{ if } L > L_c \), Well-mixed: \( \eta(D_p) = 1 - \exp \left( -\frac{L}{L_c} \right) \)

Inertial separation devices:

Terminal radial speed, inertial separation: \[ v_r = \sqrt{\frac{4 \rho_p - \rho}{3 \rho} \frac{U_d^2}{L_c} \frac{C}{C_D}}, \text{ Re} = \frac{\rho v_r D_p}{\mu} \]
where \( U_d^2 \) replaces \( g \) in the equations, \( r_m = \text{mean radius}, \theta = \frac{L_c}{r_m} \)

For Stokes flow \( \text{approx.} \) (\( \text{Re} < 0.1 \)): \[ v_r = \frac{\rho_p - \rho}{\rho} \frac{D_p^2}{18} \frac{C}{\mu} \]
Laminar settling: \( \eta(D_p) = \frac{L_c}{L} \text{ if } L < L_c; \eta(D_p) = 1 \text{ if } L > L_c \)
Well-mixed settling: \( \eta(D_p) = 1 - \exp \left( -\frac{L}{L_c} \right) \)

Standard Lapple cyclone: \[ \eta(D_p) = \frac{1}{1 + \left( \frac{D_p}{D_p, \text{cut}} \right)^2}, \text{ where } D_p, \text{cut} = \sqrt{\frac{3 \mu D_t^3}{128 \pi Q (\rho_p - \rho)}} \]

Pressure drop and required power: \[ \Delta P = 40.96 \rho \left( \frac{Q}{\pi H^2} \right) = 2621.44 \rho \left( \frac{Q^2}{D_t^4} \right), W_{\text{blower}} = \frac{Q \Delta P}{\eta_{\text{blower}}} \]
where \( W = \frac{D_t}{4}, H = \frac{D_t}{2} \)

Air cleaners in series and parallel: For \( m \) cleaners, \( j \) is cleaner number in series or parallel.

Parallel: Gases: \( \eta_{\text{overall}} = 1 - \sum_{j=1}^{m} f_j \left[ 1 - \eta_j \right] \)
Particles: \( \eta(D_p)_{\text{overall}} = 1 - \sum_{j=1}^{m} f_j \left[ 1 - \eta(D_p)_j \right] \)
where \( f_j = \frac{Q_j}{Q_{\text{total}}} \)

Series: Gases: \( \eta_{\text{overall}} = 1 - \prod_{j=1}^{m} \left[ 1 - \eta_j \right] \)
Particles: \( \eta(D_p)_{\text{overall}} = 1 - \prod_{j=1}^{m} \left[ 1 - \eta(D_p)_j \right] \)
and \( Q \) is the same through each cleaner.

Rain, Spray Chambers, and Wet Scrubbers as Air Pollution Control Systems:

C&E single-drop collection grade efficiency:
\[ \eta_d(D_p)_{\text{CE}} = \left( \frac{r_i}{R_c} \right)^2 \left( \frac{K_p}{K_p + 0.70} \right)^2, K_p = \frac{C(\rho_p - \rho)D_p^3}{9 \mu D_c} \left( V_{t,c} - V_{i,p} \right) \]

Slinn single-drop collection grade efficiency:
\[ \eta_d(D_p)_{\text{Slinn}} = \frac{2}{\text{Re}_c} \left[ 1 + 0.4 \left( \frac{\text{Re}_c}{2} \right)^{1/2} \text{Sc}^{1/3} + 0.16 \left( \frac{\text{Re}_c}{} \right)^{1/2} \right] \]

\[ \text{Re}_c = \frac{\rho D_c v_{t,c}}{\mu}, \text{ Sc = } \frac{3 \pi \nu D_p}{\rho k_b T C} \]

\[ k_b = 1.381 \times 10^{-23} \frac{\text{J}}{\text{K}} \]

Total (C&E and Slinn) single-drop collection grade efficiency: \[ \eta_d(D_p)_{\text{total}} = \eta_d(D_p)_{\text{CE}} + \eta_d(D_p)_{\text{Slinn}} \]

Overall collection grade efficiency for spray chambers and wet scrubbers:
\[ \eta(D_p) = 1 - \exp \left( -\frac{L}{L_c} \right) \]
where \( \eta(D_p) \) = spray chamber:
\[ L_c = \frac{2 Q_i}{3 Q_s} \frac{D_c}{\eta_d(D_p)_{\text{total}}} \]

Wet scrubber: \( L_c \) must be estimated or calculated, depending on size and shape of the packing material.

Other Air Pollution Control Systems: Many other APCSs have the same equation as above for grade removal efficiency, but each has a unique critical length that depends on particle diameter and needs to be calculated or measured,
\[ \eta(D_p) = 1 - \exp \left( -\frac{L}{L_c} \right) \]
**Air filters:** $\varepsilon =$ porosity, $U_0 =$ upstream air speed, $L =$ filter thickness, $\eta_f(D_p) =$ single-fiber collection efficiency

C&E single-fiber collection grade efficiency:

$\eta_{\text{CE}}(D_p) = \left( \frac{\rho_p - \rho}{18 \mu D_p} \right) D_p^2 (U_0 / \varepsilon)$

Cheng single-drop collection grade efficiency:

$D_{ap} = \frac{k_b TC}{3 \pi \mu D_p}$

$k_b = 1.381 \times 10^{-23} \text{ J/K}$

$\alpha = f_f = 1 - \varepsilon$

$\eta_f(D_p)_{\text{Cheng}} = 2.92 \left( \frac{1 - \alpha}{k_K} \right)^{1/3} \text{Pe}^{-2/3}$

Total (C&E and Cheng) single-fiber collection grade efficiency:

$\eta_f(D_p)_{\text{total}} = \eta_f(D_p)_{\text{CE}} + \eta_f(D_p)_{\text{Cheng}}$

Overall collection grade efficiency for air filters:

$L_e = \frac{\pi}{4} \frac{\varepsilon}{1 - \varepsilon} \frac{L_f}{D_f}$

$\eta(D_p)_{\text{total}} = 1 - \exp \left( -\frac{L_e}{L_f} \right)$

**Electrostatic Precipitators:** (ESPs)

$\eta(D_p) = 1 - \exp \left( -\frac{L_e}{L_f} \eta \right)$

$\eta_f(D_p)_{\text{ESP}} = \frac{\pi}{6} \rho \left( \frac{D_p}{P ap} \right)^3$

Polydisperse Aerosol Particle Statistics for Grouped Data: $j$ is bin (class) number and we assume lognormal distributions.

- $j =$ bin number, range $D_{p,\text{min},j} < D_p < D_{p,\text{max},j}$, width $\Delta D_{p,j}$, mid value $D_{p,j}$ for $j = 1$ to $J$.
- $n_j =$ number of particles in bin $j$, and $n = \text{total number of particles in the sample}$.
- $m_j =$ mass of particles in bin $j$, and $m = \text{total mass of particles in the sample}$.
- $f(D_{p,j}) =$ number fraction of particles per bin width for a number distribution.
- $g(D_{p,j}) =$ mass fraction of particles per bin width for a mass distribution.

**Histograms:** Since non-equal bin widths, plot $f(D_{p,j}) = \frac{n_j}{n_j \Delta D_{p,j}}$ vs. $D_{p,j}$ (number) or $g(D_{p,j}) = \frac{m_j}{m_j \Delta D_{p,j}}$ vs. $D_{p,j}$ (mass).

Probability (number): $\text{Prob}_j$ (number) $= f(D_{p,j}) \cdot \Delta D_{p,j} = \frac{n_j}{n_j}$

Cumulative number distribution: $F(a) = \int_0^a f(D_{p,j}) dD_p$

Probability (mass): $\text{Prob}_j$ (mass) $= g(D_{p,j}) \cdot \Delta D_{p,j} = \frac{m_j}{m_j}$

Cumulative mass distribution: $G(a) = \int_0^a g(D_{p,j}) dD_p$

Median diameter: $F(D_{p,50} \text{ (number)}) = 0.50$ for number distribution, $G(D_{p,50} \text{ (mass)}) = 0.50$ for mass distribution.

Arithmetic mean diameter:

$D_{p,\text{am}} = \int_0^\infty D_p f(D_p) dD_p = \frac{1}{n_j} \sum_{j=1}^J (n_j D_{p,j})$

Standard deviation: $\sigma = \sqrt{\frac{\sum_{j=1}^J \left[ n_j (D_{p,j} - D_{p,\text{am}})^2 \right]}{n - 1}}$

Geometric mean diameter:

$D_{p,\text{gm}} = \left( D_{p,1}^{n_1} D_{p,2}^{n_2} \cdots D_{p,J}^{n_J} \right)^{1/n} = \exp \left[ \frac{1}{n_j} \sum_{j=1}^J \left( n_j \ln(D_{p,j}) \right) \right]$  

$D_{p,\text{gm}} = D_{p,50} = D_{p,\text{median}}$

Geometric standard deviation:

$\sigma_g = e^{\ln(\sigma)}$

For lognormal distributions, $D_{p,\text{gm}} \text{ (number)} = D_{p,50} \text{ (number)}$ and $D_{p,\text{gm}} \text{ (mass)} = D_{p,50} \text{ (mass)}$

$\sigma_g = D_{p,50} \text{ (number)} = D_{p,50} \text{ (mass)}$

Conversions:

$\ln(D_{p,50} \text{ (mass)}) = \ln(D_{p,50} \text{ (number)}) + 3 \left[ \ln(\sigma_g) \right]^2$

$\sigma_g = \exp \left\{ \frac{\ln(D_{p,50} \text{ (mass)}) - \ln(D_{p,50} \text{ (number)})}{3} \right\}$