Equation Sheet for ME 433

For homework, quizzes, exams, and future reference.

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General and conversions: $g = 9.807 \dfrac{\text{m}}{\text{s}^2}$, $\dfrac{0.3048 \text{ m}}{1 \text{ ft}}$, $\dfrac{1 \text{ mile}}{1609.3 \text{ m}}$, $\dfrac{1 \text{ kPa}\cdot\text{m}^2}{1 \text{ kN}}$, $\dfrac{1 \text{ kN}\cdot\text{m}}{1 \text{ kJ}}$, $\dfrac{1 \text{ kW}\cdot\text{s}}{1 \text{ kJ}}$, $\dfrac{1 \text{ Btu}}{1.055056 \text{ kJ}}$,

$\dfrac{1 \text{ kg}}{2.205 \text{ lbm}}$, $\dfrac{1 \text{ ton}}{2000 \text{ lbm}}$, $\dfrac{1 \text{ tonne (metric ton)}}{1000 \text{ kg}}$, $\dfrac{1 \text{ g}}{10^6 \ \mu\text{g}}$, $\dfrac{1 \text{ m}}{10^6 \ \mu\text{m}}$, $\dfrac{1 \text{ m}}{10^9 \text{ nm}}$, $V_\text{sphere} = \dfrac{4}{3}\pi\left(R_p\right)^3 = \dfrac{1}{6}\pi\left(D_p\right)^3$.

Molecular weights and mols: $m = nM$, $M_\text{air} = 28.97 \text{ g/mol}$, $M_\text{water} = 18.02 \text{ g/mol}$, Avagadro's number: 6.0225×10^{23}.

Air at SATP: $P_\text{SATP} = 101.325 \text{ kPa}$, $T_\text{SATP} = 298.15 \text{ K}$, $\rho = 1.184 \text{ kg/m}^3$, $\mu = 1.849\times10^{-5} \text{ kg/(m s)}$, $\lambda = 0.06704 \ \mu\text{m}$.

Air at *P* & *T*: $\rho = \dfrac{P}{R_\text{air}T}$, $\mu \approx \mu_\text{s}\left(\dfrac{T}{T_{\text{s},0}}\right)^{3/2}\dfrac{T_{\text{s},0}+T_\text{s}}{T+T_\text{s}}$, $T_{\text{s},0} = 298.15$ K, $T_\text{s} = 110.4$ K, $\mu_\text{s} = 1.849\times10^{-5}\dfrac{\text{kg}}{\text{m}\cdot\text{s}}$, $\lambda = \dfrac{\mu}{0.499}\sqrt{\dfrac{\pi}{8\rho P}}$

Ideal gas: $PV = nR_uT$, $R = R_u/M$, $PV = mRT$, $P = \rho RT$, $R_u = 8.314\dfrac{\text{kJ}}{\text{kmol}\cdot\text{K}}$, $R_\text{air} = 0.287\dfrac{\text{kJ}}{\text{kg}\cdot\text{K}} = 287.0\dfrac{\text{J}}{\text{kg}\cdot\text{K}}$.

Volume and mass flow rate: $Q = \dot{V} = UA_c$, $\dot{m} = \rho Q = \rho\dot{V}$, $Q_\text{standard} = Q_\text{actual}\dfrac{P}{P_\text{SATP}}\dfrac{T_\text{SATP}}{T}$, $P_\text{SATP} = 101.325 \text{ kPa} = 760 \text{ mm Hg}$, $T_\text{SATP} = 25^\circ\text{C} = 298.15 \text{ K}$.

Ideal gas mixture: $m_t = \sum_{j=1}^{J} m_j$, $n_t = \sum_{j=1}^{J} n_j$, $P = \sum_{j=1}^{J} P_j$, $V = \sum_{j=1}^{J} V_j$, $f_j = \dfrac{m_j}{m_t} = y_j\dfrac{M_j}{M_t}$, $y_j = \dfrac{n_j}{n_t} = \dfrac{P_j}{P} = \dfrac{V_j}{V}$, $M_j = \dfrac{m_j}{n_j}$,

$PV = n_tR_uT$, $P_jV = n_jR_uT$, $PV_j = n_jR_uT$, $M_t = \sum_{j=1}^{J}\left(y_jM_j\right)$, $c_j = \dfrac{m_j}{V}$, $c_{\text{molar},j} = \dfrac{n_j}{V} = \dfrac{c_j}{M_j}$, $c_j = y_j\dfrac{M_j}{R_u}\dfrac{P}{T}$, $\dot{m}_j = c_jQ$.

Relative humidity & vapor pressure: $RH = \Phi = \dfrac{P_{H_2O}}{P_{v,H_2O}}\times100\% = \dfrac{P_{H_2O}}{P_{\text{sat},H_2O}}\times100\%$, $y_{H_2O} = \dfrac{P_{H_2O}}{P_\text{atm}}$, $P_v = P_\text{sat}$ for any VOC.

First-order ODE: $\dfrac{dy}{dt} = B - Ay$. For a step function change and constant *A* and *B*, $t_{1/2} = \dfrac{-\ln(1/2)}{A} = -\ln(1/2)\tau$, $y_{ss} = \dfrac{B}{A}$,

$\tau = 1/A$, and the solution at any time *t* is $y(t) = y_{ss} - \left[y_{ss} - y(0)\right]\exp(-At)$.

Emission factors (EPA AP-42): $EF = \dfrac{\dot{m}_\text{pollutant}\ \left(\text{or } m_\text{pollutant}\right)}{\text{some appropriate denominator}}$, $\dot{m}_\text{d (discharged)} = (1-\eta)\dot{m}_\text{g (generated)}$ or $m_d = (1-\eta)m_g$ where η = APCS removal efficiency.

Combustion of hydrocarbons: $C_xH_y + a\left(O_2 + 3.76N_2\right) \rightarrow bCO_2 + cH_2O + dN_2$, $a = a_\text{stoich}$ if *stoichiometric* combustion,

Dry air = 21% O_2, 79% N_2, Equivalence ratio $= \Phi = \dfrac{(\text{F/A})_n}{(\text{F/A})_{n,\text{ stoich}}} = \dfrac{1/a}{1/a_\text{stoich}} = \dfrac{a_\text{stoich}}{a}$ where a = actual molar coefficient.

Flux chamber: $\dfrac{dm_j}{dt} = V\dfrac{dc_j}{dt} = c_{j,a}Q_a + S_j - c_jQ_a$, $\dot{m}_{j,\text{ generated}} = S_j = \left(c_{j,ss} - c_{j,a}\right)Q_a$, $c_{j,ss} = c_{j,a} + \dfrac{S_j}{Q_a}$, $\tau = \dfrac{V}{Q_a}$,

$t_{1/2} = -\ln(1/2)\dfrac{V}{Q_a}$, and the solution at any time *t* is $c_j(t) = c_{j,ss} - \left[c_{j,ss} - c_j(0)\right]\exp\left(-\dfrac{Q_a}{V}t\right)$.

Tank filling: $m_{j,\text{ displaced}} = f\dfrac{M_jP_{v,j}}{R_uT}V_\text{liquid in}$, $\dot{m}_{j,\text{ displaced}} = f\dfrac{M_jP_{v,j}}{R_uT}Q_\text{liquid in}$, filling factor $f = \dfrac{P_j}{P_{v,j}}$, Loading factor $L_r = \dfrac{Q_\text{liquid in}}{V}$.

When a liquid puddle of species *k* sits at the bottom of a tank being filled with species *j*, emissions come from *both j* & *k*,

$\dot{m}_\text{total} = \dot{m}_{j,\text{ displaced}} + \dot{m}_{k,\text{ displaced}} = f_j\dfrac{M_jP_{v,j}}{R_uT}Q_\text{liquid in} + f_k\dfrac{M_kP_{v,k}}{R_uT}Q_\text{liquid in}$, where $f_j = \dfrac{P_j}{P_{v,j}}$ & $f_k = \dfrac{P_k}{P_{v,k}} = 1$ since *k* is saturated.

Lapse rate: $\Gamma = -\dfrac{dT}{dz}$. Dry adiabatic lapse rate (neutral atmosphere) is $\Gamma_a = \dfrac{gM}{R_u}\dfrac{k-1}{k}$.

Gradient diffusion of A: $J_A = -b\frac{da}{dz}$, where $a = \frac{A}{V}$ and A = mass, energy, momentum, … For mass, $M_j J_j = -D_{aj}\frac{dc_j}{dz}$.

Gaussian plume model: [h_s is actual stack height, δh is additional plume elevation due to buoyancy of the plume]

- Absorbing ground: $c_j = \frac{\dot{m}_{j,s}}{2\pi U\sigma_y\sigma_z}\exp\left\{-\frac{1}{2}\left[\left(\frac{y}{\sigma_y}\right)^2 + \left(\frac{z-H}{\sigma_z}\right)^2\right]\right\}$, where $H = h_s + \delta h$ = effective stack height.
- Reflecting ground: $c_j = \frac{\dot{m}_{j,s}}{2\pi U\sigma_y\sigma_z}\left[\exp\left\{-\frac{1}{2}\left[\left(\frac{y}{\sigma_y}\right)^2 + \left(\frac{z-H}{\sigma_z}\right)^2\right]\right\} + \exp\left\{-\frac{1}{2}\left[\left(\frac{y}{\sigma_y}\right)^2 + \left(\frac{z+H}{\sigma_z}\right)^2\right]\right\}\right]$.
- Absorbing ground w/ inversion: $c_j = \frac{\dot{m}_{j,s}}{2\pi U\sigma_y\sigma_z}\exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]\left\{\exp\left[-\frac{1}{2}\left(\frac{z-H}{\sigma_z}\right)^2\right] + \exp\left[-\frac{1}{2}\left(\frac{z-(2H_T-H)}{\sigma_z}\right)^2\right]\right\}$,

  where H = effective stack height and H_T is the elevation of the reflecting part of the inversion.
- Reflecting ground w/ elevated inversion (fumigating plume):

$$c_j = \frac{\dot{m}_{j,s}}{2\pi U\sigma_y\sigma_z}\exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]\sum_{i=-\infty}^{\infty}\left\{\exp\left[-\frac{1}{2}\left(\frac{z-H-2iH_T}{\sigma_z}\right)^2\right] + \exp\left[-\frac{1}{2}\left(\frac{z+H-2iH_T}{\sigma_z}\right)^2\right]\right\}$$

- Fumigating ***approximation***, reflecting ground, elevated inversion, far downwind: $c_{j,F} = \frac{\dot{m}_{j,s}}{\sqrt{2\pi}U\sigma_y H_T}\exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]$.

- For **all** the above, $\sigma_y = ax^b$, $\sigma_z = cx^d + f$. *Note*: x in units of km and σ_y and σ_z in units of m. Use **Tables 1** & **2** for *a*-*f*.

Table 1. Stability Classifications for Calculation of Dispersion Coefficients, Gaussian Plume Model
(adapted from Martin, 1976)

Stability increases as classification letter increases: A very unstable … C & D near neutral … F very stable

Wind Speed (m/s)[1]	Daytime Incoming Solar Radiation: Strong[2]	Moderate[3]	Slight[4]	Nighttime Cloudiness[5]: Cloudy (>1/2)	Clear (<1/2)	Day or Night: Overcast
< 2	A	A-B[6]	B	E	F	D
2-3	A-B	B	C	E	F	D
3-5	B	B-C	C	D	E	D
5-6	C	C-D	D	D	D	D
> 6	C	D	D	D	D	D

[1] Wind speed is measured at 10 m above the ground.
[2] Strong = clear summer day with sun higher than 60° above the horizon.
[3] Moderate = summer day with a few broken clouds, or a clear day with sun 35-60° above the horizon.
[4] Slight = fall afternoon, or a cloudy summer day, or a clear summer day with sun 15-35° above the horizon.
[5] Nighttime cloudiness is defined as the fraction of sky covered by clouds.
[6] For *two* stability classification letters like A-B, B-C, or C-D, ***average*** the two values obtained from Table 2.

Table 2. Curve-Fit Constants for Calculation of Dispersion Coefficients, Gaussian Plume Model
(adapted from Martin, 1976)

	All *x*		*x* < 1 km			*x* > 1 km		
Class	***a***	***b***	***c***	***d***	***f***	***c***	***d***	***f***
A	213	0.894	440.8	1.941	9.27	459.7	2.094	-9.6
B	156	0.894	106.6	1.149	3.3	108.2	1.098	2.0
C	104	0.894	61.0	0.911	0	61.0	0.911	0
D	68.0	0.894	33.2	0.725	-1.7	44.5	0.516	-13.0
E	50.5	0.894	22.8	0.678	-1.3	55.4	0.305	-34.0
F	34.0	0.894	14.35	0.740	-0.35	62.6	0.180	-48.6

Gaussian puff diffusion model for absorbing and reflecting ground:

- Absorbing: $c_j(x,y,z,t)=\dfrac{m_j}{\pi\sqrt{2\pi}\sigma_{xi}\sigma_{yi}\sigma_{zi}}\exp\left\{-\dfrac{1}{2}\left[\left(\dfrac{x-Ut}{\sigma_{xi}}\right)^2+\left(\dfrac{y}{\sigma_{yi}}\right)^2+\left(\dfrac{z-H}{\sigma_{zi}}\right)^2\right]\right\}$, but we care about ***dose*** not concentration since this is unsteady. After integration we get: $D_j(x,y,z)=\dfrac{m_j}{\pi U\sigma_{yi}\sigma_{zi}}\exp\left\{-\dfrac{1}{2}\left[\left(\dfrac{y}{\sigma_{yi}}\right)^2+\left(\dfrac{z-H}{\sigma_{zi}}\right)^2\right]\right\}$
- Reflecting: $c_j(x,y,z,t)=\dfrac{m_j}{\pi\sqrt{2\pi}\sigma_{xi}\sigma_{yi}\sigma_{zi}}\exp\left[-\dfrac{1}{2}\left(\dfrac{y}{\sigma_{yi}}\right)^2\right]\exp\left[-\dfrac{1}{2}\left(\dfrac{x-Ut}{\sigma_{xi}}\right)^2\right]\left\{\exp\left[-\dfrac{1}{2}\left(\dfrac{z-H}{\sigma_{zi}}\right)^2\right]+\exp\left[-\dfrac{1}{2}\left(\dfrac{z+H}{\sigma_{zi}}\right)^2\right]\right\}$

$$D_j(x,y,z)=\frac{m_j}{\pi U\sigma_{yi}\sigma_{zi}}\exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_{yi}}\right)^2\right]\left\{\exp\left[-\frac{1}{2}\left(\frac{z-H}{\sigma_{zi}}\right)^2\right]+\exp\left[-\frac{1}{2}\left(\frac{z+H}{\sigma_{zi}}\right)^2\right]\right\}$$

- For all the above, $\sigma_{xi}=\sigma_{yi}=ax^b$, $\sigma_{zi}=cx^d$. *Note*: x, σ_{yi}, and σ_{zi} are *all* in units of m. Use **Table 3** for *a*-*d*.

Table 3. Curve-Fit Constants for Instantaneous Dispersion Coefficients, Gaussian Puff Model
(adapted from Slade, 1968 as found in Heinsohn and Kabel, 1999)

Stability condition	*a*	*b*	*c*	*d*
Unstable	0.14	0.92	0.53	0.73
Neutral	0.06	0.92	0.15	0.70
Very Stable	0.02	0.89	0.05	0.61

Particles: $c_{\text{number},j}=\dfrac{c_j}{m_{p,\text{mean}}}$, $m_{p,\text{mean}}=\rho_p\dfrac{1}{6}\pi\left(D_{p,am}(\text{mass})\right)^3$, $\vec{F}_{\text{gravity}}=(\rho_p-\rho)\dfrac{\pi}{6}D_p^{\,3}\vec{g}$, $\vec{F}_{\text{drag}}=-\dfrac{\rho}{8}\dfrac{C_D}{C}\pi D_p^{\,2}\vec{v}_r|\vec{v}_r|$,

where v_r = **relative particle velocity**, $\vec{v}_r=\vec{v}-\vec{U}$, where $\vec{v}$ is the particle velocity and $\vec{U}$ is the air velocity.
Kn is the **Knudsen number**, λ is the **mean free path** of air molecules, and C is the **Cunningham correction factor**, $\text{Kn}=\dfrac{\lambda}{D_p}$, $\lambda=\dfrac{\mu}{0.499}\sqrt{\dfrac{\pi}{8\rho P}}$, $C=1+\text{Kn}\left[2.514+0.80\exp\left(-\dfrac{0.55}{\text{Kn}}\right)\right]$, and $C_D=C_D(\text{Re})$, where $\text{Re}=\dfrac{\rho|\vec{v}_r|D_p}{\mu}$,

Stokes: $C_D=\dfrac{24}{\text{Re}}$ for $\text{Re}<0.1$, **Morrison**: $C_D\approx\dfrac{24}{\text{Re}}+\dfrac{2.6\left(\frac{\text{Re}}{5.0}\right)}{1+\left(\frac{\text{Re}}{5.0}\right)^{1.52}}+\dfrac{0.411\left(\frac{\text{Re}}{2.63\times10^5}\right)^{-7.94}}{1+\left(\frac{\text{Re}}{2.63\times10^5}\right)^{-8.00}}+\dfrac{0.25\left(\frac{\text{Re}}{10^6}\right)}{1+\left(\frac{\text{Re}}{10^6}\right)}$ for $\text{Re}<10^6$.

Equation of particle motion: $\dfrac{d\vec{v}}{dt}=\dfrac{\rho_p-\rho}{\rho_p}\vec{g}-\dfrac{3}{4}\dfrac{\rho}{\rho_p}\dfrac{C_D}{C}\dfrac{1}{D_p}\vec{v}_r|\vec{v}_r|$.

Terminal settling speed: $V_t=\sqrt{\dfrac{4}{3}\dfrac{\rho_p-\rho}{\rho}gD_p\dfrac{C}{C_D}}$, $\text{Re}=\dfrac{\rho V_t D_p}{\mu}$. **Stokes flow *approx.*** ($\text{Re}<0.1$), $V_t=\dfrac{\rho_p-\rho}{18}D_p^{\,2}g\dfrac{C}{\mu}$.

Non-spherical: Aero: $V_t=\sqrt{\dfrac{4}{3}\dfrac{\rho_0-\rho}{\rho}gD_{ae}\dfrac{C}{C_D}}$, $\rho_0=1000\,\dfrac{\text{kg}}{\text{m}^3}$, Spherical: $V_t=\sqrt{\dfrac{4}{3}\dfrac{\rho_p-\rho}{\rho}gD_{se}\dfrac{C}{C_D}}$, Volume: $V_p=\dfrac{\pi D_{ve}^{\,3}}{6}$.

Gaussian plume with particle settling (for particles the ground is always absorbing): Set $H_0=H$ at $x=0$, and then

$$c_j=\frac{\dot{m}_{j,s}}{2\pi U\sigma_y\sigma_z}\exp\left\{-\frac{1}{2}\left[\left(\frac{y}{\sigma_y}\right)^2+\left(\frac{z-\left[H_0-(V_t x/U)\right]}{\sigma_z}\right)^2\right]\right\}.$$

Gaussian puff with particle settling (for particles the ground is always absorbing): Set $H_0=H$ at $x=0$, and then

$$D_j(x,y,z)=\frac{m_j}{\pi U\sigma_{yi}\sigma_{zi}}\exp\left\{-\frac{1}{2}\left[\left(\frac{y}{\sigma_{yi}}\right)^2+\left(\frac{z-\left[H_0-(V_t x/U)\right]}{\sigma_{zi}}\right)^2\right]\right\}.$$

Particle settling in containers, rooms, ducts, etc.:

Grade efficiency for particulate APCS: V_t (or v_r) = fnc(D_p), so η = fnc(D_p) & **Grade efficiency**: $\eta\left(D_p\right)=1-\frac{c}{c(\text{in})}$.

Settling in box, room, container of height ***H***: $t_c = H/V_t$ = critical time; **laminar and well-mixed are two extremes**:

Laminar: $\frac{c_{\text{avg}}}{c_0}=1-\frac{t}{t_c},\ \eta\left(D_p\right)=\frac{t}{t_c}\ \text{if}\ t\le t_c;\ \frac{c_{\text{avg}}}{c_0}=0,\ \eta\left(D_p\right)=1\ \text{if}\ t>t_c$ Well-mixed: $\frac{c}{c_0}=\exp\left(-\frac{t}{t_c}\right),\ \eta\left(D_p\right)=1-\exp\left(-\frac{t}{t_c}\right)$

Settling in duct: $L_c=\frac{HU}{V_t}$, Laminar: $\eta\left(D_p\right)=\frac{L}{L_c}\ \text{if}\ L\le L_c;\ \ \eta\left(D_p\right)=1\ \text{if}\ L>L_c$, Well-mixed: $\eta\left(D_p\right)=1-\exp\left(-\frac{L}{L_c}\right)$.

Inertial separation devices:

Terminal radial speed, inertial separation: $v_r=\sqrt{\frac{4}{3}\frac{\rho_p-\rho}{\rho}\frac{U_\theta^2}{r_m}D_p\frac{C}{C_D}}$, $\text{Re}=\frac{\rho v_r D_p}{\mu}$, where $\frac{U_\theta^2}{r_m}$ replaces g in the equations, r_m = mean radius, $x=r_m\theta$, $L_c=\frac{WU_\theta}{v_r}$, $\theta_c=\frac{L_c}{r_m}$. For **Stokes flow *approx***. (Re < 0.1), $v_r=\frac{\rho_p-\rho}{18}D_p^2\frac{U_\theta^2}{r_m}\frac{C}{\mu}$.

Laminar settling: $\eta\left(D_p\right)=\frac{L}{L_c}\ \text{if}\ L<L_c;\ \ \eta\left(D_p\right)=1\ \text{if}\ L>L_c$. Well-mixed settling: $\eta\left(D_p\right)=1-\exp\left(-\frac{L}{L_c}\right)$.

Standard Lapple cyclone: $\eta\left(D_p\right)=\frac{1}{1+\left(D_{p,\text{cut}}/D_p\right)^2}$, where $D_{p,\text{cut}}=\sqrt{\frac{3\mu D_2^3}{128\pi Q\left(\rho_p-\rho\right)}}$, D_2 = overall cyclone diameter.

Pressure drop and required power: $\Delta P=40.96\rho\left(\frac{Q}{WH}\right)^2=2621.44\rho\frac{Q^2}{D_2^4}$, $\dot{W}_{\text{blower}}=\frac{Q\Delta P}{\eta_{\text{blower}}}$, where $W=\frac{D_2}{4}$ & $H=\frac{D_2}{2}$.

Air cleaners in series and parallel: For m cleaners, j is cleaner number in series or parallel.

Parallel: Gases: $\eta_{\text{overall}}=1-\sum_{j=1}^{m}f_j\left[1-\eta_j\right]$ Particles: $\eta\left(D_p\right)_{\text{overall}}=1-\sum_{j=1}^{m}f_j\left[1-\eta\left(D_p\right)_j\right]$, where $f_j=\frac{Q_j}{Q_{\text{total}}}$.

Series: Gases: $\eta_{\text{overall}}=1-\prod_{j=1}^{m}\left[1-\eta_j\right]$ Particles: $\eta\left(D_p\right)_{\text{overall}}=1-\prod_{j=1}^{m}\left[1-\eta\left(D_p\right)_j\right]$, and Q is the same through each cleaner.

Rain, Spray Chambers, and Wet Scrubbers as Air Pollution Control Systems:

C&E single-drop collection grade efficiency: $\eta_d\left(D_p\right)_{\text{CE}}=\left(\frac{r_1}{R_c}\right)^2=\left(\frac{K_p}{K_p+0.70}\right)^2$, $K_p=\frac{C_p\left(\rho_p-\rho\right)D_p^2\left|V_{t,c}-V_{t,p}\right|}{9\mu D_c}$.

Slinn single-drop collection grade efficiency: $\eta_d\left(D_p\right)_{\text{Slinn}}=\frac{2}{\text{Re}_c\,\text{Sc}}\left[1+0.4\left(\frac{\text{Re}_c}{2}\right)^{1/2}\text{Sc}^{1/3}+0.16\left(\frac{\text{Re}_c\,\text{Sc}}{2}\right)^{1/2}\right]$,

$\text{Re}_c=\frac{\rho D_c V_{t,c}}{\mu}$, $\text{Sc}=\frac{3\pi\mu^2 D_p}{\rho k_B T C_p}$, $k_B=1.381\times10^{-23}\ \frac{\text{J}}{\text{K}}$.

Total (C&E *and* Slinn) single-drop collection grade efficiency: $\eta_d\left(D_p\right)_{\text{total}}=\eta_d\left(D_p\right)_{\text{CE}}+\eta_d\left(D_p\right)_{\text{Slinn}}$.

Overall collection grade efficiency for spray chambers and wet scrubbers: $\eta\left(D_p\right)=1-\exp\left(-L/L_c\right)$, where

Spray chamber: $L_c=\frac{2}{3}\frac{Q_a}{Q_s}\frac{V_c}{V_{t,c}}\frac{D_c}{\eta_d\left(D_p\right)_{\text{total}}}$, $V_c=V_{t,c}-U_a$.

Wet scrubber: L_c must be estimated or calculated, depending on size and shape of the ***packing material***.

Other Air Pollution Control Systems: Many other APCSs have the same equation as above for grade removal efficiency, but each has a unique critical length that depends on particle diameter and needs to be calculated or measured,

$\eta\left(D_p\right)=1-\exp\left(-L/L_c\right)$.

Air filters: (ε = porosity, U_0 = upstream air speed, L = filter thickness, $\eta_f(D_p)$ = single-fiber collection efficiency)

C&E single-fiber collection grade efficiency: $Stk = \frac{(\rho_p - \rho) D_p^2 (U_0/\varepsilon)}{18 \mu D_f}$, $\eta_f(D_p)_{CE} = \left(\frac{Stk}{Stk + 0.425}\right)^2$.

Cheng single-drop collection grade efficiency: $D_{ap} = \frac{k_B T C_p}{3\pi\mu D_p}$, $k_B = 1.381 \times 10^{-23}\ \frac{\text{J}}{\text{K}}$, $\alpha = f_f = 1 - \varepsilon$,

$k_K = -0.5\ln(\alpha) + \alpha - 0.25\alpha^2 - 0.75$, $\text{Pe} = \frac{D_f U_0}{D_{ap}}$, $\eta_f(D_p)_{\text{Cheng}} = 2.92\left(\frac{1-\alpha}{k_K}\right)^{1/3} \text{Pe}^{-2/3}$.

Total (C&E *and* Cheng) single-fiber collection grade efficiency: $\eta_f(D_p)_{\text{total}} = \eta_f(D_p)_{\text{CE}} + \eta_f(D_p)_{\text{Cheng}}$.

Overall collection grade efficiency for air filters: $L_c = \frac{\pi}{4}\frac{\varepsilon}{1-\varepsilon}\frac{D_f}{\eta_f(D_p)_{\text{total}}}$, $\eta(D_p) = 1 - \exp\left(-L/L_c\right)$.

Electrostatic Precipitators: (ESPs) $\eta(D_p) = 1 - \exp\left(-\frac{L}{L_c}\right) = 1 - \exp\left(-\frac{w_t A_{s,\text{plates}}}{Q_{\text{ESP}}}\right)$, w_t = terminal drift speed or migration speed, $A_{s,\text{plates}}$ = total surface area of collector plates, Q_{ESP} = total ***actual*** volume flow rate through the ESP.

Polydisperse Aerosol Particle Statistics for Grouped Data: j is bin (class) number and we assume lognormal distributions.

- j = bin number, range $D_{p,\min,j} < D_p \le D_{p,\max,j}$, width $\Delta D_{p,j}$, mid value $D_{p,j}$ for $j = 1$ to J.
- n_j = number of particles in bin j, and n_t = total number of particles in the sample, $n_t = \Sigma n_j$.
- m_j = mass of particles in bin j, and m_t = total mass of particles in the sample, $m_t = \Sigma m_j$.
- $f(D_{p,j})$ = number fraction of particles per bin width for a **number distribution**.
- $g(D_{p,j})$ = mass fraction of particles per bin width for a **mass distribution**.

$m_j = n_j \rho_p \frac{\pi (D_{p,j})^3}{6}$

Histograms: Since non-equal bin widths, plot $f(D_{p,j}) = \frac{n_j}{n_t \Delta D_{p,j}}$ vs. $D_{p,j}$ (number) or $g(D_{p,j}) = \frac{m_j}{m_t \Delta D_{p,j}}$ vs. $D_{p,j}$ (mass).

Probability (number): $\text{Prob}_j\ (\text{number}) = f(D_{p,j}) \cdot \Delta D_{p,j} = \frac{n_j}{n_t}$, **Cumulative number distribution**: $F(a) = \int_0^a f(D_{p,j}) dD_p$.

Probability (mass): $\text{Prob}_j\ (\text{mass}) = g(D_{p,j}) \cdot \Delta D_{p,j} = \frac{m_j}{m_t}$, **Cumulative mass distribution**: $G(a) = \int_0^a g(D_{p,j}) dD_p$.

Median diameter: $F(D_{p,50}\ (\text{number})) = 0.50$ for number distribution, $G(D_{p,50}\ (\text{mass})) = 0.50$ for mass distribution.

Arithmetic mean diameter: $D_{p,am} = \int_0^\infty D_p f(D_p) dD_p = \frac{1}{n_t}\sum_{j=1}^{J}(n_j D_{p,j})$. **Standard deviation**: $\sigma = \sqrt{\frac{\sum_{j=1}^{J}\left[n_j (D_{p,j} - D_{p,am})^2\right]}{n_t - 1}}$.

Geometric mean diameter: $D_{p,gm} = \left(D_{p,1}^{\ n_1} D_{p,2}^{\ n_2} \ldots D_{p,j}^{\ n_j} \ldots D_{p,J}^{\ n_J}\right)^{\frac{1}{n_t}} = \exp\left[\frac{1}{n_t}\sum_{j=1}^{J}\left(n_j \ln(D_{p,j})\right)\right]$, $D_{p,gm} = D_{p,50} = D_{p,\text{median}}$.

Geometric standard deviation: $\sigma_g = e^{\ln(\sigma_g)}$, $\ln(\sigma_g) = \sqrt{\frac{\sum_{j=1}^{J}\left\{n_j\left[\ln(D_{p,j}) - \ln(D_p)_{,am}\right]^2\right\}}{n_t - 1}}$, $\ln(D_p)_{,am} = \frac{\sum_{j=1}^{J}\left[n_j \ln(D_{p,j})\right]}{n_t}$,

For ***lognormal*** distributions, $D_{p,gm}(\text{number}) = D_{p,50}(\text{number})$ and $D_{p,gm}(\text{mass}) = D_{p,50}(\text{mass})$,

$$\sigma_g = \frac{D_{p,50}(\text{number})}{D_{p,15.9}(\text{number})} = \frac{D_{p,50}(\text{mass})}{D_{p,15.9}(\text{mass})} \text{ or } \sigma_g = \frac{D_{p,84.1}(\text{number})}{D_{p,50}(\text{number})} = \frac{D_{p,84.1}(\text{mass})}{D_{p,50}(\text{mass})} \text{ or } \sigma_g = \sqrt{\frac{D_{p,84.1}(\text{number})}{D_{p,15.9}(\text{number})}} = \sqrt{\frac{D_{p,84.1}(\text{mass})}{D_{p,15.9}(\text{mass})}}.$$

Conversions: $\ln(D_{p,50}\ (\text{mass})) = \ln(D_{p,50}\ (\text{number})) + 3\left[\ln(\sigma_g)\right]^2$, $\sigma_g = \exp\left\{\sqrt{\frac{\left[\ln(D_{p,50}\ (\text{mass})) - \ln(D_{p,50}\ (\text{number}))\right]}{3}}\right\}$.