ME 433
Spring Semester, 2020
Homework Set # 6
Professor J. M. Cimbala

1. (10 pts) In class we discussed the Gaussian puff diffusion model, but equations were given only for the ground-absorbing case. In terms of mass concentration, our equation for the perfectly ground-absorbing case was

\[
c_j(x, y, z, t) = \frac{m_j}{\pi \sigma_x \sigma_y \sigma_z} \exp \left\{ -\frac{1}{2} \left[ \frac{(x-Ut)^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} + \frac{(z-H)^2}{\sigma_z^2} \right] \right\}
\]

(a) Think about how this equation would change if the ground were perfectly reflecting rather than perfectly absorbing, and write out the equation.

(b) As a check, show from your equation that at ground level (\(z = 0\)), the ground-reflecting case has twice the mass concentration as the ground-absorbing case.

2. (10 pts) A small particle is moving through the air at velocity \(\vec{v}\) as sketched to the right. At the location of the particle, and at a given instant in time, the air velocity is \(\vec{U}\) as also sketched.

(a) Carefully analyze and draw the direction of the aerodynamic drag force on the particle at this instant in time. [The magnitude is arbitrary – the direction is what is important.]

(b) Now suppose that the gravitational force on the particle (weight minus buoyancy force) in the downward direction is half the magnitude of the drag force. To scale, analyze and draw the direction of the acceleration of the particle at this instant in time. [In the end, all we really care about is the direction of the acceleration vector – the magnitude is arbitrary, but make sure that the gravitational force is half the magnitude of the aerodynamic drag force.]

3. (20 pts) Here is a fun problem. We all know that with each breath, a human being inhales air (mostly nitrogen and oxygen with a small amount of CO\(_2\)) and then exhales air (again mostly nitrogen but with less oxygen and more CO\(_2\) since our bodies convert oxygen into carbon dioxide as part of our metabolic process). In this problem we analyze how much weight (actually mass) a person loses each day due to breathing. As a reasonable approximation, consider that an average person at rest inhales 18 L/min of air, and exhales the same amount. The mol fraction of CO\(_2\) in the atmosphere right now is around 415 PPM; this is what we inhale. Experiments show that the mol fraction of CO\(_2\) exhaled by the average person is about 3.6 \times 10^4 PPM. Give all answers to three significant digits.

(a) Calculate the mass concentration (in units of mg/m\(^3\)) of CO\(_2\) in inhaled and exhaled air. Assume SATP conditions.

(b) Briefly explain why we lose weight just by sitting there breathing.

(c) Estimate the net loss of mass (in kg and lbm) of a typical person in one day due to the conversion of O\(_2\) into CO\(_2\) due to breathing.

(d) Repeat for a case where the person is exercising, with a breathing rate of 45 L/min, all else being equal.

(e) Finally, what other gas(es) besides CO\(_2\) do we exhale that contributes to a net weight loss due to breathing?

Note: There is another page. →
4. (25 pts) Maintenance workers are applying some nasty herbicides on the grass at a park. The tank containing the herbicide suddenly explodes at ground level, the herbicide vaporizes, and a puff of poisonous gas gets carried downwind. The total mass of herbicide vapor released is 0.72 kg. The atmosphere is very stable on this particular day, and a gentle breeze is blowing at 2.0 m/s. The ground absorbs the gas immediately upon contact. Our goal is to estimate the area along the ground where people are likely to have serious respiratory problems due to inhaling the gas. For this particular gas, the median hazardous dose is 7000 mg s/m³, which is defined as the dose at which half of the people would have serious respiratory problems from inhaling this dose. In notation consistent with our class notes, let’s call this $D_{\text{median hazardous}} = 7000 \text{ mg s/m}^3$.

(a) For this accident, estimate the maximum downwind distance $x$ (in meters) at ground level along the centerline such that 50% of people closer than this distance are likely to have serious respiratory problems.

(b) Plot the hazardous zone ($y$ vs. $x$) downwind of the release in which 50% of the people in that area are likely to suffer severe respiratory problems. To verify your results, at $x = 500$ m, you should get a $y$ value between 4 and 5 m (and between -5 and -4 m due to symmetry). Your plot of the hazardous area (looking down from above) should be shaped somewhat like an ellipse.

5. (35 pts) This is a continuation of a previous homework problem (Homework 5, Problem 4). Use the same plume properties, the same atmosphere, the same safe exposure level, etc. as in the previous problem. Consider only the ground-reflecting case.

(a) Consider at the lateral direction, $y$. The goal is to plot an area on the ground ($y$ vs. $x$) downwind of the plant in which the ground-level mass concentration of the air pollutant exceeds the safe limit. As explained in class, we expect this to be some kind of an ellipse or perhaps a football-shaped area. This will be considered the “hazardous area,” in which people living there are exposed to potentially harmful concentrations of the air pollutant. Do some algebra to solve for $y$ as a function of the other variables, setting the mass concentration equal to the safe limit, and generate an explicit equation for $y$ as a function of the other variables and parameters. Be sure to show your work (algebra) along with your final equation. As a check before continuing, calculate the $y$ values at $x = 5.0$ km. You should get a $y$ value between 600 and 700 m (0.60 to 0.70 km) and the negative of this since the hazardous area is symmetric about the $x$-axis. Calculate $y$ for various values of $x$, and then generate and print out a plot of the hazardous area. For consistency, plot $x$ (km) and $y$ (km) and adjust the scales as necessary to show a nice plot.

(b) Discuss your results, especially in light of all the assumptions and approximations inherent in the Gaussian plume model. Is the chemical plant company in danger of lawsuits? Discuss a little bit of ethics, namely, how you could modify your calculations to make things look better or worse? [I am not suggesting that you would do this, but there are a lot of assumptions and approximations in these kinds of calculations, and I am looking for a discussion of how the results could be made to sway (“spun”) one way or the other.]