Today, we will:

- Continue discussing the Coriolis effect and do an example problem
- Discuss lapse rate and how it affects atmospheric stability

Coriolis effect (continued) – Summary from previous lecture:

The Coriolis force is an apparent force that a moving object “feels” when observed in a rotating reference frame. The Coriolis force is perpendicular to the direction of motion. When the roundabout is rotating counterclockwise (mathematically positive when looking from above), the Coriolis force appears, to someone in the rotating reference frame, to pull the object to the right. In reality, however, the object is actually moving in a straight line – as observed by someone in a stationary (non-rotating) reference frame.

Math and equations:

It turns out that the Coriolis acceleration $\tilde{a}_c$ and the Coriolis force $\tilde{F}_c$ are expressed as vectors with cross products as follows:

$$\tilde{a}_c = -2(\tilde{\Omega} \times \tilde{U})$$

and from Newton’s Law,

$$\tilde{F}_c = m\tilde{a}_c = -2m(\tilde{\Omega} \times \tilde{U})$$

Cross product illustrated for our counterclockwise rotating roundabout: Consider the case where we throw a ball from the middle of the roundabout towards the outside rim.

We apply the right-hand rule to get the direction of the cross product. But due to the negative sign in the above equations, the Coriolic force is in the opposite direction:
Coriolis forces on the earth are more complex because Earth is a sphere, but same principles apply and we use the same right-hand rule to figure out which direction moving objects veer as they travel.

Bottom line:

- Objects in the **Northern Hemisphere** veer *to the right* [clockwise] due to the Coriolis force.
- Objects in the **Southern Hemisphere** veer *to the left* [counterclockwise] due to the Coriolis force.
- This includes any objects moving relative to the earth’s surface – balls, airplanes, bullets, even parcels of air itself (wind!) as in the above sketch.

The actual wind patterns on the earth are quite complicated, due to three-dimensional effects (the earth is nearly spherical with a thin atmosphere relative to the earth’s radius), instabilities caused by temperature effects (convection cells called *Hadley cells* form), and Coriolis forces.
Global wind patterns:

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Recall this picture of smoke plumes with tracers [when we discussed acid rain]. This shows that plumes in the USA veer to the north and east due to the prevailing westerly winds:

The Science Behind the Polar Vortex

The polar vortex is a large area of low pressure and cold air surrounding the Earth’s North and South poles. The term vortex refers to the counterclockwise flow of air that helps keep the colder air close to the poles (left globe). Often during winter in the Northern Hemisphere, the polar vortex will become less stable and expand, sending cold Arctic air southward over the United States with the jet stream (right globe). The polar vortex is nothing new — in fact, it’s thought that the term first appeared in an 1853 issue of E. Littell’s Living Age.
Example: Coriolis Force on a merry-go-round

Given: Punxsutawney Phil ($m = 7.50$ kg, $16.5$ lbm) is riding a merry-go-round on Groundhog Day. The merry-go-round’s radius is $R = 15.0$ m and it rotates at $\Omega = 14.0$ rpm. He stands at the edge of the ride ($r = R$) and holds on to a rail to keep from flying off.

(a) To do: Calculate the magnitude of the Coriolis acceleration in g’s experienced by Phil ($“g” = \frac{a_c}{g}$, where $g = 9.807$ m/s$^2$).

Solution: The Coriolis force is $\vec{F}_c = m\vec{a}_c = -2m(\vec{\Omega} \times \vec{U})$.

(b) To do: Calculate the radial force (in units of lbf to 3 significant digits) that Phil needs to exert on the rail to keep from flying off. Note: $1$ N = $0.2248$ lbf.

Solution:

(c) To do: Phil is angry about all the attention and chucks a reporter’s camera horizontally at $67.11$ mph ($30$ m/s). Calculate the initial value of the magnitude of the Coriolis acceleration acting on the camera from Phil’s perspective, in g’s.

Solution: Recall, the Coriolis force is $\vec{F}_c = m\vec{a}_c = -2m(\vec{\Omega} \times \vec{U})$.

In this problem, $m = 7.50$ kg, $\Omega = 1.4661$ rad/s, and one “g” = $9.807$ m/s$^2$. 
The stability of the atmosphere is determined in exactly the same way as the “ball on the floor” analogy. Namely,

- If the air parcel \textit{returns to} its original location (there is a restoring force), this is a \textit{stable atmosphere}.
- If the air parcel \textit{diverges from} its original location (there is no restoring force, but rather a diverging force that makes it move even farther from the original location), this is an \textit{unstable atmosphere}.
- If the air parcel \textit{stays put} at its new (perturbed) location (there is neither a restoring force nor a diverging force), this is a \textit{neutrally stable atmosphere}.

To figure out which of these stability conditions holds, we look at a \textit{free-body diagram} of the air parcel at its perturbed location:

\begin{center}
\includegraphics[width=0.7\textwidth]{free_body_diagram}
\end{center}

Bottom line for stability of this air parcel that has been perturbed \textit{upward}:

- If $\rho_p > \rho_a$, the net force is \textit{down} and the air parcel \textit{returns to} its original location. This is a \textit{stable atmosphere}.
- If $\rho_p < \rho_a$, the net force is \textit{up} and the air parcel \textit{diverges from} its original location. This is an \textit{unstable atmosphere}.
- If $\rho_p = \rho_a$, the net force is \textit{zero} and the air parcel \textit{stays put} at its perturbed location. This is a \textit{neutrally stable atmosphere}. 


Atmospheric stability based on lapse rate:

- Dry adiabatic lapse rate (neutrally stable, 9.8°C/km)
- Normal (standard) lapse rate (6.5°C/km)
- Subadiabatic lapse rate (stable)
- Superadiabatic lapse rate (unstable)
- Temperature inversion (extremely stable)

$T$ (temperature) vs. $z$ (elevation)