Today, we will:
- Continue discussing the **Coriolis effect** and do an example problem
- Discuss **lapse rate** and how it affects **atmospheric stability**

**Coriolis effect (continued) – Summary from previous lecture:**

The **Coriolis force** is an *apparent* force that a moving object “feels” when observed in a **rotating reference frame**. The Coriolis force is perpendicular to the direction of motion. When the roundabout is rotating **counterclockwise** (mathematically positive when looking from above), the Coriolis force appears, to someone in the **rotating reference frame**, to pull the object **to the right**. In reality, however, the object is actually moving in a **straight line** – as observed by someone in a **stationary** (non-rotating) **reference frame**.

**Math and equations:**

It turns out that the **Coriolis acceleration** $\vec{a}_c$ and the **Coriolis force** $\vec{F}_c$ are expressed as **vectors** with **cross products** as follows:

$$\vec{a}_c = -2(\vec{\Omega} \times \vec{U})$$

and from Newton’s Law,

$$\vec{F}_c = m\vec{a}_c = -2m(\vec{\Omega} \times \vec{U})$$

Cross product illustrated for our counterclockwise rotating roundabout: Consider the case where we throw a ball from the middle of the roundabout towards the outside rim.

We apply the right-hand rule to get the direction of the cross product. But due to the **negative sign** in the above equations, **the Coriolis force is in the opposite direction**: 

**Angular velocity vector along the z-axis**

**Counter clockwise rotation**

**Our setup**

**Apply the right-hand rule**
Coriolis forces on the earth are more complex because Earth is a sphere, but same principles apply and we use the same right-hand rule to figure out which direction moving objects veer as they travel.

Bottom line:
- Objects in the Northern Hemisphere veer to the right [clockwise] due to the Coriolis force.
- Objects in the Southern Hemisphere veer to the left [counterclockwise] due to the Coriolis force.
- This includes any objects moving relative to the earth’s surface – balls, airplanes, bullets, even parcels of air itself (wind!) as in the above sketch.

The actual wind patterns on the earth are quite complicated, due to three-dimensional effects (the earth is nearly spherical with a thin atmosphere relative to the earth’s radius), instabilities caused by temperature effects (convection cells called Hadley cells form), and Coriolis forces.
Global wind patterns:

**HADLEY CELL**

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Recall this picture of smoke plumes with tracers [when we discussed acid rain]. This shows that plumes in the USA veer to the north and east due to the prevailing westerly winds:

**The Science Behind the Polar Vortex**

The polar vortex is a large area of low pressure and cold air surrounding the Earth's North and South poles. The term vortex refers to the counterclockwise flow of air that helps keep the colder air close to the poles (left globe). Often during winter in the Northern Hemisphere, the polar vortex will become less stable and expand, sending cold Arctic air southward over the United States with the jet stream (right globe). The polar vortex is nothing new — in fact, it's thought that the term first appeared in an 1853 issue of E. Littell's Living Age.

Air pressure and winds around the Arctic switch between these two phases (Arctic Oscillation) and contribute to winter weather patterns.
Example: Coriolis Force on a merry-go-round

**Given:** Punxsutawney Phil ($m = 7.50 \text{ kg}, 16.5 \text{ lbm}$) is riding a merry-go-round on Groundhog Day. The merry-go-round’s radius is $R = 15.0 \text{ m}$ and it rotates at $\Omega = 14.0 \text{ rpm}$. He stands at the edge of the ride ($r = R$) and holds on to a rail to keep from flying off.

\[ \Omega = \left( 14.0 \text{ rpm} \right) \left( \frac{2\pi \text{ rad}}{60 \text{ s}} \right) = 1.4661 \text{ rad/s} \]

(a) **To do:** Calculate the magnitude of the Coriolis acceleration in g’s experienced by Phil [“g” = $a_c/g$, where $g = 9.807 \text{ m/s}^2$].

**Solution:** The Coriolis force is

\[ \vec{F}_c = m\vec{a}_c = -2m\left( \vec{\Omega} \times \vec{U} \right) \]

(b) **To do:** Calculate the radial force (in units of lbf to 3 significant digits) that Phil needs to exert on the rail to keep from flying off. *Note: 1 N = 0.2248 lbf.*

**Solution:**

\[ \vec{F} = ma = m\vec{a} = \frac{V^2}{R} = \frac{(30 \text{ m/s})^2}{15.0 \text{ m}} = 60 \text{ N} \]

Centripetal Force

\[ \vec{F} = \frac{N}{0.2248 \text{ lbf}} = 54.4 \text{ lbf} \]

(c) **To do:** Phil is angry about all the attention and chucks a reporter’s camera horizontally at 67.11 mph (30 m/s). Calculate the initial value of the magnitude of the Coriolis acceleration acting on the camera from Phil’s perspective, in g’s.

**Solution:** Recall, the Coriolis force is

\[ \vec{F}_c = m\vec{a}_c = -2m\left( \vec{\Omega} \times \vec{U} \right) \]

In this problem, $m = 7.50 \text{ kg}$, $\Omega = 1.4661 \text{ rad/s}$, and one “g” = 9.807 m/s$^2$.

\[ |\vec{a}_c| = -2r\Omega \times \vec{U} \]

\[ g's = \frac{2r\Omega}{g} = \frac{2(1.4661 \text{ rad/s})(30 \text{ m/s})}{9.807 \text{ m/s}^2} = 8.97 \text{ g's} \]

\[ \text{DO NOT CONFUSE CORIOLIS FORCE WITH CENTRIFUGAL (CENTRIPETAL) FORCE!} \]
Some basics of meteorology (continued):

- Coriolis effect and global wind patterns
- Atmospheric stability

In the atmosphere, stability of the atm depends on slope of $T$ vs $z$.
The stability of the atmosphere is determined in exactly the same way as the “ball on the floor” analogy. Namely,

- If the air parcel returns to its original location (there is a restoring force), this is a **stable atmosphere**.
- If the air parcel diverges from its original location (there is no restoring force, but rather a diverging force that makes it move even farther from the original location), this is an **unstable atmosphere**.
- If the air parcel stays put at its new (perturbed) location (there is neither a restoring force nor a diverging force), this is a **neutrally stable atmosphere**.

To figure out which of these stability conditions holds, we look at a *free-body diagram* of the air parcel at its perturbed location:

Bottom line for stability of this air parcel that has been perturbed upward:
- If $\rho_p > \rho_a$, the net force is down and the air parcel returns to its original location. This is a **stable atmosphere**.
- If $\rho_p < \rho_a$, the net force is up and the air parcel diverges from its original location. This is an **unstable atmosphere**.
- If $\rho_p = \rho_a$, the net force is zero and the air parcel stays put at its perturbed location. This is a **neutrally stable atmosphere**.

For a neutral atmosphere, $\rho_p = \rho_a$. 