Today, we will:
- Finish talking about gradient diffusion and discuss the Reynolds analogy
- Begin to discuss the Gaussian plume model  

Examples of the One-Dimensional Diffusion Equation:

<table>
<thead>
<tr>
<th>A and ( a ) (the property with a gradient)</th>
<th>( J_A ) (the amount of ( A ) diffused per unit area per unit time)</th>
<th>( b ) (diffusion coefficient)</th>
<th>1-D gradient diffusion equation</th>
</tr>
</thead>
</table>
| \( A = \) energy \( = mC_pT \)
\[ a = \frac{m}{V} C_p T = \rho C_p T \]
\[ a = \rho C_p T \]
So, we have a gradient of temperature | \( J_A = q = \) heat flux
\[ q = \text{rate of heat (energy) transfer per unit area} \]
\( \{ q \} = \left\{ \frac{\text{energy}}{\text{area} \cdot \text{time}} \right\} \)
\( \kappa = \) thermal diffusivity
\[ \kappa = \frac{k}{\rho C_p} \]
\( \{ \kappa \} = \left\{ \frac{\text{length}^2}{\text{time}} \right\} \)
\[ q = -\kappa \frac{d}{dz} \left( \rho C_p T \right) \]
or
\[ q = -k \frac{dT}{dz} \]
Heat diffusion equation |

| A = magnitude of momentum \( = mU \)
\[ a = \frac{A}{V} = \frac{m}{V} U = \rho U \]
\[ a = \rho U \]
So, we have a gradient of magnitude of velocity or speed | \( J_A = -\tau = \) shear stress
\[ -\tau = \text{rate of momentum transfer per unit area} \]
\( \{-\tau\} = \left\{ \frac{\text{momentum}}{\text{area} \cdot \text{time}} \right\} \)
\( \nu = \) kinematic viscosity
\[ \nu = \frac{\mu}{\rho} \]
\( \{ \nu \} = \left\{ \frac{\text{length}^2}{\text{time}} \right\} \)
\[ -\tau = -\nu \frac{d}{dz} \left( \rho U \right) \]
or
\[ -\tau = -\mu \frac{dU}{dz} \]
Equation for 1-D shear stress
Note: The negative sign on \( \tau \) is due to the fluid mechanics sign convention |

| A = number of mols of species \( j = n_j \)
\[ a = \frac{n_j}{V} = c_{\text{molar},j} \]
\[ a = c_{\text{molar},j} \]
So, we have a gradient of species (mols, but we can also think of it as a gradient of species mass) | \( J_A = J_j = \) molar flux
\[ J_j = \text{rate of transfer of mols of species} j \text{ per unit area} \]
\( \{ J_j \} = \left\{ \frac{\text{mols}}{\text{area} \cdot \text{time}} \right\} \)
\( D_{aj} = \) binary diffusion coefficient between air and species \( j \)
\[ D_{aj} = \left\{ \frac{\text{length}^2}{\text{time}} \right\} \]
\[ J_j = -D_{aj} \frac{d}{dz} \left( c_{\text{molar},j} \right) \]
Fick’s law in terms of molar concentration
Note: Can instead be written in terms of mass concentration |

\[ M_j J_j = -D_{aj} \frac{d}{dz} \left( c_j \right) \]
**Nondimensional Ratios of Diffusion Coefficients:**

Recall, when the one-dimensional diffusion equations for energy, momentum, and mass (species) are written properly, the diffusion coefficient for each case has the same dimensions. Namely:

Energy (heat):

\[ q = -\kappa \frac{d}{dz} \left( \rho C_p T \right) \]

\[ \{\kappa\} = \begin{cases} \text{length}^2 & \text{time} \end{cases} \]

Momentum:

\[ -\tau = -\nu \frac{d}{dz} (\rho U) \]

\[ \{\nu\} = \begin{cases} \text{length}^2 & \text{time} \end{cases} \]

Species:

\[ J_j = -D_{aj} \frac{d}{dz} (c_{\text{molar},j}) \]

\[ \{D_{aj}\} = \begin{cases} \text{length}^2 & \text{time} \end{cases} \]

So… since all three of these coefficients have the same dimensions, we can create nondimensional parameters out of them by taking ratios of any two of them:

We define the following nondimensional **laminar** diffusion ratios:

- **Sc** = Schmidt number: \( Sc = \frac{\nu}{D_{aj}} \) (ratio of momentum diffusion to species diffusion)
- **Le** = Lewis number: \( Le = \frac{\kappa}{D_{aj}} \) (ratio of heat energy diffusion to species diffusion)
- **Pr** = Prandtl number: \( \text{Pr} = \frac{\nu}{\kappa} \) (ratio of momentum diffusion to heat energy diffusion)

How to interpret these ratios? Take Schmidt number as an example:

- If \( Sc \) is **small**, momentum diffuses much more **slowly** than species \( (\nu << D_{aj}) \).
- If \( Sc \) is **large**, momentum diffuses much more **rapidly** than species \( (\nu >> D_{aj}) \).

**Example: Physical significance of Prandtl number**

**Given:** A laminar boundary layer on a flat plate with velocity and thermal BLs sketched.

**To do:** Choose the correct description of Prandtl number for this fluid:

\[ Pr = \frac{\nu}{\kappa} \]

\[ \text{e.g. water} \rightarrow Pr \approx 7 \]

\[ \text{air} \rightarrow Pr = 0.7 \]

A. \( Pr < 1 \)
B. \( Pr \approx 1 \)
C. \( Pr > 1 \)
Reynolds analogy for laminar cases – Energy, momentum, and mass, all diffuse in similar fashion, due to molecular diffusion. Compare:

Suddenly heated wall [$T = T_0 = 0^\circ C$ everywhere, then suddenly $T = T_i$ at the wall.]

- **Energy** is diffused upward.
- Rate of diffusion depends on $\kappa$ (thermal diffusivity).

Suddenly moving wall [$U = U_0 = 0$ m/s everywhere, then suddenly $U = U_i$ at the wall.]

- **Momentum** is diffused upward.
- Rate of diffusion depends on $\nu$ (kinematic viscosity).

Sudden removal of a membrane [$c_{\text{molar}} = c_{\text{molar},0} = 0$ mol/m$^3$ everywhere, then suddenly $c_{\text{molar}} = c_{\text{molar},i}$ at the location of the membrane, and the membrane disappears suddenly).]

- **Mass** (species) is diffused upward.
- Rate of diffusion depends on $D_{ij}$ (binary diffusion coefficient).

Reynolds Analogies – all 3 are similar – diffusion rate depends on diffusion coeff.
RESULT: Laminar diffusion is very slow
Turbulent diffusion (when turn on the fan) is much more rapid.
Example: Review of basic chemistry and conversions

**Given:** We know that we emit carbon dioxide into the atmosphere when we drive our cars. Approximate gasoline as mostly **octane, C_{8}H_{18}**. The molecular weight of gasoline is about 110 kg/kmol and its specific gravity is about 0.75. The molecular weight of CO\textsubscript{2} is about 44.0 kg/kmol.

**To do:** Approximating the combustion of gasoline as ideal combustion of octane, estimate how many kg of CO\textsubscript{2} you emit into the atmosphere for every gallon of gasoline you burn.

**Solution:** This conversion factor is useful: \(1 \text{ m}^3 = 264.17 \text{ gallons}\).

\[
\text{Stoichiometric chemistry} \rightarrow \begin{array}{c}
8 \text{CO}_2 + \text{air} \rightarrow 8 \text{CO}_2 + \text{N}_2
\end{array}
\]

\[
\text{8 CO}_2 \text{ mols emitted for every mol of octane burned}
\]

\[
\frac{\text{m}_{\text{CO}_2 \text{ emitted}}}{\text{gal of octane}} = \frac{\text{m}_{\text{CO}_2}}{\text{gal out}} = \frac{n_{\text{CO}_2} M_{\text{CO}_2}}{\lambda} = 8 \frac{n_{\text{oct}} M_{\text{CO}_2}}{\lambda} = 8 \frac{M_{\text{CO}_2}}{n_{\text{oct}}}
\]

\[
\frac{\text{m}_{\text{CO}_2}}{\text{gal of octane}} = 8 \frac{M_{\text{CO}_2}}{M_{\text{oct}}}
\]

\[
= 8 \left( 750 \frac{\text{kg}}{\text{m}^3} \right) \left( \frac{110 \text{ kg/mol}}{\text{mol}} \right) \left( \frac{1 \text{ m}^3}{264.17 \text{ gal}} \right) = 9.09 \frac{\text{kg}}{\text{gal}}
\]

\[
1 \text{ gal of gasoline \rightarrow m}_{\text{CO}_2} = \left( 750 \frac{\text{kg}}{\text{m}^3} \right) \left( \frac{1 \text{ m}^3}{264.17 \text{ gal}} \right) = 2.84 \frac{\text{kg}}{\text{gal}}
\]

\[
\text{Emit} = 3 \times \text{the m}_{\text{CO}_2} \text{ of the gasoline is CO}_2
\]