Today, we will:
- Modify the Gaussian solution for a **buoyant plume**, and do some examples/applications
- Compare a ground that **absorbs** the pollutant vs. one that does **not absorb** (**reflects**) the pollutant

**Gaussian plume model** (steady with no buoyancy and no ground effect):

\[
\begin{align*}
    c_j &= \frac{m_{j,s}}{2\pi U \sigma_y \sigma_z} \exp\left\{-\frac{1}{2} \left[ \left( \frac{y}{\sigma_y} \right)^2 + \left( \frac{z}{\sigma_z} \right)^2 \right]\right\}, \\
    \sigma_y &= ax^b, \quad \sigma_z = cx^d + f, \quad x \text{ in units of km and } \sigma_y \text{ and } \sigma_z \text{ in units of m.}
\end{align*}
\]

Coordinate transformation to make \( z = 0 \) on the ground:

We simply use \( z - h_s \) in place of \( z \) in the above equation! We get:

\[
\begin{align*}
    c_j &= \frac{m_{j,s}}{2\pi U \sigma_y \sigma_z} \exp\left\{-\frac{1}{2} \left[ \left( \frac{y}{\sigma_y} \right)^2 + \left( \frac{z - h_s}{\sigma_z} \right)^2 \right]\right\}.
\end{align*}
\]

Modification for a **buoyant plume**:
- Most plumes from smoke stacks are hot, and rise before they level off.
- Notation: Let \( \delta h = \) the **additional** elevation (above the stack exit) due to plume buoyancy.
- Thus, the total plume height, measured from the ground (once the plume levels off), is the sum of stack height \( h_s \) and this additional buoyancy height \( \delta h \). We define \( H = h_s + \delta h \), where \( H \) is the **effective stack height** of the plume, accounting for buoyancy, as sketched.
Coordinate transformation to keep \( z = 0 \) on the ground, and move the source up as shown:

We simply use \( z - H \) in place of \( z - h_s \) in the above equation! We get:

Modified solution for a buoyant plume:

\[
\sigma_y
\sigma_y
\frac{m_{j,s}}{2\pi U \sigma_y \sigma_z} \exp \left\{ -\frac{1}{2} \left[ \frac{y}{\sigma_y} \right]^2 + \left[ \frac{z - H}{\sigma_z} \right]^2 \right\}
\]

Note: The above equation is for the case of a ground that absorbs the pollutant. We call this an absorbing ground.

People at A do not breathe the pollution

People at B breather some pollution

The portion of plume under the ground

This part is "absorbed" by the ground
Example: Atmospheric stability classification, Martin Model

Given: It is a clear summer day at noon. The average wind speed is 6 m/s.

To do:
(a) What stability class (A, B, C, D…) would we use for the Gaussian plume model?
(b) What value of constant $c$ should we use at $x = 2.0$ km to determine the dispersion coefficient?

Solution:

Table 1. Stability Classifications for Calculation of Dispersion Coefficients, Gaussian Plume Model
(adapted from Martin, 1976)

<table>
<thead>
<tr>
<th>Stability Classifications</th>
<th>A: Very unstable</th>
<th>B: Moderately unstable</th>
<th>C: Slightly unstable</th>
<th>D: Neutrally stable</th>
<th>E: Slightly stable</th>
<th>F: Stable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind Speed (m/s)$^1$</td>
<td>Daytime</td>
<td>Incoming Solar Radiation</td>
<td>Nighttime Cloudiness$^3$</td>
<td>Day or Night Overcast</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&lt; 2</td>
<td>A</td>
<td>A-B$^6$</td>
<td>B</td>
<td>E</td>
<td>F</td>
<td>D</td>
</tr>
<tr>
<td>2-3</td>
<td>A-B</td>
<td>B</td>
<td>C</td>
<td>E</td>
<td>F</td>
<td>D</td>
</tr>
<tr>
<td>3-5</td>
<td>B</td>
<td>B-C</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>D</td>
</tr>
<tr>
<td>5-6</td>
<td>C</td>
<td>C-D</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>&gt; 6</td>
<td>C</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
</tr>
</tbody>
</table>

$^1$ Wind speed is measured at 10 m above the ground.
$^2$ Corresponds to a clear summer day with sun higher than 60° above the horizon.
$^3$ Corresponds to a summer day with a few broken clouds, or a clear day with sun 35-60° above the horizon.
$^4$ Corresponds to a fall afternoon or a cloudy summer day with the sun higher 15-35° above the horizon.
$^5$ Cloudiness is defined as the fraction of sky covered by clouds.
$^6$ For two stability classification letters like A-B, B-C, or C-D, average the two values obtained from Table 2.

Calc. $c$ @ $x = 2.0$ km, Class C

Table 2. Curve-Fit Constants for Calculation of Dispersion Coefficients, Gaussian Plume Model
(adapted from Martin, 1976)

<table>
<thead>
<tr>
<th>Class</th>
<th>All x</th>
<th>$x &lt; 1$ km</th>
<th>$x &gt; 1$ km</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a$</td>
<td>$b$</td>
<td>$c$</td>
</tr>
<tr>
<td>A</td>
<td>213</td>
<td>0.894</td>
<td>440.8</td>
</tr>
<tr>
<td>B</td>
<td>156</td>
<td>0.894</td>
<td>106.6</td>
</tr>
<tr>
<td>C$^5$</td>
<td>104</td>
<td>0.894</td>
<td>61.0</td>
</tr>
<tr>
<td>D</td>
<td>68.0</td>
<td>0.894</td>
<td>33.2</td>
</tr>
<tr>
<td>E</td>
<td>50.5</td>
<td>0.894</td>
<td>22.8</td>
</tr>
<tr>
<td>F</td>
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<td>0.894</td>
<td>14.35</td>
</tr>
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</table>

$c = 61.0$ — repeat for $a$, $b$, $d$, $f$
Ground effects (reflecting vs. absorbing ground):

We expect \( C_j \) to be higher at ground level.

We use the method of images.

With ground absorption,

\[
\frac{c_j}{2\pi U \sigma_y \sigma_z} \exp \left\{ -\frac{1}{2} \left[ \frac{y}{\sigma_y} \right]^2 + \left( \frac{z-H}{\sigma_z} \right)^2 \right\}
\]

At centerline, \( y=0 \)
At ground, \( z=0 \)

With ground reflection,

\[
\frac{c_j}{2\pi U \sigma_y \sigma_z} \left[ \exp \left\{ -\frac{1}{2} \left[ \frac{y}{\sigma_y} \right]^2 + \left( \frac{z-H}{\sigma_z} \right)^2 \right\} + \exp \left\{ -\frac{1}{2} \left[ \frac{y}{\sigma_y} \right]^2 + \left( \frac{z+H}{\sigma_z} \right)^2 \right\} \right]
\]

Summary of Gaussian plume model: (where \( H = h_s + \delta h \) = effective stack height)

At centerline, \( y=0 \)
At ground, \( z=0 \)
Example: Gaussian plume dispersion coefficients

Given: A plant emits air pollution from a stack under the following conditions:
- daytime with moderate solar radiation (summer day with a few clouds)
- average wind speed = 4.0 m/s

To do:
(a) What stability class is this?
(b) What value of constant $d$ should we use at $x = 2.0$ km to determine the dispersion coefficient?

Solution:
(a) Use Table 1 to determine the stability class (see table below).

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<td>Day or Night Overcast</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1-2</td>
<td>Strong$^2$</td>
<td>A</td>
<td>Cloudy (&gt;/2)</td>
<td>D</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-3</td>
<td>Moderate$^3$</td>
<td>A-B</td>
<td>E</td>
<td>F</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-5</td>
<td>Slight$^4$</td>
<td>B</td>
<td>D</td>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5-6</td>
<td>Strong</td>
<td>C</td>
<td>D</td>
<td>D</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&gt; 6</td>
<td>Strong</td>
<td>C-D</td>
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Example: Gaussian plume dispersion coefficients

Given: A plant emits air pollution from a stack under the following conditions:

- Daytime with moderate solar radiation (summer day with a few clouds)
- Average wind speed = 4.0 m/s

To do:
(a) What stability class is this?
(b) What value of constant \( d \) should we use at \( x = 2.0 \) km to determine the dispersion coefficient?

Solution:
(b) Use Table 2 to determine constant \( d \) at \( x = 2.0 \) km (see table below).

### Table 2. Curve-Fit Constants for Calculation of Dispersion Coefficients, Gaussian Plume Model
(adapted from Martin, 1976)

<table>
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\[
\bar{d} = \frac{(1.098 + 0.911)}{2} = \frac{2.0099}{2} = 1.0049 = d
\]

Repeat for \( a, b, c, e \)

Then, we have to calculate:

\[
\begin{align*}
\sigma_{y} &= a \times b \\
\sigma_{z} &= c \times d + f
\end{align*}
\]