Today, we will:

- Discuss the **Gaussian puff diffusion model** (sudden burst of air pollution from a point)
- Do some example problems.

**Example: Fumigating Gaussian plume**

**Given:** A buoyant plume emitting air pollution, under the following conditions:

- Stack height = 80 m. Buoyant plume rise = 20 m above stack exit.
- The stack emits the air pollutant at a rate of 110 g/s.
- An elevated temperature inversion is present, extending from 120 m to 140 m.
- The average wind speed is a gentle 1.4 m/s.
- Both above and below the temperature inversion, the atmosphere is very unstable, and is classified as Class A.
- Far downstream, the mass concentration of the air pollutant is *well mixed* (constant) vertically between the ground and the bottom of the elevated temperature inversion, and people who are downwind of the plume are fumigated, as sketched.
- The ground *reflects* (does not absorb) the air pollutant.

**To do:** At the centerline of the plume ($y = 0$), and at a downwind distance of 2.0 km, estimate the mass concentration of the pollutant experienced by people near the ground.

**Solution:**

- Use Table 2 to obtain the coefficients for calculation of dispersion coefficients: For Class A, we have $a = 213, \ b = 0.894$.
- At a given $x$ location, calculate the dispersion coefficient in the $y$ direction:
  \[
  \sigma_y = ax^b, \quad \text{with } x \text{ in units of km and } \sigma_y \text{ and } \sigma_z \text{ in units of m.}
  \]

- Use the reflecting ground fumigating Gaussian plume equation at $y = 0$ (centerline) to calculate the well-mixed mass concentration at this particular value of $x$:

\[
C_{j,F} = \frac{\dot{m}_{j,s}}{\sqrt{2\pi U \sigma_y H_T}} \exp \left[ -\frac{1}{2} \left( \frac{y}{\sigma_y} \right)^2 \right] \quad \text{at } y = 0, \quad C_{j,F} = \frac{\dot{m}_{j,s}}{\sqrt{2\pi U \sigma_y H_T}}
\]

*BE CAREFUL WITH UNITS. YOU SHOULD GET $C_{j,F} = 660 \ \text{mg} \ \text{m}^{-3}$*
Gaussian puff diffusion, absorbing ground:

Top view:

Diffusion in x direction

= \ldots \ y \ldots

WE CARE ABOUT THE DOSE THAT YOU ARE EXPOSED TO

D_{eq} = \text{integrate over time}
Equations for the Gaussian puff diffusion model (also on equation sheet):

Mass concentration of species \( j \), absorbing ground:

\[
c_j(x, y, z, t) = \frac{m_j}{\pi \sqrt{2 \pi} \sigma_x \sigma_y \sigma_z} \exp \left\{ -\frac{1}{2} \left[ \frac{x - Ut}{\sigma_x} \right]^2 + \left( \frac{y}{\sigma_y} \right)^2 + \left( \frac{z - H}{\sigma_z} \right)^2 \right\}
\]

where \( \sigma_x = \sigma_y = ax^b \), \( \sigma_z = cx^d \).

The analysis is similar to Gaussian plumes. Note, however, \( \sigma_x \) and \( \sigma_y \) and \( \sigma_z \) are in units of m, but \( x \) is in units of m (not km).

Use these empirical values for the instantaneous diffusion coefficients, depending on atmospheric stability conditions:

We use a simpler atmospheric stability model than the one used for the Gaussian plume:

\[
\text{Only 3 stability cases: } a, b, c \text{ and } d \text{ work: } - (n+1)
\]

Table 3. Curve-Fit Constants for Instantaneous Dispersion Coefficients, Gaussian Puff Model
(adapted from Slade, 1968 as found in Heinsohn and Kabel, 1999)

<table>
<thead>
<tr>
<th>Stability condition</th>
<th>( a )</th>
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<tbody>
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Consider a case in which \( H \) is small compared to \( x \) (explosion is at or near the ground).

If you are standing on the ground at some location \((x, y, z) = (x, y, 0)\), what is your exposure?

\[
\text{DOSE} = \int_{0}^{\infty} \text{Integral exposure over time } \, dt
\]
Define $D_j = \text{total dose}$ of species $j$ at some location $(x,y,z)$; $D_j = D_j(x,y,z)$:

$$D_j(x,y,z) = \int_{t=0}^{t=\infty} c_j(x,y,z,t) \, dt$$

We consider the dose on the ground ($z = 0$) only. Plugging in (1) into this integral yields

Gaussian puff diffusion – ground level dose of species $j$, absorbing ground:

$$D_j(x,y,0) = \frac{m_j}{\pi U \sigma_y \sigma_z} \exp \left\{ -\frac{1}{2} \left[ \frac{y}{\sigma_y} \right]^2 + \frac{H^2}{\sigma_z^2} \right\}$$  \hspace{1cm} (2a)

For a reflecting ground, we add a mirror image source under the ground, just like we did for the Gaussian plume model.

By symmetry, the dose along the ground is simply doubled for the ground reflecting case:

Gaussian puff diffusion – ground level dose of species $j$, reflecting ground:

$$D_j(x,y,0) = \frac{2m_j}{\pi U \sigma_y \sigma_z} \exp \left\{ -\frac{1}{2} \left[ \frac{y}{\sigma_y} \right]^2 + \frac{H^2}{\sigma_z^2} \right\}$$  \hspace{1cm} (2r)
Example: Gaussian puff diffusion

Given: A ground-level tank containing 10 kg of hydrogen cyanide (HCN) ruptures at a chemical plant early in the morning. The atmosphere is very stable, and a gentle breeze is blowing at \( U = 1.5 \text{ m/s} \). The ground absorbs the HCN on contact. Workers downwind of the explosion are exposed to the HCN.

To do:

(a) Estimate the dose of HCN that would constitute hazardous conditions for the workers. In other words, estimate the maximum safe dose \( D_{j,\text{max}} \) in units of mg \( \cdot \) s/m\(^3\).

Solution:

Look up HCN on NIOSH Pocket Guide (SDS) -

\[ ST = \frac{5}{m^3} \rightarrow ST \text{ defined for } 15\text{ min exposure} \]

Calc. \( D_{j,\text{max}} = \left( \frac{5}{m^3} \right) \left( 15 \text{ min} \right) \left( \frac{60}{1\text{ min}} \right) = 4500 \text{ mg} \cdot \text{s/m}^3 \)

(b) Predict the ground level dose directly downwind at \( x = 1.5 \text{ km} \).

Solution:

First determine what instantaneous dispersion coefficients to use.

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\[ \sigma_y \approx 4x^b = 0.02 \left( 1500 \right)^{0.83} = 13.42 \text{ m} \]

\[ \sigma_z \approx cx^d = x^{3.23} = 4.329 \text{ m} \]

Then use the equation for ground level dose, absorbing ground:

\[ D_j(x,y,0) = \frac{m_j}{\pi U \sigma_y \sigma_z} \exp \left\{ -\frac{1}{2} \left[ \frac{(x - x_0)^2}{\sigma_y^2} + \left( \frac{H - y}{\sigma_z} \right)^2 \right] \right\} \]

\[ D(x,0,0) = 36,750 \frac{\text{mg} \cdot \text{s}}{\text{m}^3} \gg 4500 \text{ mg} \cdot \text{s/m}^3 \]

This is hazardous.
(b) (continued) Repeat at various x locations. For example, predict the ground-level dose directly downwind at x = 2.5 km.

Solution:
Here we have:
- A very stable atmosphere
- Total mass of the chemical released = 10 kg
- Wind speed = 1.5 m/s
- Explosion is at ground level at time zero

Equations and tables for ground-level dose, absorbing ground:

\[
D_j(x, y, 0) = \frac{m_j}{\pi U \sigma_{yi} \sigma_{zi}} \exp \left\{ -\frac{1}{2} \left[ \left( \frac{y}{\sigma_{yi}} \right)^2 + \left( \frac{H}{\sigma_{zi}} \right)^2 \right] \right\}, \quad \sigma_{xi} = \sigma_{yi} = ax^b, \quad \sigma_{zi} = cx^d.
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\[ y = 0 \quad H = 0 \]

\[ G_{yi} = ax^b = 0.02(2500)^{0.89} = 21.444 \text{ m} \]
\[ G_{zi} = cx^d = 0.05(2500)^{0.61} = 5.91171 \text{ m} \]

\[ D_j = \frac{m_j}{\pi U G_{yi} G_{zi}} \]

\[ D_j = \frac{10 \text{ kg}}{\pi (1.5 \frac{\text{ m}}{\text{s}})(21.444 \text{ m})(5.91171 \text{ m})} \]

\[ D_j = 17,000 \frac{\text{ mg} \cdot \text{s}^{1/2}}{\text{ m}^2} \quad \text{at} \ x = 2.5 \text{ km} \]

\[ \text{ANS} \]

Notes: 1) \( D_j \) still > \( D_j \text{ safe (4000)} \)
2) \( D_j \) smaller @ 2.5 km than at 1.5 km (36,500)

Now repeat at various x values to make a plot!
(c) Plot dose vs. $x$. How far downstream is this hazardous to people on the ground?

**Solution:**
Repeat for a range of $x$ locations downstream. Note that for each $x$, you need to re-calculate the instantaneous dispersion coefficients. I used Excel and here is my plot:

**Final Answers**
- For **absorbing ground**, need to be @ $x \geq 6$ km to be “safe” @ centerline.
- For **reflecting ground**, need to be @ $x \geq 9.5$ km to be “safe” @ centerline.

We can also repeat for non-zero $y$ values. Expect you will plot a hazardous area for homework.