Today, we will:

- Discuss **The Coriolis Effect** and how it influences plumes or air pollution
- Do an example problem

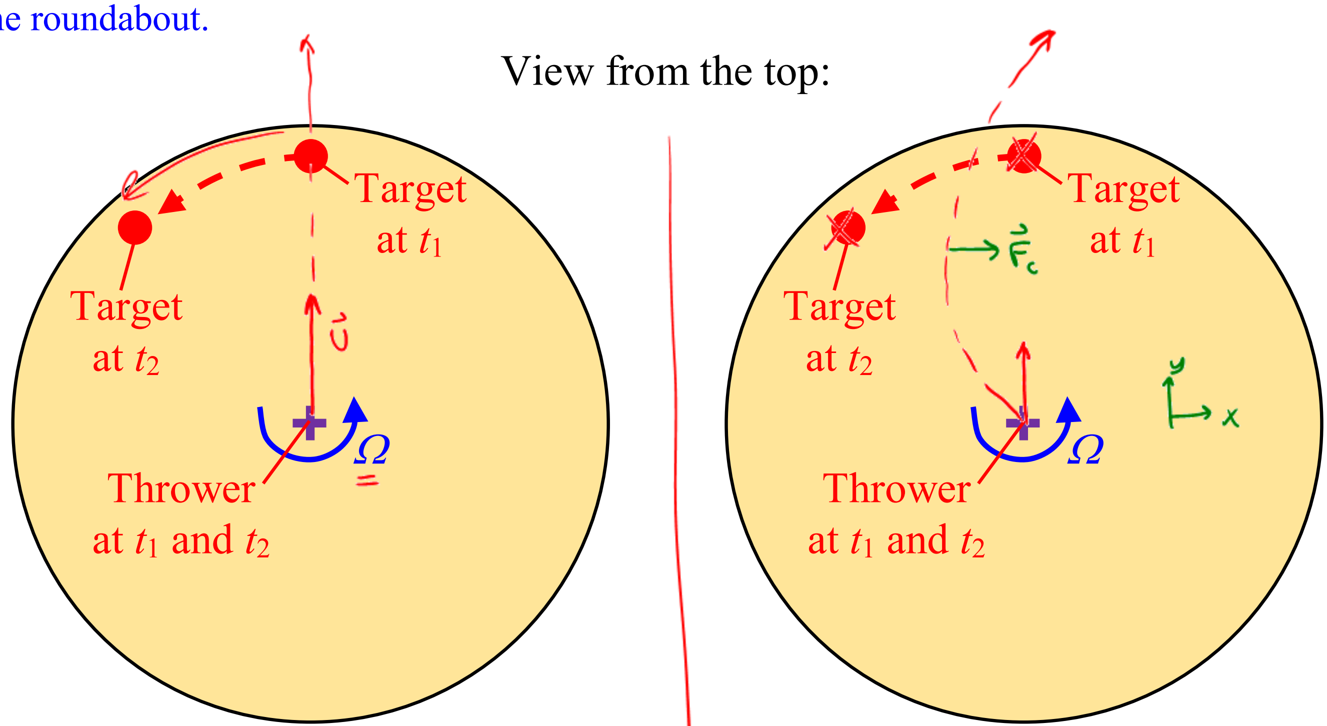
The Coriolis Effect:

Coriolis force is an ***apparent*** force that an object "feels" when moving in a ***rotating reference frame***.

Simplest example: a merry-go-round or "roundabout."

Case 1: Throw a ball from the middle of the roundabout towards a target at the outer part of the roundabout.

View from the top:



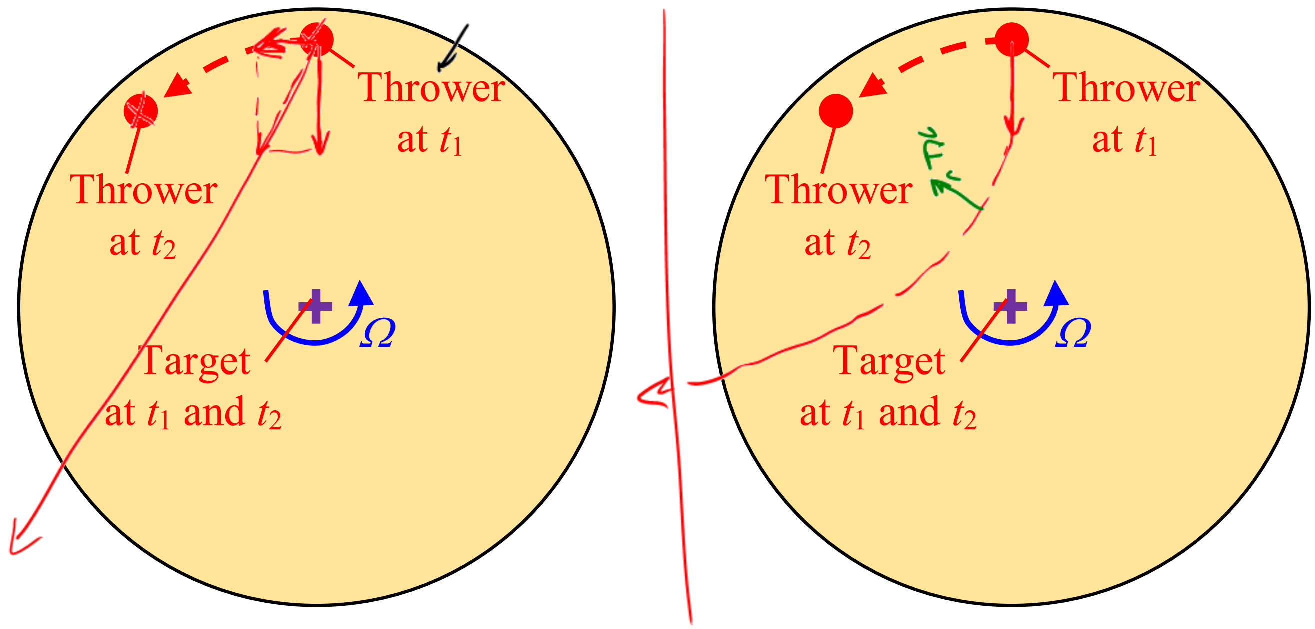
Stationary (absolute) reference frame Rotating reference frame

$\vec{F}_c$ = Coriolis force
= an apparent force that veers the object to the right when $\vec{\Omega}$ is ⊕

WHAT HAPPENS IF WE THROW THE BALL THE OPPOSITE WAY?

Case 2: Throw a ball from the outer part of the roundabout towards a target in the middle of the roundabout.

View from the top:



Stationary (absolute) reference frame

Rotating reference frame

If $\vec{\Omega} > 0$ (counterclockwise from top), the ball veers to the right no matter the direction of motion

If $\vec{\Omega} < 0$ (clockwise ... left ...

There are several videos on YouTube that illustrate the Coriolis Effect. See, for example, https://www.youtube.com/watch?v=mcPs_OdQOYU

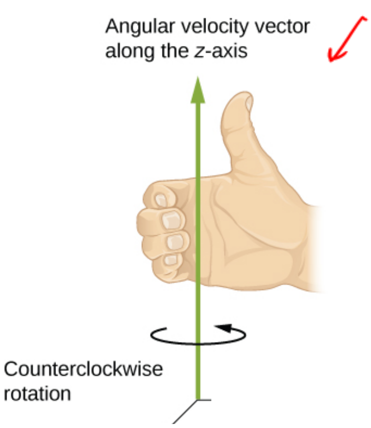
The ***Coriolis force*** is an ***apparent*** force that a moving object "feels" when observed in a ***rotating reference frame***. The Coriolis force is perpendicular to the direction of motion. When the roundabout is rotating ***counterclockwise*** (mathematically positive when looking from above), the Coriolis force appears, to someone in the ***rotating reference frame***, to pull the object ***to the right***. In reality, however, the object is actually moving in a ***straight line*** – as observed by someone in a ***stationary*** (non-rotating) ***reference frame***.

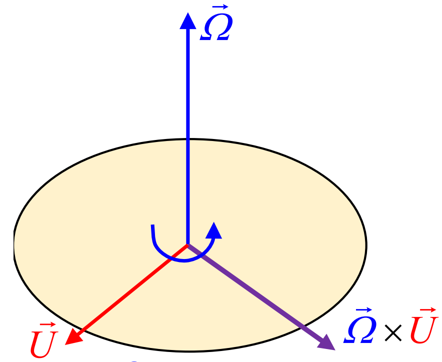
Math and equations:

It turns out that the ***Coriolis acceleration*** $\vec{a}_c$ and the ***Coriolis force*** $\vec{F}_c$ are expressed as ***vectors*** with ***cross products*** as follows:

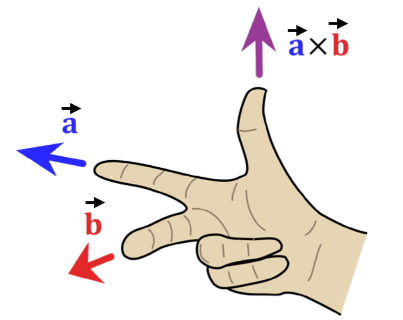
$$\vec{a}_c = -2\left(\vec{\Omega}\times\vec{U}\right) \text{ and from Newton's Law, } \vec{F}_c = m\vec{a}_c = -2m\left(\vec{\Omega}\times\vec{U}\right)$$

<u>Cross product illustrated for our counterclockwise rotating roundabout</u>: Consider the case where we throw a ball from the middle of the roundabout towards the outside rim.



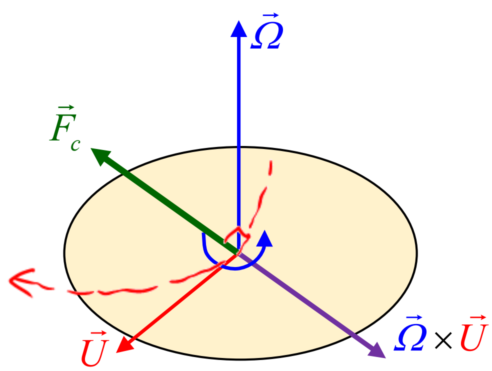


Our setup



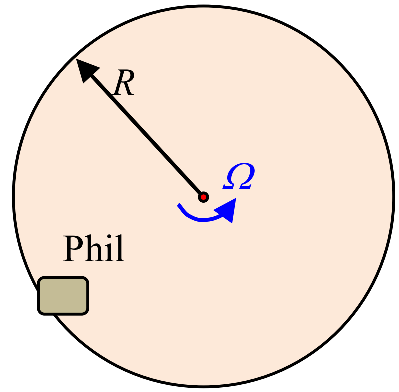
Apply the right-hand rule

We apply the right-hand rule to get the direction of the cross product. But due to the ***negative sign*** in the above equations, **the Coriolic force is in the *opposite* direction**:



Example: Coriolis Force on a merry-go-round

Given: Punxsutawney Phil (m = 7.50 kg, 16.5 lbm) is riding a merry-go-round on Groundhog Day. The merry-go-round's radius is R = 15.0 m and it rotates at Ω = 14.0 rpm. He stands at the edge of the ride ($r = R$) and holds on to a rail to keep from flying off.



$$\Omega = \left(14.0\ \frac{\text{rot}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{\text{rot}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 1.4661\ \frac{\text{rad}}{\text{s}} = \boxed{1.4661\ \frac{1}{\text{s}}}$$

(a) To do: Calculate the magnitude of the Coriolis acceleration in g's experienced by Phil ["g" = a_c/g, where g = 9.807 m/s^2].

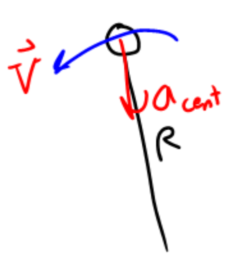
Solution: The Coriolis force is $\vec{F}_c = m\vec{a}_c = -2m\left(\vec{\Omega}\times\vec{U}\right)$.

THIS $\vec{U}$ IS RELATIVE TO THE OBSERVER ON THE ROUND-ABOUT

Answer $\boxed{0\ \text{g's}}$ (Trick question)

(b) To do: Calculate the radial force (in units of **lbf** to 3 significant digits) that Phil needs to exert on the rail to keep from flying off. *Note*: 1 N = 0.2248 lbf.

Solution:



$$a_{cent} = \frac{V^2}{R} = \frac{(\Omega R)^2}{R} = R\Omega^2$$

$$F = ma = m\Omega^2 R = (7.50\ \text{kg})\left(1.4661\ \frac{1}{\text{s}}\right)^2(15.0\ \text{m})\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)\left(\frac{0.2248\ \text{lbf}}{\text{N}}\right) = \boxed{54.4\ \text{lbf}}$$

(c) To do: Phil is angry about all the attention and chucks a reporter's camera horizontally at 67.11 mph (30 m/s). Calculate the initial value of the magnitude of the Coriolis acceleration acting on the camera from Phil's perspective, in g's.

Solution: Recall, the Coriolis force is $\vec{F}_c = m\vec{a}_c = -2m\left(\vec{\Omega}\times\vec{U}\right)$.

In this problem, m = 7.50 kg, Ω = 1.4661 rad/s, and one "g" = 9.807 m/s^2.

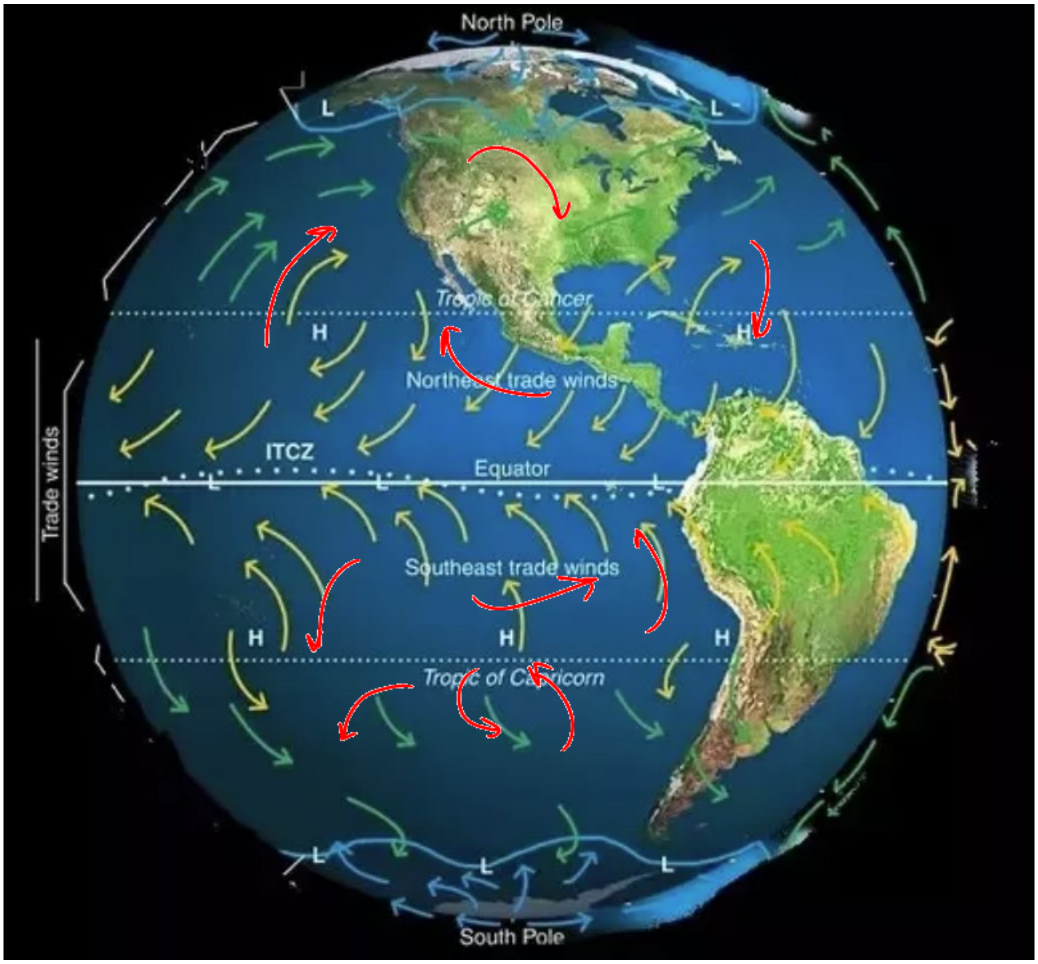
$$|\vec{a}_c| = \left|-2\left(\vec{\Omega}\times\vec{U}\right)\right| = \underline{2\Omega U}$$

$$\text{g's}: \frac{|\vec{a}_c|}{g} = \frac{2\Omega U}{g} = \frac{2\left(1.4661\ \frac{1}{\text{s}}\right)\left(30.0\ \frac{\text{m}}{\text{s}}\right)}{9.807\ \text{m/s}^2} = \boxed{8.97\ \text{g's}}$$

Veer to right

DO NOT CONFUSE CORIOLIS FORCE WITH CENTRIPETAL FORCE

Coriolis forces on the earth are more complex because Earth is a sphere, but same principles apply and we ***use the same right-hand rule*** to figure out which direction moving objects veer as they travel.

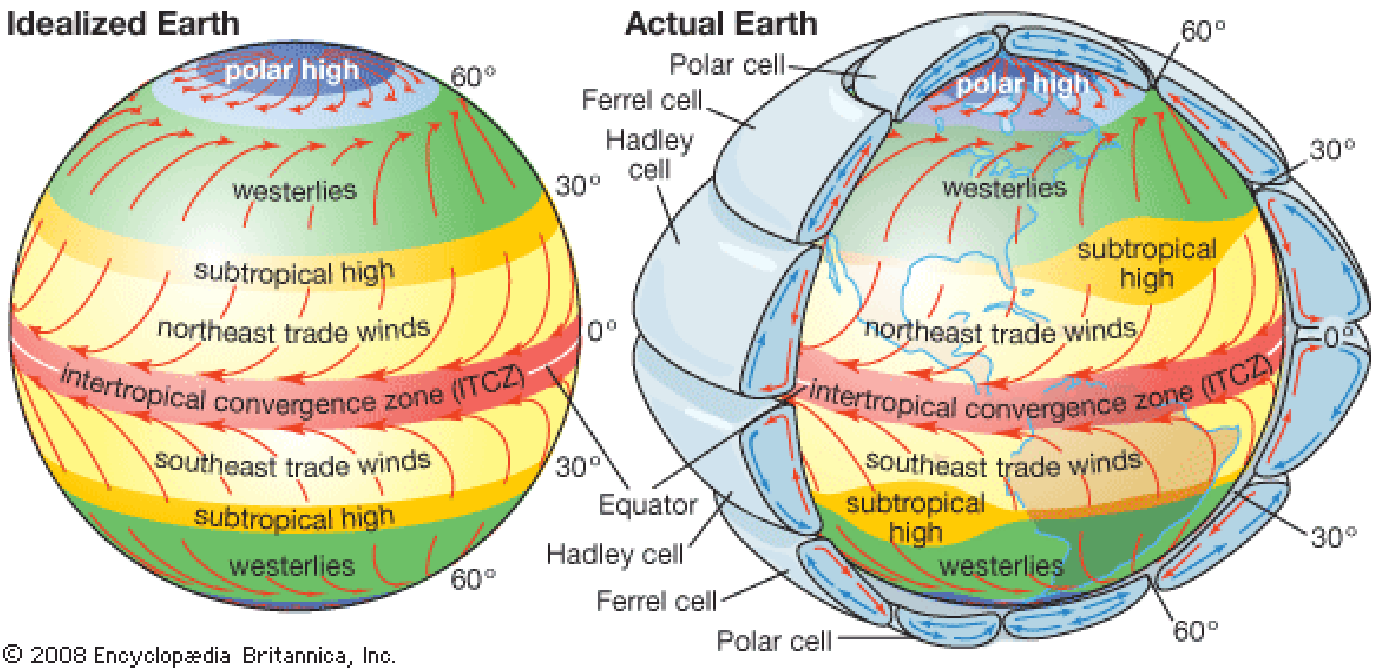


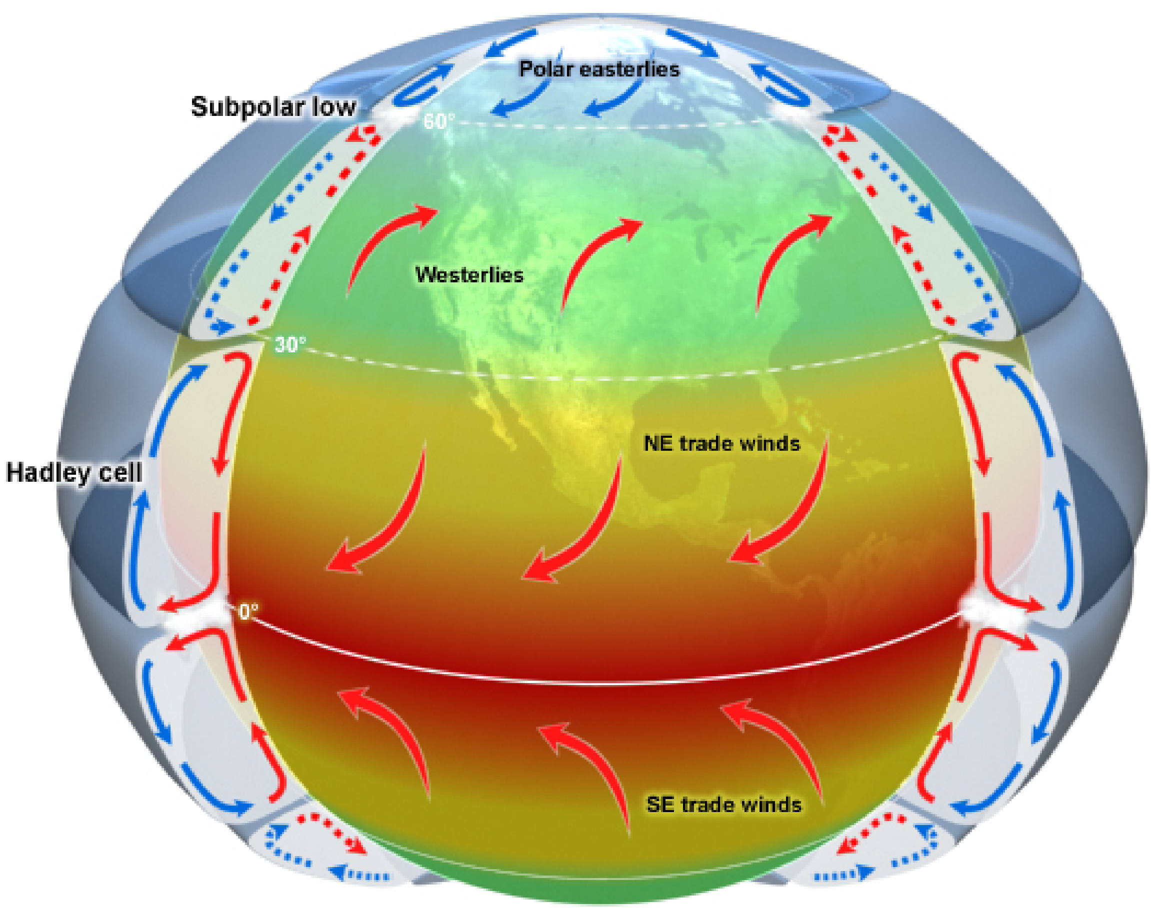
Bottom line:

- Objects in the **Northern Hemisphere** veer ***to the right*** [**clockwise**] due to the Coriolis force.
- Objects in the **Southern Hemisphere** veer ***to the left*** [**counterclockwise**] due to the Coriolis force.
- This includes ***any*** objects moving relative to the earth's surface – balls, airplanes, bullets, even parcels of air itself (wind!) as in the above sketch.

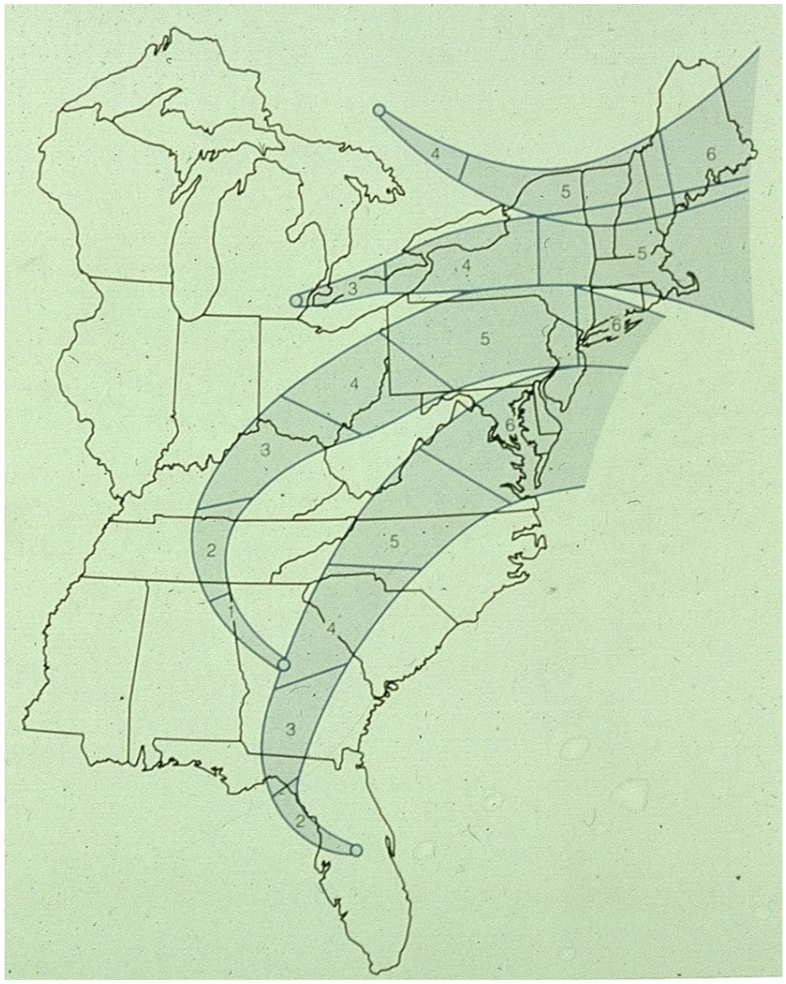
The actual wind patterns on the earth are quite complicated, due to three-dimensional effects (the earth is nearly spherical with a thin atmosphere relative to the earth's radius), ***instabilites*** caused by temperature effects (convection cells called ***Hadley cells*** form), and Coriolis forces.

Global wind patterns:





Recall this picture of smoke plumes with tracers [when we discussed acid rain]. This shows that plumes in the USA veer to the north and east due to the prevailing westerly winds:



The Science Behind the Polar Vortex

The polar vortex is a large area of low pressure and cold air surrounding the Earth's North and South poles. The term vortex refers to the counterclockwise flow of air that helps keep the colder air close to the poles (left globe). Often during winter in the Northern Hemisphere, the polar vortex will become less stable and expand, sending cold Arctic air southward over the United States with the jet stream (right globe). The polar vortex is nothing new — in fact, it's thought that the term first appeared in an 1853 issue of E. Littell's *Living Age*.