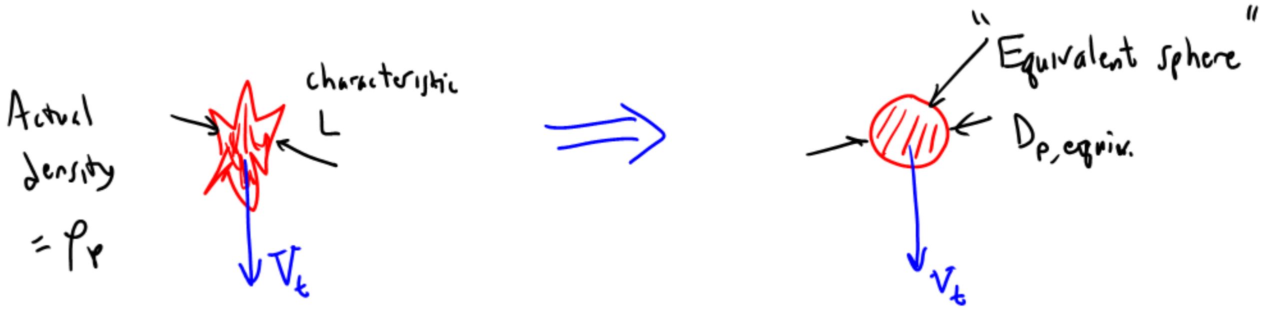
ME 433 Professor J. M. Cimbala **Lesson 10 E: Equivalent Diameters**

Today, we will:

- Discuss **Aerodynamic Equivalent Diameter**, **Spherical Equivalent Diameter**, and **Volume Equivalent Diameter**
- Do an example problem for each of the above three equivalent diameters

Non-Spherical Particles and Equivalent Diameters:

Since all our equations of motion are based on *spherical* particles, the easiest way to deal with non-spherical particles is to create some kind of ***equivalent*** spherical diameter so that we can use the same equations.



Here are definitions for three common types of equivalent spherical diameters:

(a) We define ***D_{ae}*** **= aerodynamic equivalent diameter** as the diameter of a sphere of **unit density** (pretend $\rho_p = \rho_0$ = 1000 kg/m^3) that would settle at the *same* (measured) terminal settling speed as the *actual* (non-spherical) particle at its *actual* density.

Procedure: 1) Measure V_t of actual particle

2) Use our eq. for settling speed V_t except
- Use D_{ae} instead of D_p
- Use ρ_0 " " ρ_p

$$V_t = \sqrt{\frac{4}{3}\frac{\rho_0 - \rho}{\rho} g D_{ae} \frac{C}{C_D}}$$ where $\rho_0 = 1000 \frac{\text{kg}}{\text{m}^3}$ = unit density

3) Solve for D_{ae} IMPLICIT! → $C = \text{fnc}(D_{ae})$, $C_D = C_D(D_{ae})$ (unknown)

I use Method of False Position to Iterate – will show in Excel

(b) We define ***D_{se}*** **= spherical equivalent diameter** as the diameter of a sphere of *actual* particle density (ρ_p) that would settle at the *same* (measured) terminal settling speed as the *actual* (non-spherical) particle of the same actual density.

PROCEDURE IS IDENTICAL TO ABOVE FOR D_{ae} except:

- Use D_{se} instead D_{ae}
- Use ρ_p " " ρ_0 (actual particle density)

Eq ↓

$$V_t = \sqrt{\frac{4}{3}\frac{\rho_p - \rho}{\rho} g D_{se} \frac{C}{C_D}}$$

IMPLICIT

MUST ITERATE !! since $C = C(D_{se})$ & $C_D = C_D(D_{se})$

[I use M.F.P.]

(**c**) We define ***D**$_{ve}$* **= volume equivalent diameter** as the diameter of a sphere of the *same volume* as the actual particle.

Procedure

1) Count N (or n_t) particles & weigh them = m_{total}
2) Calc. m_p = avg. mass per particle = $\frac{m_{total}}{N}$
3) Calc. $\forall_p$ = avg. volume per particle = $\frac{m_p}{\rho_p}$
4) Use eq. for volume of a sphere

$$V_p = \frac{\pi D_{ve}^3}{6}$$

5) Solve for D_{ve} → $D_{ve} = \left(\frac{6\forall_p}{\pi}\right)^{1/3}$

explicit eq.

NO ITERATION REQUIRED! ☺

Example: Equivalent Diameters

Given: Particles of density 2010 kg/m^3 are being studied by a manufacturer of air pollution control systems (APCSs). The particles are of various (unknown) shapes and diameters, so the engineers devise an optical technique to measure the terminal settling speed V_t of the actual particles and then plan to calculate some kind of equivalent spherical diameter to use in their APCS performance simulations, which are all based on spheres. The tests are at SATP conditions, and the average V_t is found to be 3.46×10^{-4} m/s. The engineers also use their optical technique to count all the particles that fall during a period of time, which they then weigh using a sensitive scale. The result is a mass of 0.158 mg per 8.22×10^6 particles.

V_t m_{total} N

To do:

(a) Calculate D_{ae}, the ***aerodynamic*** equivalent diameter.

(b) Calculate D_{se}, the ***spherical*** equivalent diameter.

(c) Calculate D_{ve}, the ***volume*** equivalent diameter.

Solution:

Useful equations:

For Part (a), $V_t = \sqrt{\frac{4}{3}\frac{\rho_0 - \rho}{\rho} g D_{ae} \frac{C}{C_D}}$ where $\rho_0 = 1000 \frac{\text{kg}}{\text{m}^3}$

For Part (b), $V_t = \sqrt{\frac{4}{3}\frac{\rho_p - \rho}{\rho} g D_{se} \frac{C}{C_D}}$

For Part (c), $V_p = \frac{\pi D_{ve}^3}{6}$

I did all my calculations in Excel. Iteration is required for the first two cases. *Note*: The **False Position Method** is used for the iteration in (a) and (b). Watch Professor Cimbala's short video about this method at https://youtu.be/RTY7QyxBMNA if you are not familiar with it.

* I used Excel for all 3 equiv. dia calculations

(a) Use $\rho_p = \rho_0$ — First Guess — Second guess — from far right → — Iteration

(a) Aerodynamic equivalent diameter using unit density (must iterate):

Particle density to use: ρ_p = 1000 kg/m^3

Set Goal for $f(x)$: 0

Iterations are in each row - I just added columns to the right:

False Postion Method:	D_{ae} (µm)	D_{ae} (m)	$f(x)$	Converged V_t (m/s)	Converged Re (-)	Kn	C	Guess V_t (m/s)	Re (-)	C_D (-)	New V_t (m
Guess:	10	0.00001	0.002646709	0.002992709	0.001916669	6.7034E-03	1.016852454	3.46E-04	0.000221595	108305.9338	0.001017
Guess again:	4	0.000004	0.000144744	0.000490744	0.000125718	1.6759E-02	1.042131134	2.99E-03	0.000766669	31304.37929	0.0012118
Linearly interpolate:	3.652887248	3.65289E-06	6.48395E-05	0.000410839	9.61152E-05	1.8351E-02	1.046134612	4.91E-04	0.000114809	209043.5797	0.0004490
(always use latest two values)	3.371218632	3.37122E-06	5.21314E-06	0.000351213	7.583E-05	1.9884E-02	1.049989204	4.11E-04	8.87038E-05	270563.3611	0.0003798
↓	3.346592307	3.34659E-06	2.21952E-07	0.000346222	7.42063E-05	2.0031E-02	1.050357056	3.51E-04	7.52761E-05	318826.5411	0.0003487
↓	3.345497204	3.3455E-06	8.26622E-10	0.000346001	7.41346E-05	2.0037E-02	1.050373539	3.46E-04	7.4182E-05	323528.6568	0.0003461
↓	3.34549311	3.34549E-06	-6.07089E-14	0.000346	7.41344E-05	2.0037E-02	1.050373601	3.46E-04	7.41345E-05	323735.8166	0.0003
↓	3.34549311	3.34549E-06	-8.03827E-16	0.000346	7.41344E-05	2.0037E-02	1.050373601	3.46E-04	7.41344E-05	323736.5901	0.0003
↓	3.34549311	3.34549E-06	1.0571E-17	0.000346	7.41344E-05	2.0037E-02	1.050373601	3.46E-04	7.41344E-05	323736.59	0.0003
↓	3.34549311	3.34549E-06	0	0.000346	7.41344E-05	2.0037E-02	1.050373601	3.46E-04	7.41344E-05	323736.59	0.0003
↓	3.34549311	3.34549E-06	0	0.000346	7.41344E-05	2.0037E-02	1.050373601	3.46E-04	7.41344E-05	323736.59	0.0003
↓	3.34549311	3.34549E-06	0	0.000346	7.41344E-05	2.0037E-02	1.050373601	3.46E-04	7.41344E-05	323736.59	0.0003

D_{ae} = 3.34549311 µm — aerodynamic equivalent diameter

$D_{ae} = 3.35\ \mu m$

(b) Set $\rho = \rho_p$ (actual density)

(b) Spherical equivalent diameter using actual density (must iterate):

Particle density to use: ρ_p = 2010 kg/m³

Set Goal for $f(x)$: 0

Iterations are in each row - I just added columns to the right:

False Postion Method:	D_{se} (µm)	D_{se} (m)	$f(x)$	Converged V_t (m/s)	Converged Re (-)	Kn	C	Guess V_t (m/s)	Re (-)	C_D (-)	New V_t (m
Guess:	0.3	0.0000003	-0.000337599	8.40122E-06	1.61417E-07	2.2345E-01	1.57699897	3.46E-04	6.64784E-06	3610193.975	5.39149E-
Guess again:	1	0.000001	-0.000276831	6.91686E-05	4.42988E-06	6.7034E-02	1.168539195	8.40E-06	5.38054E-07	44605205.41	2.41061E-
Linearly interpolate:	4.188911481	4.18891E-06	0.000734432	0.001080432	0.000289855	1.6003E-02	1.040231104	6.92E-05	1.85564E-05	1293354.115	0.0002733
(always use latest two values)	1.872958509	1.87296E-06	-0.00011967	0.00022633	2.7149E-05	3.5791E-02	1.089977726	1.08E-03	0.000129601	185183.6647	0.0004945
↓	2.197452459	2.19745E-06	-3.8251E-05	0.000307749	4.33111E-05	3.0506E-02	1.076690868	2.26E-04	3.18526E-05	753471.8662	0.0002639
↓	2.349900339	2.3499E-06	4.30406E-06	0.000350304	5.27203E-05	2.8526E-02	1.07171561	3.08E-04	4.63158E-05	518181.4382	0.0003283
↓	2.334481621	2.33448E-06	-1.2501E-07	0.000345875	5.17122E-05	2.8715E-02	1.072189275	3.50E-04	5.23744E-05	458239.3249	0.0003480
↓	2.334916814	2.33492E-06	-3.88953E-10	0.000346	5.17405E-05	2.8710E-02	1.07217582	3.46E-04	5.17218E-05	464020.7566	0.0003459
↓	2.334918173	2.33492E-06	1.67067E-13	0.000346	5.17406E-05	2.8710E-02	1.072175778	3.46E-04	5.17405E-05	463853.3569	0.0003
↓	2.334918172	2.33492E-06	4.2208E-16	0.000346	5.17406E-05	2.8710E-02	1.072175778	3.46E-04	5.17406E-05	463852.8353	0.0003
↓	2.334918172	2.33492E-06	-1.0842E-18	0.000346	5.17406E-05	2.8710E-02	1.072175778	3.46E-04	5.17406E-05	463852.8356	0.0003
↓	2.334918172	2.33492E-06	0	0.000346	5.17406E-05	2.8710E-02	1.072175778	3.46E-04	5.17406E-05	463852.8356	0.0003

D_{se} = 2.334918172 µm spherical equivalent diameter

$$D_{se} = 2.33\ \mu m$$

(c) Volume equivalent diameter using actual density (no need to iterate):

total mass =	0.158	mg	for	8.22E+06 particles
average mass per particle = mass/(number of particles) =			1.92E-08	mg
		=	1.92E-14	kg
average volume per particle = (average mass per particle) / (particle density) =			9.56E-18	m³
volume equivalent diameter =	$(6V_p/\pi)^{1/3}$ =		2.633E-06	m

D_{ve} = 2.633481044 µm **volume equivalent diameter**

$$m_p = \frac{m_{total}}{N} = \frac{0.158\ mg}{8.22 \times 10^6} = 1.922 \times 10^{-8}\ mg$$

$$\forall_p = \frac{m_p}{\rho} = \frac{1.922 \times 10^{-8}\ mg}{(2010\ kg/m^3)\left(\frac{10^6\ mg}{kg}\right)} = 9.563 \times 10^{-18}\ m^3$$

$$D_{ve} = \left(\frac{6\forall_p}{\pi}\right)^{1/3} = \left[\frac{6(9.563 \times 10^{-18}\ m^3)}{\pi}\right]^{1/3}\left(\frac{10^6\ \mu m}{m}\right) = 2.63\ \mu m$$

$$D_{ve} = 2.63\ \mu m$$

All 3 are different