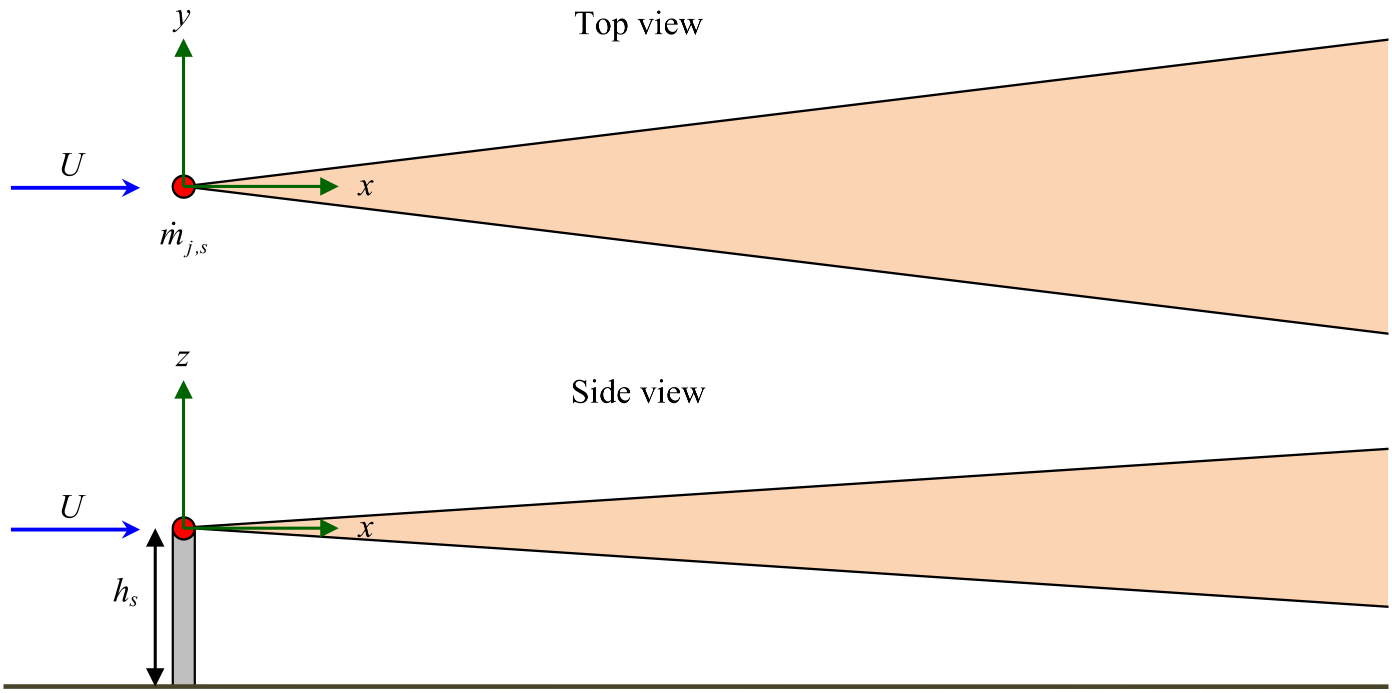
Today, we will:

- Continue derivation of the **Gaussian plume model** – equation and solution
- Modify the solution for a *buoyant* plume, and do some examples/applications
- Compare a ground that absorbs the pollutant vs. one that does *not* absorb the pollutant



From the previous lecture, we derived the differential equation for a non-buoyant Gaussian plume, assuming gradient diffusion, constant U, and constant diffusion coefficients:

$$\frac{\partial c_j}{\partial t} = -\frac{\partial}{\partial x}\left(Uc_j\right) + \frac{\partial}{\partial x}\left(D_x \frac{\partial c_j}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_y \frac{\partial c_j}{\partial y}\right) + \frac{\partial}{\partial z}\left(D_z \frac{\partial c_j}{\partial z}\right)$$

where our notation is $D_x = D_{aj,x}$, $D_y = D_{aj,y}$, and $D_z = D_{aj,z}$ for simplicity.

Assumptions and Approximations:

Simplified equation for a steady Gaussian plume without buoyancy:

$$U\frac{\partial c_j}{\partial x} = D_y \frac{\partial^2 c_j}{\partial y^2} + D_z \frac{\partial^2 c_j}{\partial z^2} \qquad (1)$$

Now we apply boundary conditions (BCs) and solve (1) for $c_j(x,y,z)$ in this plume.

Dispersion coefficients: Tables scanned from Cooper, C. D. and Alley, F. C. *Air Pollution Control: A Design Approach*, Edition 4, Waveland Press, Inc., Long Grove, IL, 2011, pp. 662-663.

Table 20.1 Stability Classifications*

Surface Wind Speed[a] m/s	Day Incoming Solar Radiation: Strong[b]	Moderate[c]	Slight[d]	Night Cloudiness[e]: Cloudy (≥4/8)	Clear (≤3/8)
<2	A	A–B[f]	B	E	F
2–3	A–B	B	C	E	F
3–5	B	B–C	C	D	E
5–6	C	C–D	D	D	D
>6	C	D	D	D	D

[a] Surface wind speed is measured at 10 m above the ground.
[b] Corresponds to clear summer day with sun higher than 60° above the horizon.
[c] Corresponds to a summer day with a few broken clouds, or a clear day with sun 35-60° above the horizon.
[d] Corresponds to a fall afternoon, or a cloudy summer day, or clear summer day with the sun 15–35°.
[e] Cloudiness is defined as the fraction of sky covered by clouds.
[f] For A–B, B–C, or C–D conditions, average the values obtained for each.

* A = Very unstable; D = Neutral
B = Moderately unstable; E = Slightly stable
C = Slightly unstable; F = Stable
Regardless of wind speed, Class D should be assumed for overcast conditions, day or night.

Table 20.2 Values of Curve-Fit Constants for Calculating Dispersion Coefficients as a Function of Downwind Distance and Atmospheric Stability

			$x < 1$ km			$x > 1$ km		
Stability	*a*	*b*	*c*	*d*	*f*	*c*	*d*	*f*
A	213	0.894	440.8	1.941	9.27	459.7	2.094	–9.6
B	156	0.894	106.6	1.149	3.3	108.2	1.098	2.0
C	104	0.894	61.0	0.911	0	61.0	0.911	0
D	68	0.894	33.2	0.725	–1.7	44.5	0.516	–13.0
E	50.5	0.894	22.8	0.678	–1.3	55.4	0.305	–34.0
F	34	0.894	14.35	0.740	–0.35	62.6	0.180	–48.6

Adapted from Martin, 1976.