Today, we will:

- Summarize our equations for particle motion
- Discuss **terminal settling velocity** and equations for its prediction in quiescent air
- Discuss **aerodynamic equivalent diameter** and how to apply it

Review of equations for particle motion so far:

Relative particle velocity = $\boxed{\vec{v}_r = \vec{v} - \vec{U}}$, where $\vec{v}$ = particle velocity and $\vec{U}$ = air velocity.

Drag force on a spherical particle = $\boxed{\vec{F}_{\text{drag}} = -\frac{1}{8}\rho \frac{C_D}{C}\pi D_p^{\,2}\vec{v}_r\left|\vec{v}_r\right|}$, where

$\boxed{C = 1 + \text{Kn}\left[2.514 + 0.80\exp\left(-\frac{0.55}{\text{Kn}}\right)\right]}$, $\boxed{\text{Kn} = \frac{\lambda}{D_p}}$, $\boxed{\lambda = \frac{\mu}{0.499}\sqrt{\frac{\pi}{8\rho P}}}$, $\boxed{\text{Re} = \frac{\rho\left|\vec{v}_r\right|D_p}{\mu}}$, and

$\boxed{C_D = \frac{24}{\text{Re}} \text{ for Re} < 0.1}$, $\boxed{C_D = \frac{24}{\text{Re}}\left(1 + 0.0916\,\text{Re}\right) \text{ for } 0.1 < \text{Re} < 5}$.

Equation of motion for spherical particle: $\boxed{m_p\frac{d\vec{v}}{dt} = \frac{\pi D_p^{\,3}}{6}\left(\rho_p - \rho\right)\vec{g} - \frac{1}{8}\rho\frac{C_D}{C}\pi D_p^{\,2}\vec{v}_r\left|\vec{v}_r\right|}$.

But for a spherical particle, we also know its mass, $\boxed{m_p = \rho_p V_p = \rho_p\frac{\pi D_p^{\,3}}{6}}$. Plugging this mass into the equation of motion (after a little algebra), we get our final expression,

$$\boxed{\frac{d\vec{v}}{dt} = \frac{\rho_p - \rho}{\rho_p}\vec{g} - \frac{3}{4}\frac{\rho}{\rho_p}\frac{C_D}{C}\frac{1}{D_p}\vec{v}_r\left|\vec{v}_r\right|}.$$

Simplest application – <u>Terminal settling velocity in quiescent air</u>

Terminal settling velocity: $\boxed{V_t = \sqrt{\frac{4}{3}\frac{\rho_p - \rho}{\rho}gD_p\frac{C}{C_D}}}$, but $C_D = C_D(\text{Re})$, where $\boxed{\text{Re} = \frac{\rho V_t D_p}{\mu}}$.

Example: Terminal settling velocity as a function of particle diameter

Given: Air at STP with Cunningham correction factors from previous calculations. For Stokes flow (Re less than about 0.1), $\boxed{C_D = 24/\mathrm{Re}}$, and $\boxed{V_t = \frac{\rho_p - \rho}{18}\left(D_p^{\ 2}\right) g \frac{C}{\mu}}$.

To do:

Calculate V_t for various values of particle diameter D_p. Also calculate Re to test our Stokes flow approximation. At STP conditions, ρ = 1.184 kg/m^3, and μ = 1.849 × 10^{-5} kg/(m s). Use g = 9.807 m/s^2 (gravitational constant), and ρ_p = 1000 kg/m^3 (unit density spheres). [Give your answers to 3 significant digits, and be careful with units.]

Solution:

Table to be filled in during class:

D_p (μm)	C	V_t (m/s)	Re
0.001	222.7		
0.0025	89.43		
0.006	37.60		
0.01	22.79		
0.025	9.489		
0.06	4.355		
0.1	2.921		
0.25	1.702		

D_p (μm)	C	V_t (m/s)	Re
0.6	1.282		
1	1.169		
2.5	1.067		
6	1.028		
10	1.017		
25	1.007		
60	1.003		
100	1.0017		