Today, we will:

- Continue to discuss **inertial separation** (particles in curved flows)
- Discuss the analogy between gravimetric settling and inertial separation
- Compare laminar vs. well-mixed inertial separation in curved ducts

Review of equations so far for inertial separation in a curved duct:

$$\vec{F}_{\text{centrifugal}} = \left(m_p - m_{\text{air}}\right)\frac{U_\theta^{\,2}}{r} = \pi\frac{D_p^{\,3}}{6}\left(\rho_p - \rho\right)\frac{U_\theta^{\,2}}{r}$$ = centrifugal force, radially outward

$$\vec{F}_{\text{drag}} = -\frac{1}{8}\rho\frac{C_D}{C}\pi D_p^{\,2}\vec{v}_r\left|\vec{v}_r\right| = -\frac{1}{8}\rho\frac{C_D}{C}\pi D_p^{\,2}v_r^{\,2}$$ = aerodynamic drag force, radially inward

(same aerodynamic drag force that we used previously)

Consider the simplest case in which v_r is *constant*, and the two above forces must balance:

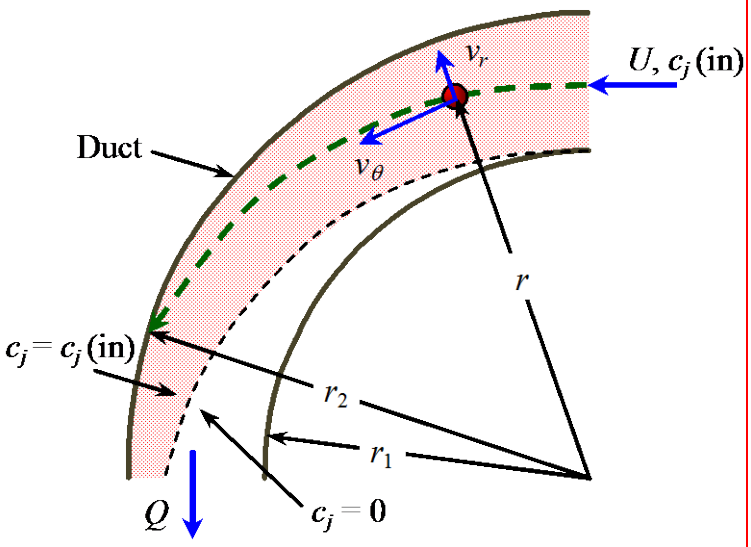
$$v_r = \sqrt{\frac{4}{3}\frac{\rho_p - \rho}{\rho}\frac{U_\theta^{\,2}}{r}D_p\frac{C}{C_D}}$$

Compare to gravimetric settling in quiescent air (from a previous lecture):

$$V_t = \sqrt{\frac{4}{3}\frac{\rho_p - \rho}{\rho}gD_p\frac{C}{C_D}}$$

Example: Comparison of Centrifugal and Gravitational Settling

Given: Dusty air enters a curved duct at average speed U. Aerosol particles of a certain diameter D_p have a terminal settling speed of $V_t = 0.00025$ m/s in quiescent air. At the instant of time shown, a particle of diameter D_p is at radius $r = 0.32$ m.



To do: Calculate the air speed U such that the radial velocity v_r of the particle is the *same* as its terminal settling velocity. Give your answer in m/s to three significant digits.

Solution:

Laminar vs. well-mixed settling:

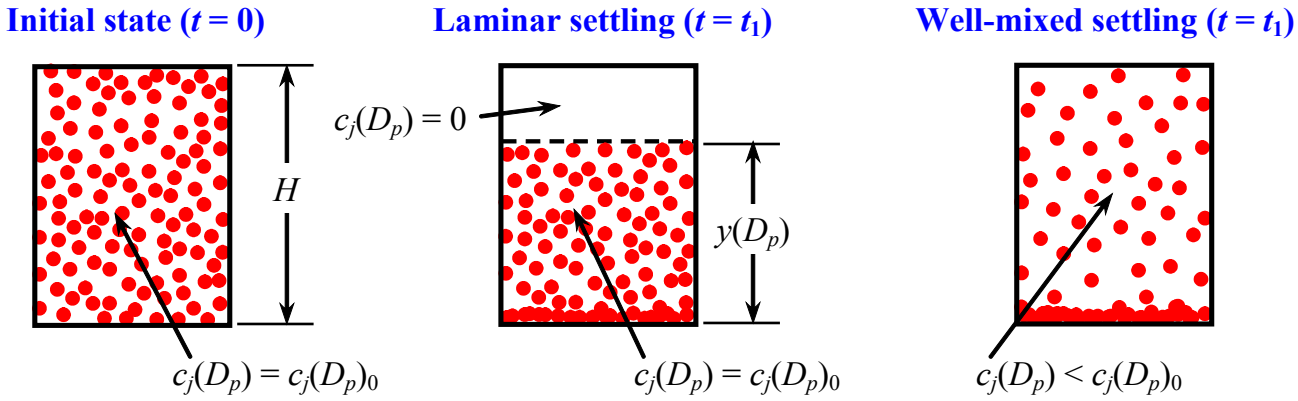
We discussed this twice previously:

1. Gravimetric settling in a room or container
2. Gravimetric settling in a horizontal duct

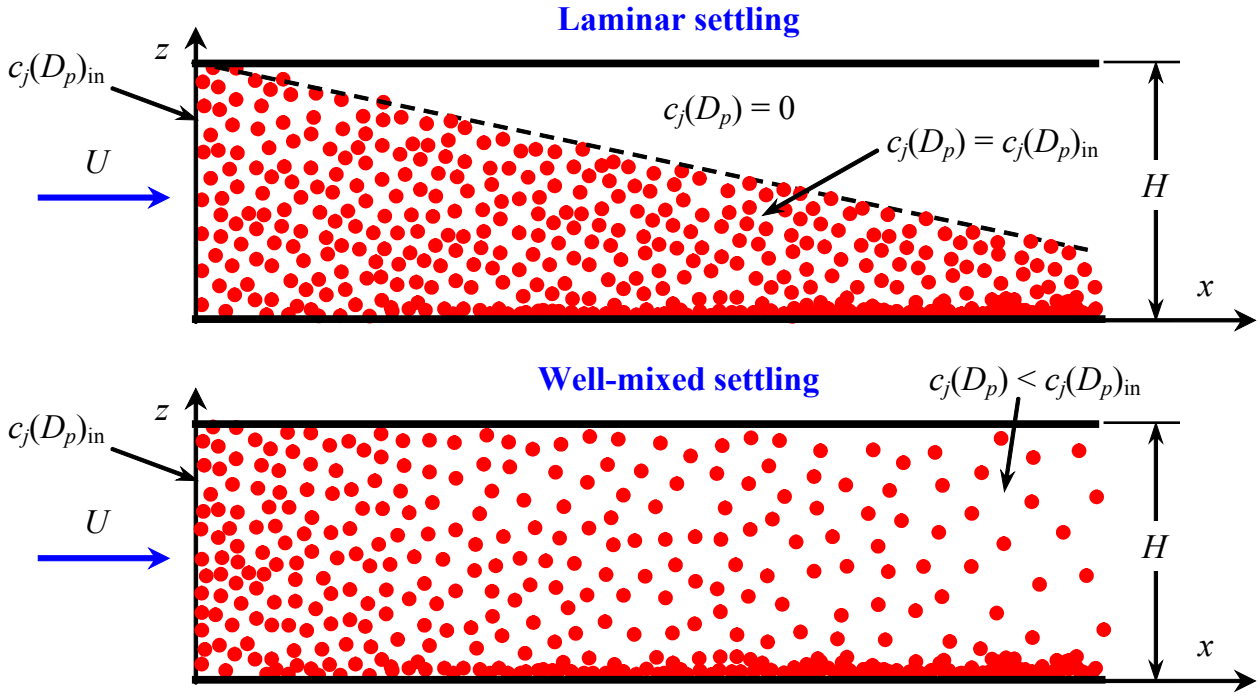
Now we apply the same principles to

3. **Inertial separation in a curved duct**.

1. Gravimetric settling in a room or container:

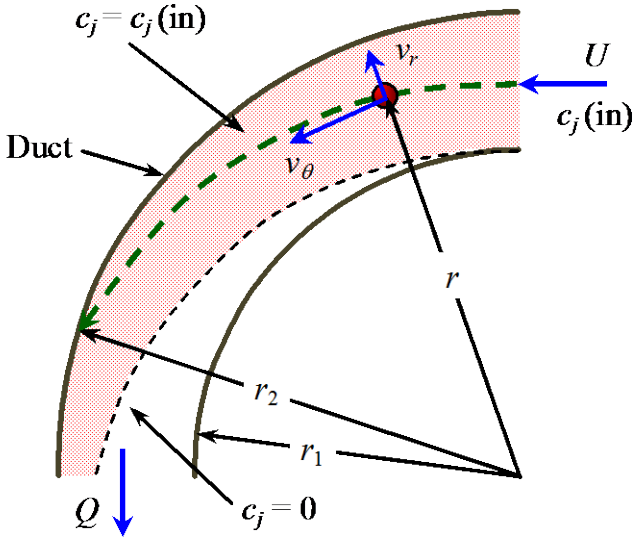


2. Gravimetric settling in a horizontal duct:

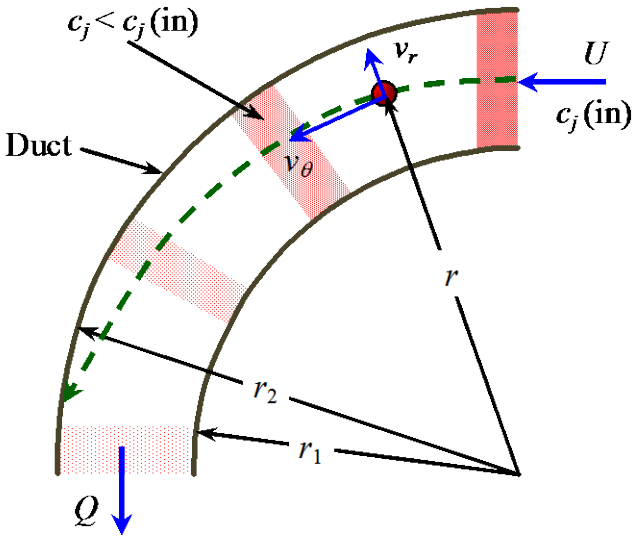


3. Inertial separation in a curved duct:

Laminar settling

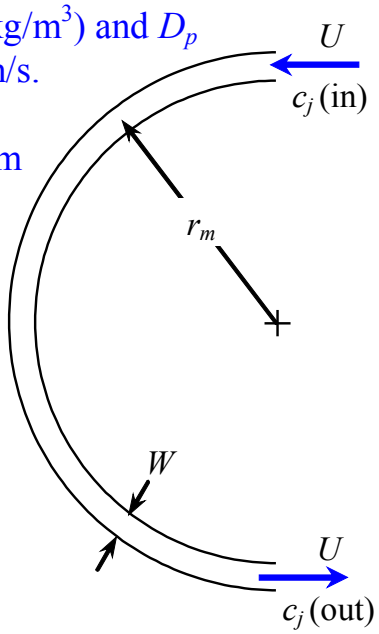


Well-mixed settling



Example: Removal efficiency of a curved duct as a function of particle diameter

Given: Monodisperse aerosol particles of unit density (ρ_p = 1000 kg/m^3) and D_p = 15.0 microns enter a 180^o curved duct at average speed U = 10.3 m/s. The air is at STP (ρ = 1.184 kg/m^3, μ = 1.849 × 10^{-5} kg/(m s)). The middle duct radius is r_m = 0.500 m and the duct width is W = 22.5 mm (0.0225 m). We assume laminar settling, and we assume that the particles stick to the outer wall when they hit, and are therefore removed from the flow.



To do: Calculate your best estimate of the predicted removal efficiency E of these particles (% to three significant digits).

Solution: The Cunningham correction factor is C = 1.0168, and Stokes flow is appropriate (C_D = 24/Re).

Equations: $$v_r = \sqrt{\frac{4}{3}\frac{\rho_p - \rho}{\rho}\frac{U_\theta^2}{r}D_p\frac{C}{C_D}}, \quad \mathrm{Re} = \frac{\rho v_r D_p}{\mu}, \quad C_D = \frac{24}{\mathrm{Re}},$$

Laminar settling model: $$E = 1 - \frac{c_j}{c_j(\mathrm{in})} = \frac{x}{L_c}, \text{ where } L_c = W\frac{U_\theta}{v_r}, \quad x = r_m\theta.$$