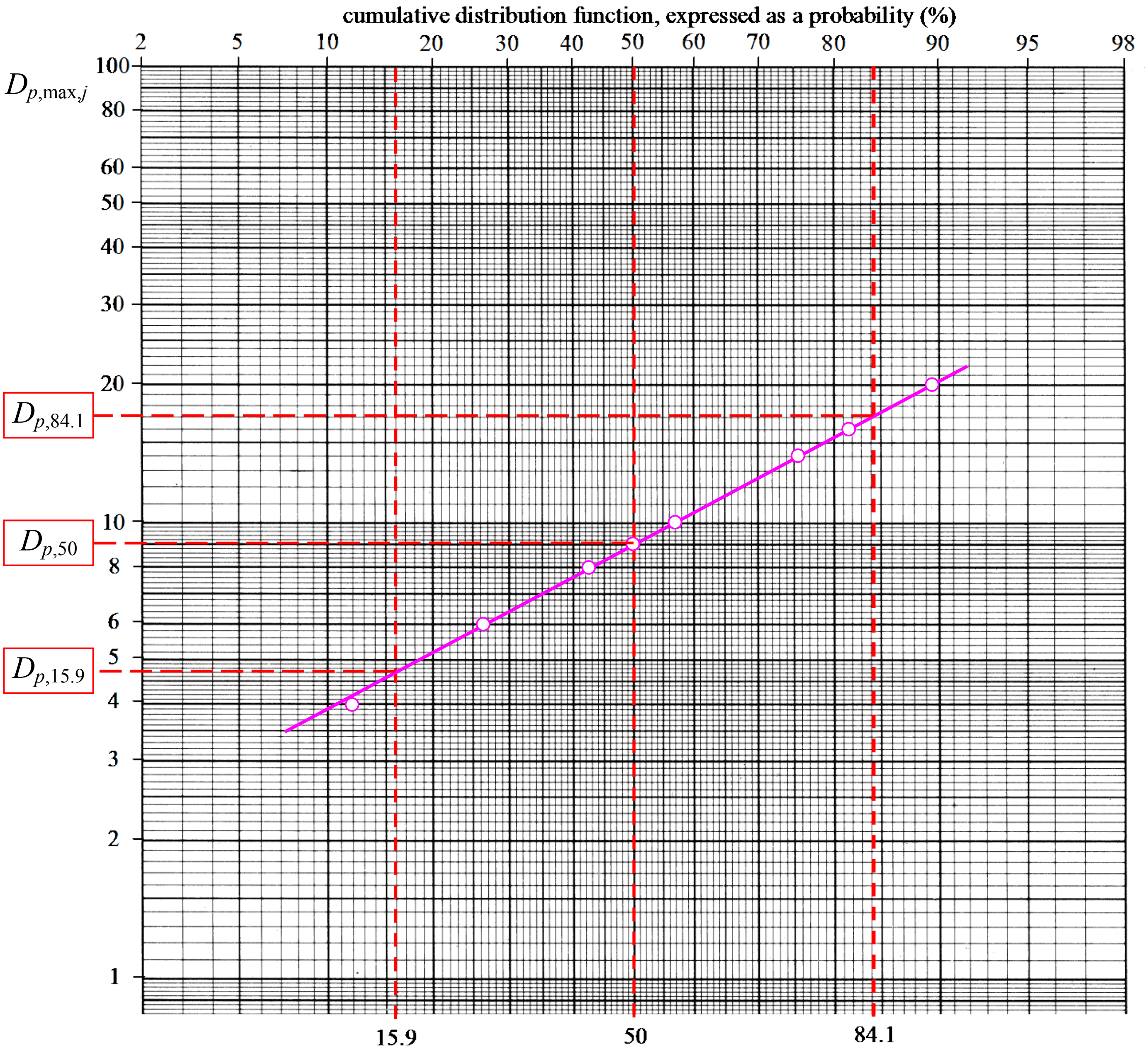
Today, we will:

- Continue discussing aerosol particle statistics: **arithmetic mean diameter**, **geometric mean diameter**, **geometric standard deviation**, **mass distribution**.
- Show how to (carefully!) plot cumulative distribution functions on log-probability paper.

Log-probability paper for particle distributions (created by J. M. Cimbala)



Example: Log-probability plots

Given: A sample (in terms of grouped data) is shown below.

j = class (bin number)	$D_{p,\min,j}$ (lower limit)	$D_{p,\max,j}$ (upper limit)	$D_{p,j}$ (middle value)	$\Delta D_{p,j}$ = class width	n_j = count per class	probability in this class = n_j/n_t	$F(D_{p,\max,j})$ = cumulative dist. func.
1	0.4	0.8	0.6	0.4	26	0.0052	0.52
2	0.8	1.2	1	0.4	67	0.0134	1.86
3	1.2	2	1.6	0.8	255	0.051	6.96
4	2	3	2.5	1	447	0.0894	15.9
5	3	4	3.5	1	477	0.0954	25.44
6	4	5	4.5	1	456	0.0912	34.56
7	5	6	5.5	1	411	0.0822	42.78
8	6	8	7	2	674	0.1348	56.26
9	8	10	9	2	503	0.1006	66.32
10	10	12	11	2	372	0.0744	73.76
11	12	16.3	14.15	4.3	517	0.1034	84.1
12	16.3	20	18.15	3.7	255	0.051	89.2
13	20	30	25	10	325	0.065	95.7
14	30	50	40	20	215	0.043	100

To do: Calculate the geometric standard deviation σ_g (to 3 significant digits).

Solution:

Mass Distribution

Additional analysis of the sample particle data (class handout; also see Excel spreadsheet):

j = class (bin number)	$D_{p,\min,j}$ (lower limit)	$D_{p,\max,j}$ (upper limit)	$D_{p,j}$ (middle value)	$m_{j,\text{in}} = n_j \rho_p \pi D_{p,j}^3/6$ = mass in class j (mg)	$m_{j,\text{in}} / \Delta(D_{p,j})$ (mass in class j divided by class width)	$g(D_{p,j})_{\text{in}} = m_j/m_t$ (mass fraction of original aerosol)	$G(D_p)_{\text{in}} = (M_j/m_t)_{\text{in}}$ = cumulative mass distribution (%)
1	1	4	2.5	8.50848E-07	2.83616E-07	0.000373985	0.037398476
2	4	6	5	1.0472E-05	5.23599E-06	0.004602889	0.497687408
3	6	8	7	2.89147E-05	1.44573E-05	0.012709268	1.768614192
4	8	9	8.5	2.41166E-05	2.41166E-05	0.01060031	2.828645219
5	9	10	9.5	3.00777E-05	3.00777E-05	0.013220447	4.150689966
6	10	14	12	0.000168289	4.20722E-05	0.073970273	11.54771722
7	14	16	15	0.000107796	5.38979E-05	0.047380992	16.28581641
8	16	20	18	0.000241237	6.03092E-05	0.106034	26.88921637
9	20	35	27.5	0.00098003	6.53353E-05	0.430765712	69.96578759
10	35	50	42.5	0.000683305	4.55536E-05	0.300342124	100
11	50	100	75	0	0	0	100

Mass of particles in class j: m_j

Mass of particles in class j divided by class width: $m_j/(\Delta D_{p,j})$

Mass fraction of particles in class j: $g(D_{p,j}) = m_j/m_t$

Cumulative mass distribution function: $G(D_p)$ [sometimes written as M_j/m_t]

When plot this on log-probability paper, remember to use $D_{p,\max,j}$, *not* $D_{p,j}$ since the cumulative distribution function includes the *entire bin* from min to max.

Number distribution and mass distribution for the sample data of the class handout