M E 433 Professor John M. Cimbala Lecture 40

Today, we will:

- Continue discussing aerosol particle statistics: **mass distribution**, and **cumulative mass distribution**.
- Finish discussing aerosol particle statistics: Predict how the particle statistics change after passing through an APCS.
- News article presentation by Isaac Moore

Mass Distribution

Additional analysis of the sample particle data (class handout; also see Excel spreadsheet):

j = class (bin number)	$D_{p,\min,j}$ (lower limit)	$D_{p,\max,j}$ (upper limit)	$D_{p,j}$ (middle value)	$m_{j,\text{in}} = n_j \rho_p \pi D_{p,j}^3/6$ = mass in class j (mg)	$m_{j,\text{in}} / \Delta(D_{p,j})$ (mass in class j divided by class width)	$g(D_{p,j})_{\text{in}} = m_j/m_t$ (mass fraction of original aerosol)	$G(D_p)_{\text{in}} = (M_j/m_t)_{\text{in}}$ = cumulative mass distribution (%)
1	1	4	2.5	8.50848E-07	2.83616E-07	0.000373985	0.037398476
2	4	6	5	1.0472E-05	5.23599E-06	0.004602889	0.497687408
3	6	8	7	2.89147E-05	1.44573E-05	0.012709268	1.768614192
4	8	9	8.5	2.41166E-05	2.41166E-05	0.01060031	2.828645219
5	9	10	9.5	3.00777E-05	3.00777E-05	0.013220447	4.150689966
6	10	14	12	0.000168289	4.20722E-05	0.073970273	11.54771722
7	14	16	15	0.000107796	5.38979E-05	0.047380992	16.28581641
8	16	20	18	0.000241237	6.03092E-05	0.106034	26.88921637
9	20	35	27.5	0.00098003	6.53353E-05	0.430765712	69.96578759
10	35	50	42.5	0.000683305	4.55536E-05	0.300342124	100
11	50	100	75	0	0	0	100

Mass of particles in class j: m_j

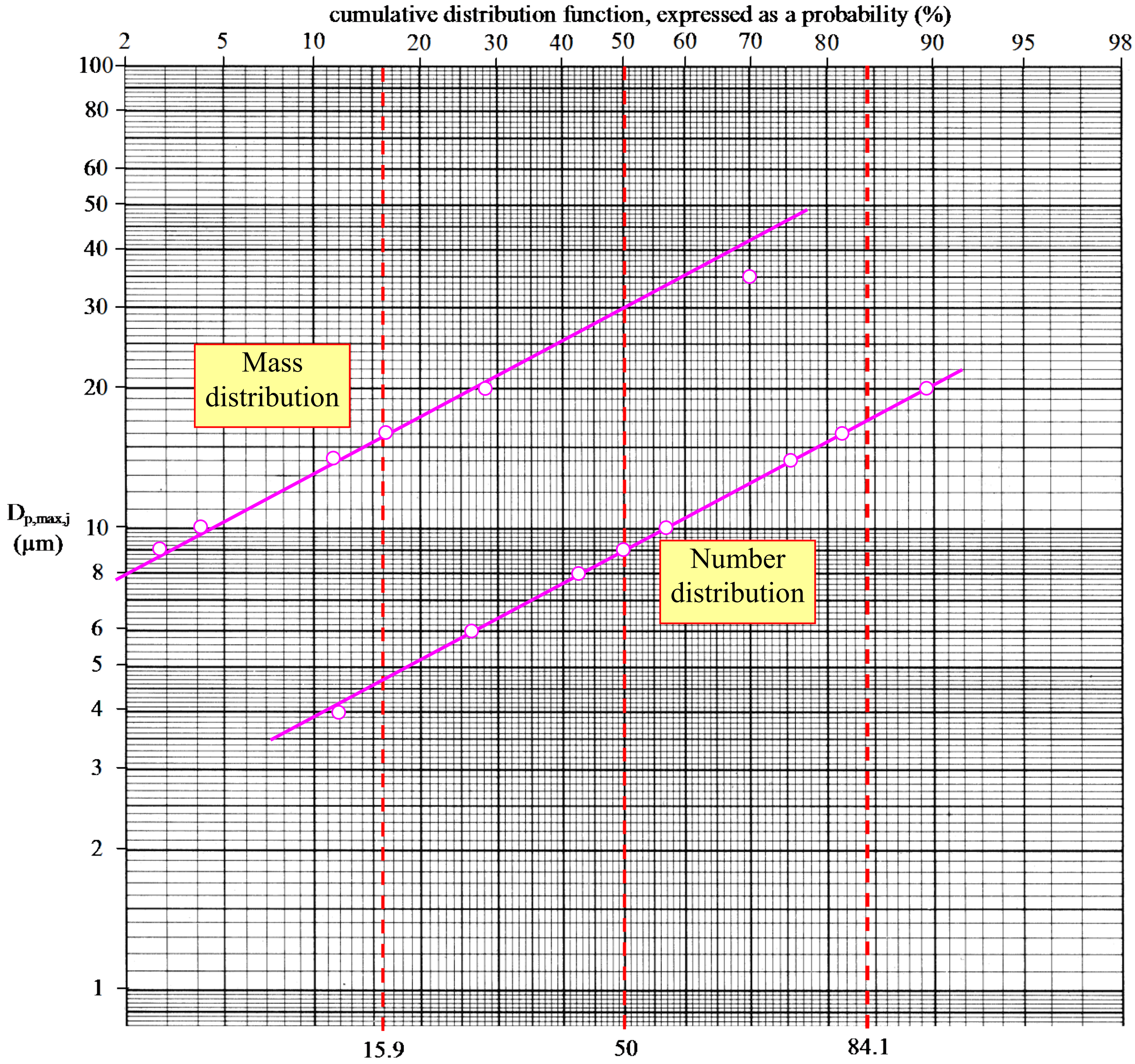
Mass of particles in class j divided by class width: $m_j/(\Delta D_{p,j})$

Mass fraction of particles in class j: $g(D_{p,j}) = m_j/m_t$

Cumulative mass distribution function: $G(D_p)$ [sometimes written as M_j/m_t]

When plot this on log-probability paper, remember to use $D_{p,\max,j}$, *not* $D_{p,j}$ since the cumulative distribution function includes the *entire bin* from min to max.

Number distribution and mass distribution for the sample data of the class handout



Example: Converting from mass distribution to number distribution

Given: Mingshou uses a cascade impactor to sample the air quality in Beijing. After carefully weighing all the trays before and after the sample, he plots the cumulative mass distribution on log-probability paper. The data fit fairly nicely into a straight line. From the plot, he determines that $D_{p,15.9}$ (mass) = 2.9 microns, and $D_{p,50}$ (mass) = 5.8 microns.

To do: For the air quality study that Mingshou is performing, he needs to know the median particle diameter based on *number*, not mass. From the given data, estimate $D_{p,50}$ (number). Give your answer in microns to two significant digits.

Solution: Some equations: $D_{p,gm}(\text{number}) = D_{p,50}(\text{number})$, $D_{p,gm}(\text{mass}) = D_{p,50}(\text{mass})$,

$\sigma_g = \dfrac{D_{p,50}(\text{number})}{D_{p,15.9}(\text{number})} = \dfrac{D_{p,50}(\text{mass})}{D_{p,15.9}(\text{mass})}$, $\ln\left(D_{p,50}\ (\text{mass})\right) = \ln\left(D_{p,50}\ (\text{number})\right) + 3\left[\ln\left(\sigma_g\right)\right]^2$