Today, we will:

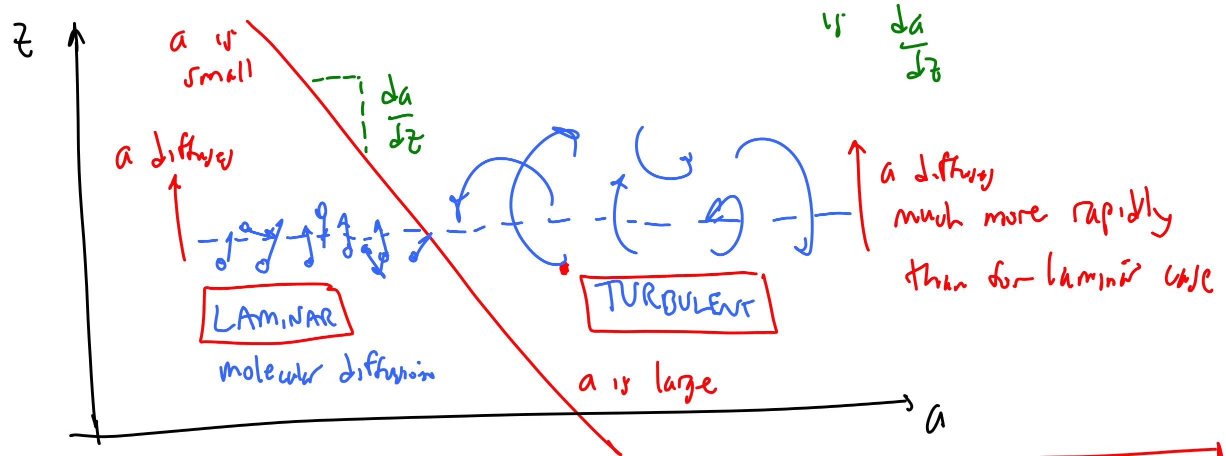
- Discuss **gradient diffusion** and the **Reynolds analogy**

Goal – To discuss & predict plume dispersion in the atmosphere

Gradient Diffusion Let a = some concentration of a property A

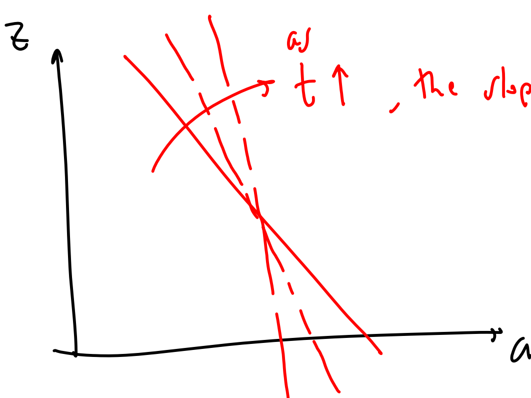
Let $a = \frac{A}{V}$ (A per unit volume) (a can be any property)

Simple 1-D diffusion Let $a = a(z)$ → Gradient (slope) of a w.r.t. z is $\frac{da}{dz}$



Either way, laminar or turbulent, $\boxed{\text{a will diffuse from high concentration of a to low concentration of a}}$

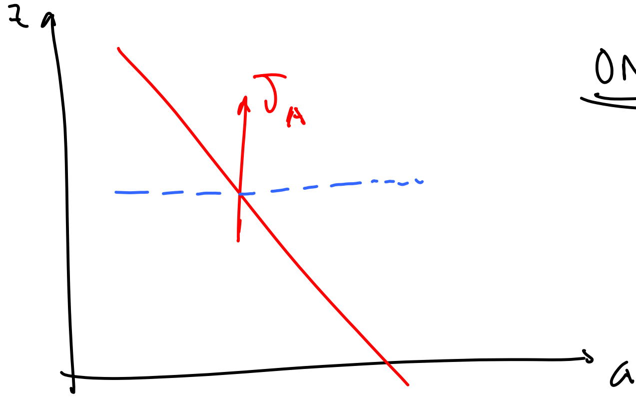
1-D GRADIENT DIFFUSION



as $t\uparrow$, the slope gets smaller (a tries to get uniform)

Mathematically, let $\boxed{J_A = \text{net amount of property A transported per unit time per unit area in the z-direction}}$

let $\boxed{b = \text{a diffusion coefficient}}$ → b determines how rapidly A diffuses



ONE-D DIFFUSION EQ. for any property (A or a)

$$\boxed{J_A = -b\,\frac{da}{dz}}$$

$-$ because J_A is $+$ when $\frac{da}{dz}$ $-$

J_A is $-$ when da/dz $+$

Dimensions:

A $\frac{1}{t}$

B $\frac{L}{t}$

C L^2/t

D L^3/t

E $1/(Lt)$

$$\{a\} = \left\{\frac{A}{Vol}\right\} = \left\{\frac{A}{L^3}\right\}$$

$$\{J_A\} = \left\{\frac{A}{area \cdot time}\right\} = \left\{\frac{A}{L^2 \cdot t}\right\}$$

In M, L, t, T system of primary dimensions

$$\{b\} = \left\{\frac{J_A}{da/dz}\right\} = \left\{\frac{\cancel{A}}{L^2 \cdot t}\,\frac{L^3\,L}{\cancel{A}}\right\} = \left\{\frac{L^2}{t}\right\}$$

units typ. m^2/s

EXAMPLES OF 1-D DIFFUSION EQ

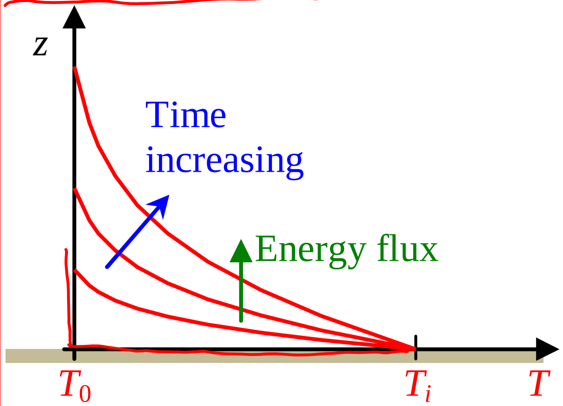
$$\boxed{J_A = -b\, \frac{da}{dz}}$$

(A), (a) Property w/ a gradient	(J_A) amt of A per unit area, per unit time	(b) Diffusion coeff	1-D gradient diffusion eq.
$\boxed{\text{ENERGY}}$ $A = \text{energy} = mC_pT$ $a = \frac{A}{\forall} = \frac{m}{\forall} C_pT$ $\boxed{a = \rho C_p T}$ We have a gradient of T when $\rho, C_p = \text{const.}$	q = heat flux = rate of heat (energy) transfer per unit area $\left\{ \frac{\text{energy}}{\text{area} \cdot \text{time}} \right\}$	K [Some books use α] (thermal diffusivity) $K = \rho C_p k$ k = thermal conductivity $\{K\} = \left\{\frac{L^2}{t}\right\}$ (m^2/s)	$\boxed{q = -K \frac{d}{dz}(\rho C_p T)}$ or $\boxed{q = -k \frac{dT}{dz}}$ ✱ HEAT DIFFUSION EQ (one-d)
$\boxed{\text{MOMENTUM}}$ $A = \text{momentum} = mU$ $a = \frac{mU}{\forall} = \boxed{a = \rho U}$ We have a gradient of U (velocity) when $\rho = \text{const}$	$-\tau$ = shear stress Fluid mechanics convention for defining τ $-\tau$ = rate of momentum transfer per unit area $\frac{d}{dt}\left(\frac{\text{mom}}{\text{area}}\right) = \left\{\frac{\text{Force}}{\text{area}}\right\}$ [Force = rate of change of momentum]	ν Kinematic viscosity $\left[\nu = \frac{\mu}{\rho}\right]$ $\{\nu\} = \left\{\frac{L^2}{t}\right\}$ (m^2/s)	$-\tau = -\nu \frac{d(\rho U)}{dz}$ or $\boxed{-\tau = -\mu \frac{dU}{dz}}$ $\boxed{\tau = \mu \frac{dU}{dz}}$
$\boxed{\text{MASS}}$ → mols of species j Let $A = n_j$ = # mols $a = \frac{n_j}{\forall} = C_{molar,j}$ molar conc. of species j	J_j = molar flux = rate of transfer of mols of j per unit area $\left\{\frac{\text{mols}}{\text{area} \cdot \text{time}}\right\}$	D_{aj} = binary diffusion coefficient between air & j $\left\{\frac{L^2}{t}\right\}$ (m^2/s)	$\boxed{J_j = -D_{aj} \frac{dC_{molar,j}}{dz}}$ FICK'S LAW

Alternate version of Fick's Law → Multiply by M_j → $\boxed{M_j J_j = -D_{aj} \frac{dc_j}{dz}}$ ✱

[Since $c_j = C_{molar,j} M_j$]

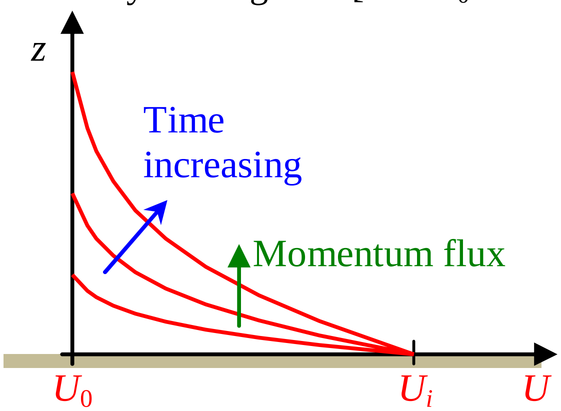
Reynolds Analogy – Energy, momentum, and mass, all diffuse in similar fashion. Compare:
Suddenly heated wall [$T = T_0 = 0^{o}$C everywhere, then suddenly $T = T_i$ at the wall.]



• energy is diffused upward

• rate of diffusion depends on κ (thermal diffusivity)

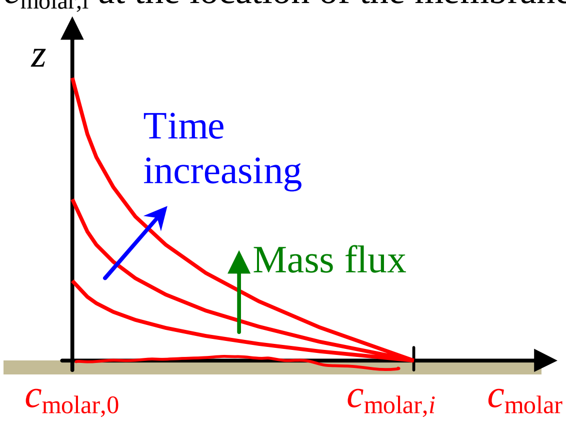
Suddenly moving wall [$U = U_0 = 0$ m/s everywhere, then suddenly $U = U_i$ at the wall.]



• momentum is diffused upward

• rate of diffusion depends on ν (kinematic viscosity)

Sudden removal of a membrane [$c_{molar} = c_{molar,0} = 0$ mol/m^3 everywhere, then suddenly $c_{molar} = c_{molar,i}$ at the location of the membrane, and the membrane disappears suddenly).]



• mass (or mols) is diffused upward

• rate of diffusion depends on $D_{a,j}$ (binary diffusion coeff.)

Also a model of vapor evaporating from a liquid surface

THIS SIMILAR BEHAVIOR IS THE BASIS OF THE REYNOLDS ANALOGY