Today, we will:

- Continue our discussion of **gradient diffusion** and the **Reynolds analogy**
- Begin to discuss the **Gaussian plume model**: how pollutants diffuse in plumes

Nondimensional ratios of diffusion coefficients

recall $\{b\} = \left\{\frac{L^2}{t}\right\}$ (m^2/s) ← any b (diffusion coeff) has same units (m^2/s)

Energy → K
Momen → ν
Species → D_{aj}

all have units m^2/s ∴ their ratios are nondimensional

These ratios are important & significant

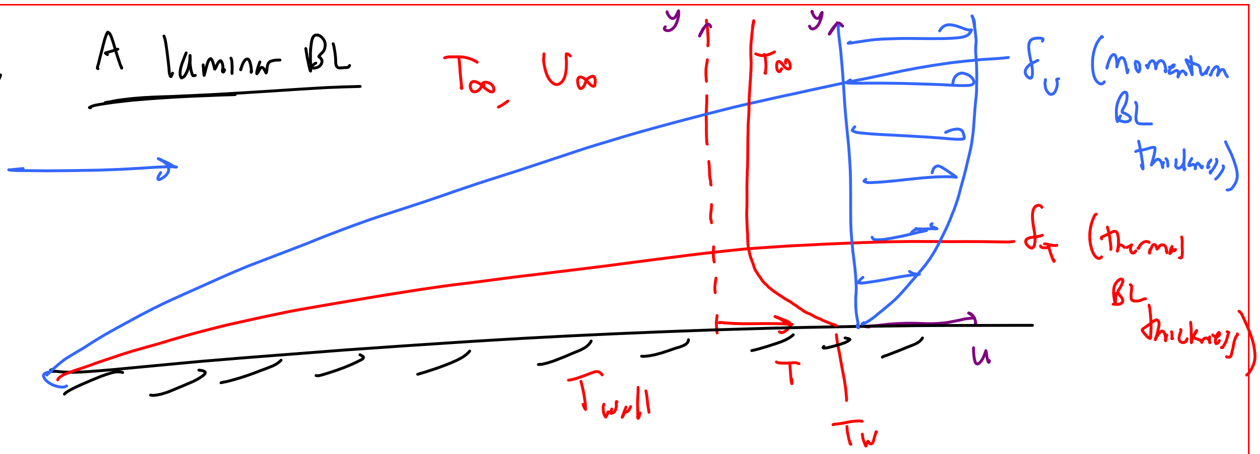
$\boxed{Sc = \text{Schmidt \#} = \frac{\nu}{D_{aj}} = \text{ratio of momentum diffusion to species diffusion}}$

Small Sc means that species diffuses faster than momentum
large Sc " " momen. " " " species

$\boxed{Pr = \text{Prandtl \#} = \frac{\nu}{K} = \text{ratio of momentum diffusion to heat energy diffusion}}$

$\boxed{Le = \text{Lewis \#} = \frac{K}{D_{aj}} = \text{ratio of heat energy diffusion to species diffusion}}$

Example A laminar BL



We can conclude

(a) $P_r < 1$

(b) $P_r \approx 1$

$\boxed{\text{(c) } P_r > 1}$

$$P_r = \frac{\nu}{K}$$

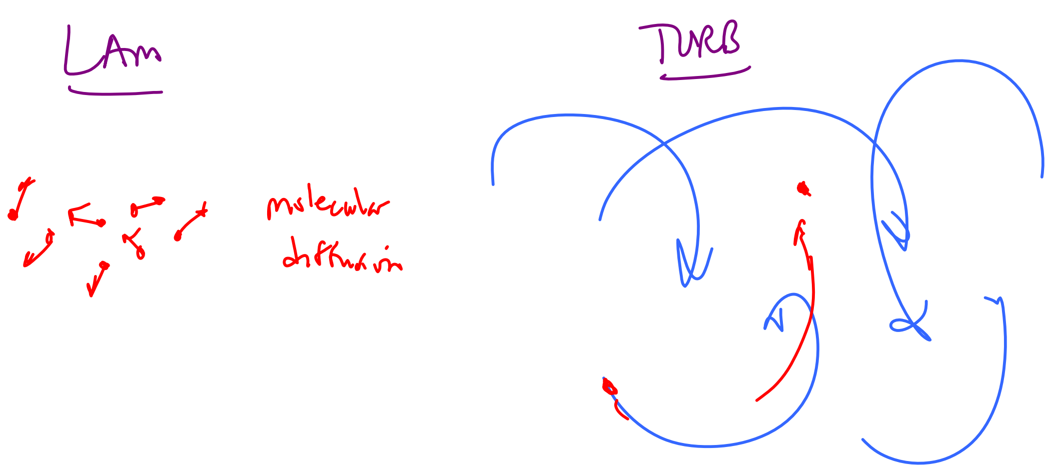
ν is bigger than K since momentum diffuses faster than heat energy

Eg. water $\rightarrow P_r \approx 7$

air $\rightarrow P_r \approx 0.7$ (opposite)

Reynolds Analogy

- For turbulent flow, diffusion is dominated by large eddies mixing up the flow



Diffusion coefficients for turbulent flow >> those of laminar flow

Reynolds analogy → energy, momentum, & mass diffusion all occur at similar rates in turbulent flow (due to the eddies)

Usefulness → easy to measure heat transfer characteristics
hard " " species " "

We can use heat transfer correlations to predict mass transfer (species) diffusion

<u>Define</u> <u>turbulent Prandtl #</u> $\boxed{Pr_t = \frac{\nu_t}{K_t}}$

<u>" Schmidt #</u> $\boxed{Sc_t = \frac{\nu_t}{D_{ij\,t}}}$

<u>turb. Lewis #</u> $\boxed{Le_t = \frac{K_t}{D_{ij\,t}}}$

Mixing or Diffusion is dominated by the turbulent eddies

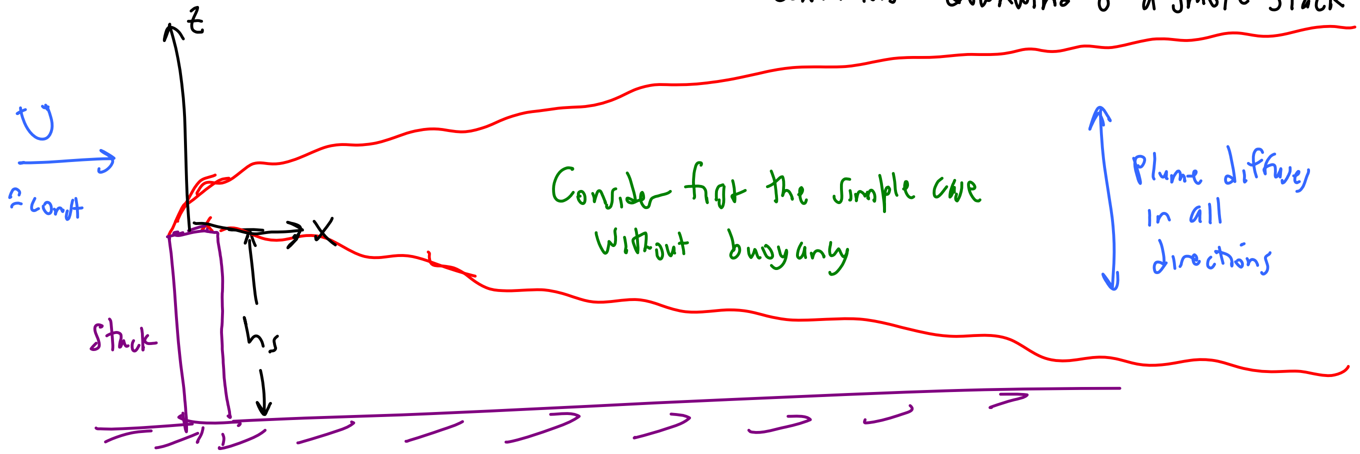
What is the approx. value of $\boxed{Pr_t \approx Sc_t \approx Le_t \approx 1}$

Mass, momen., & heat are diffused by the turbulent eddies, all at ≈ same rate

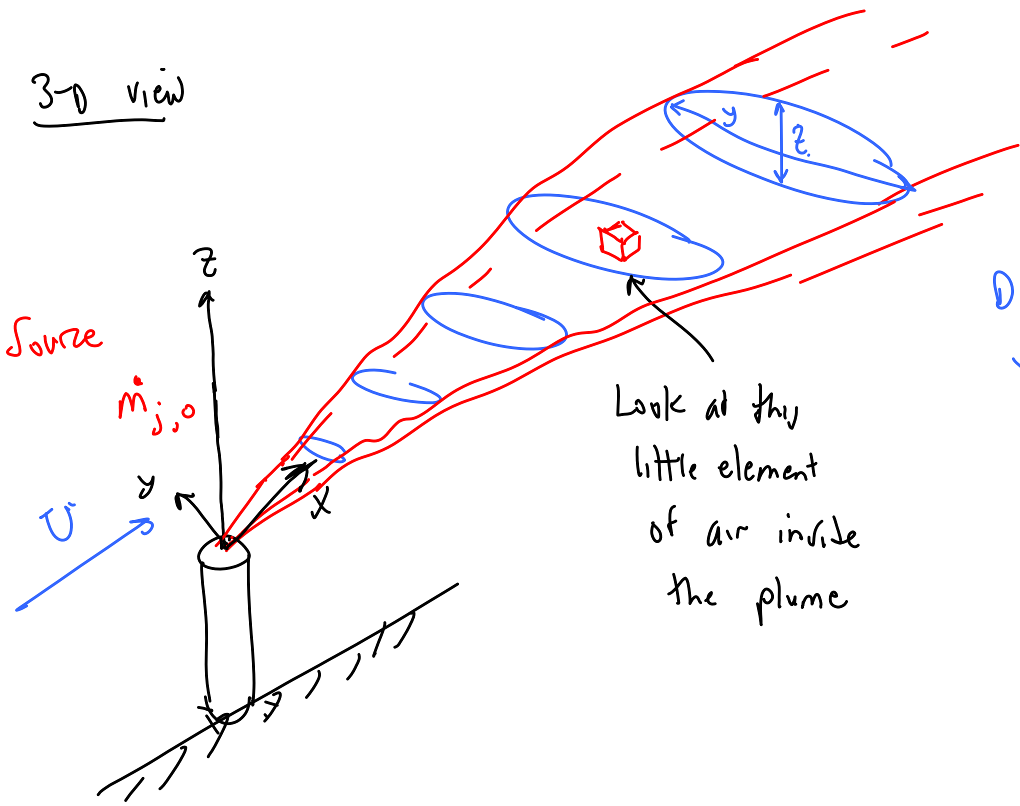
<u>Reynolds analogy</u>

Gaussian Plume Model

→ useful model to predict air pollution concentrations downwind of a smoke stack



3-D view



Normal atmosphere is stable

Diffusion in y-direction > (faster) than Diffusion in z-direction

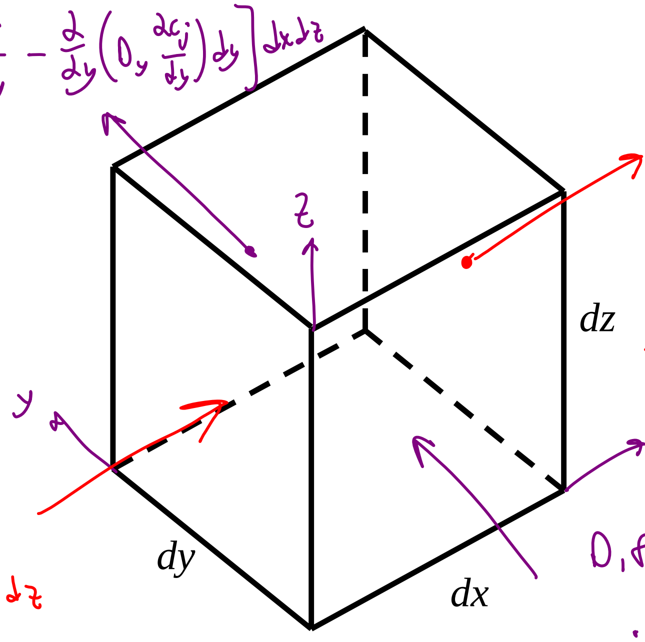
Consider a small element of air in the plume of dimensions *dx*, *dy*, and *dz* as sketched here.

Cons. of mass balance of species j → Use Taylor series expansion, truncate to first-order

$\boxed{\text{Notation} \rightarrow D_y = D_{aj,y} \rightarrow \text{Diffusion coeff. in } y\text{-direction}}$

$$\dot m_{out} = \left[-D_y \frac{\partial c_j}{\partial y} - \frac{\partial}{\partial y}\left(D_y \frac{\partial c_j}{\partial y}\right) dy\right] dx\,dz$$

$$\dot m_{out} = \left[U c_j + \frac{\partial}{\partial x}(U c_j)\,dx\right] dy\,dz + \left[-D_x \frac{\partial c_j}{\partial x} - \frac{\partial}{\partial x}\left(D_x \frac{\partial c_j}{\partial x}\right) dx\right] dy\,dz$$



"Advection"

$$\dot m_{in} = U c_j\, dy\, dz$$

$+$

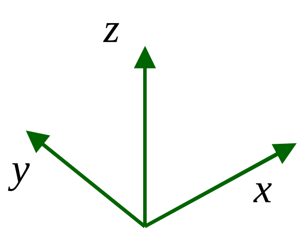
Diffusion

$$-D_x \frac{\partial c_j}{\partial x}\, dy\, dz$$

$$\frac{m^2}{s} \quad \frac{mg}{m^3\, m} \quad m \quad m$$

Diffusion only (no advection)

$$\dot m_{in} = -D_y \frac{\partial c_j}{\partial y}\, dx\, dz$$



+ Similar terms in z-direction

Let's sum up all the mass flow rates into our little element

$$\frac{dm_j}{dt} = \forall \frac{dc_j}{dt} = dx\,dy\,dz\,\frac{dc_j}{dt} = \overset{IN_x}{\cancel{U c_j\, dy\, dz}} - \overset{OUT_x}{\left[\cancel{U c_j} + \frac{\partial}{\partial x}(U c_j)\,dx\right] dy\,dz}$$

$$- \cancel{D_x \frac{\partial c_j}{\partial x}\, dy\, dz} - \left[\cancel{-D_x \frac{\partial c_j}{\partial x}} - \frac{\partial}{\partial x}\left(D_x \frac{\partial c_j}{\partial x}\right) dx\right] dy\,dz$$

$+$ y-terms $+$ z-terms

Add up all terms (all 6 of them) (6 faces), $\div\, dx\,dy\,dz$

$$\boxed{\frac{\partial c_j}{\partial t} = -\frac{\partial}{\partial x}\left(U c_j\right) + \frac{\partial}{\partial x}\left(D_x \frac{\partial c_j}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_y \frac{\partial c_j}{\partial y}\right) + \frac{\partial}{\partial z}\left(D_z \frac{\partial c_j}{\partial z}\right)}$$