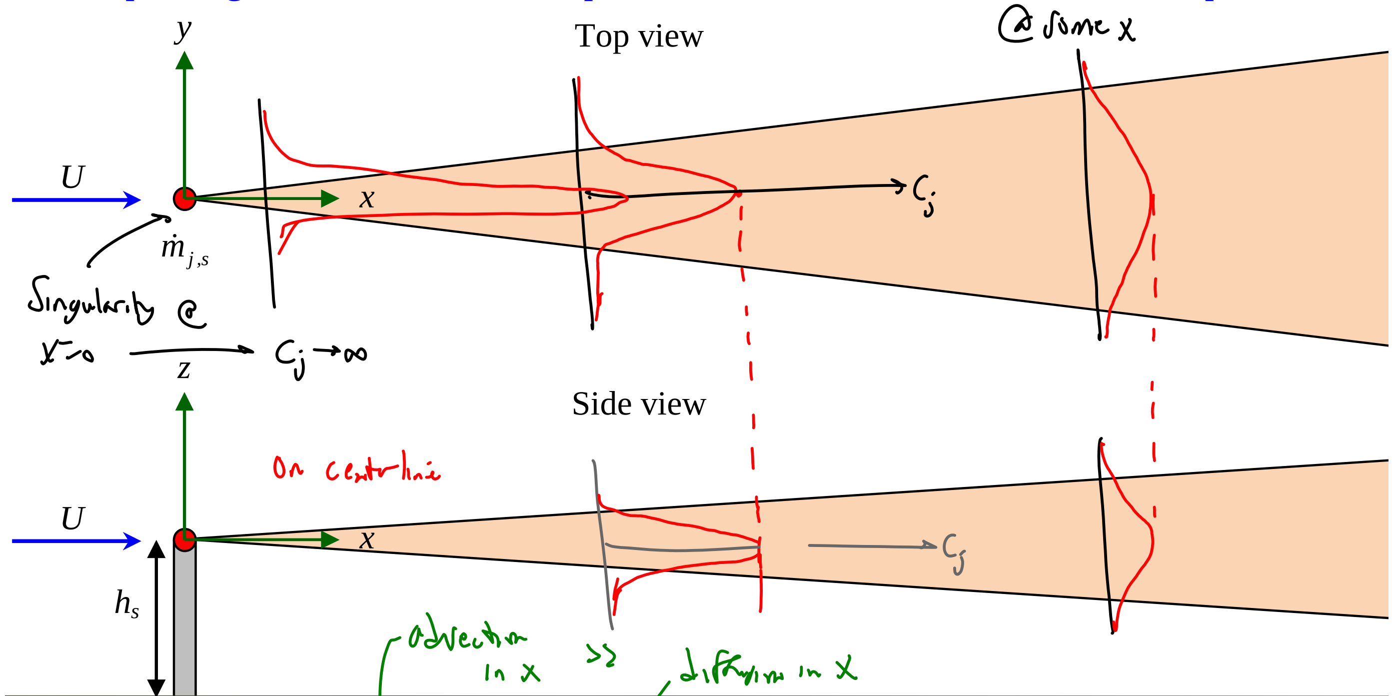
Today, we will:

- Continue derivation of the **Gaussian plume model** – equation and solution
- Modify the solution for a *buoyant* plume, and do some examples/applications
- Compare a ground that absorbs the pollutant vs. one that does *not* absorb the pollutant



advection in x >> diffusion in x

From the previous lecture, we derived the differential equation for a non-buoyant Gaussian plume, assuming gradient diffusion, constant U, and constant diffusion coefficients:

U=const

$$\cancel{\frac{\partial c_j}{\partial t}} = -\frac{\partial}{\partial x}\left(Uc_j\right) + \cancel{\frac{\partial}{\partial x}\left(D_x \frac{\partial c_j}{\partial x}\right)} + \frac{\partial}{\partial y}\left(D_y \frac{\partial c_j}{\partial y}\right) + \frac{\partial}{\partial z}\left(D_z \frac{\partial c_j}{\partial z}\right)$$

where our notation is $D_x = D_{aj,x}$, $D_y = D_{aj,y}$, and $D_z = D_{aj,z}$ for simplicity.

Assumptions and Approximations:

- Steady (LHS = 0)
- U = const (U comes out of derivative)
- Advection in x >> diffusion in x since U dominates the x-direction
- D_y = const w.r.t. y & D_z = const w.r.t. z, but D_y & D_z = fncs of x

Eq. reduces to:

Simplified equation for a steady Gaussian plume without buoyancy:

$$\boxed{c_j = c_j(x,y,z)}$$

$$U\frac{\partial c_j}{\partial x} = D_y \frac{\partial^2 c_j}{\partial y^2} + D_z \frac{\partial^2 c_j}{\partial z^2} \qquad (1)$$

Now we apply boundary conditions (BCs) and solve (1) for $c_j(x,y,z)$ in this plume.

BC's (need 5)

(1) $\boxed{C_j \to 0}$ as $y \to \infty$ (far field, air is clean)

(2) $\boxed{C_j \to 0}$ as $z \to \infty$ "

In other words, $C_j = max$ @ $y=0$ & $z=0$

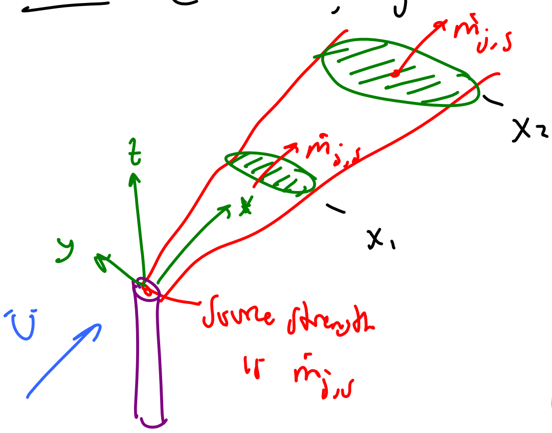
(3) Symmetry about y axis @ $y=0$ $\to$ $\boxed{\dfrac{\partial C_j}{\partial y} = 0 \text{ @ } y=0}$

(4) " " z " @ $z=0$ $\quad \boxed{\dfrac{\partial C_j}{\partial z} = 0 \text{ @ } z=0}$

for any $x>0$

(5) $\to$ need a BC in x.

Note: @ $x=0$, $C_j \to \infty$ (this is a point source)



$$\boxed{\int_{z=-\infty}^{\infty} \int_{y=-\infty}^{\infty} U C_j \, dy \, dz = \dot{m}_{j,s}}$$

Soln:

$$\boxed{C_j = \frac{\dot{m}_{j,s}}{2\pi x \sqrt{D_y D_z}} \exp\left[-\frac{U y^2}{4 x D_y} - \frac{U z^2}{4 x D_z} \right]} \quad (2)$$

Define:

$$\boxed{\sigma_y = \sqrt{\frac{2 x D_y}{U}}} \qquad \boxed{\sigma_z = \sqrt{\frac{2 x D_z}{U}}}$$

D_y & D_z = diffusion coeff's (m^2/s)

Dispersion coefficients (m)

Note: σ_y & σ_z are different

(2) becomes

$$\boxed{C_j = \frac{\dot{m}_{j,s}}{2\pi U \sigma_y \sigma_z} \exp\left\{ -\frac{1}{2} \left[\left(\frac{y}{\sigma_y}\right)^2 + \left(\frac{z}{\sigma_z}\right)^2 \right] \right\}} \quad (3)$$

No ground effect, no buoyancy

Need eq'n for σ_y & σ_z → Get these empirically

Depend on
- Type of terrain (rural, urban, forest)
- Stability of atmosphere (meteorology, lapse rate)
- x (distance from source)

There are many empirical eq'n for σ_y & σ_z – can be complicated

We will use this model:

$$\boxed{\begin{aligned} \sigma_y &= a x^b \\ \sigma_z &= c x^d + f \end{aligned}}$$

x is in km

σ_y & σ_z are in m

Be careful of units

See Tables 20.1 & 20.2 – Cooper & Alley

Example

Calc σ_y & σ_z @ $x = 2.0$ km

$U = 4.8$ m/s

on an overcast morning

Sol'n: Table 20.1 → $\boxed{\text{class D}}$

Table 20.2 → $a = 68 \quad b = 0.894$

$x > 1$ km → $c = 44.5 \quad d = 0.516, \quad f = -13.0$

$$\sigma_y = a x^b \rightarrow 68(2)^{0.894} = \boxed{126.4 \text{ m}}$$

put x in as km

$$\sigma_z = c x^d + f \rightarrow 44.5(2)^{0.516} - 13.0 = \boxed{50.63 \text{ m}}$$

notice $\sigma_y > \sigma_z$