Today, we will:

- Discuss Gaussian plume with a reflecting ground and within a temperature inversion
- Discuss the **Gaussian puff diffusion model** (sudden burst of air pollution from a point)

Gaussian plume with a reflecting ground and an elevated temperature inversion

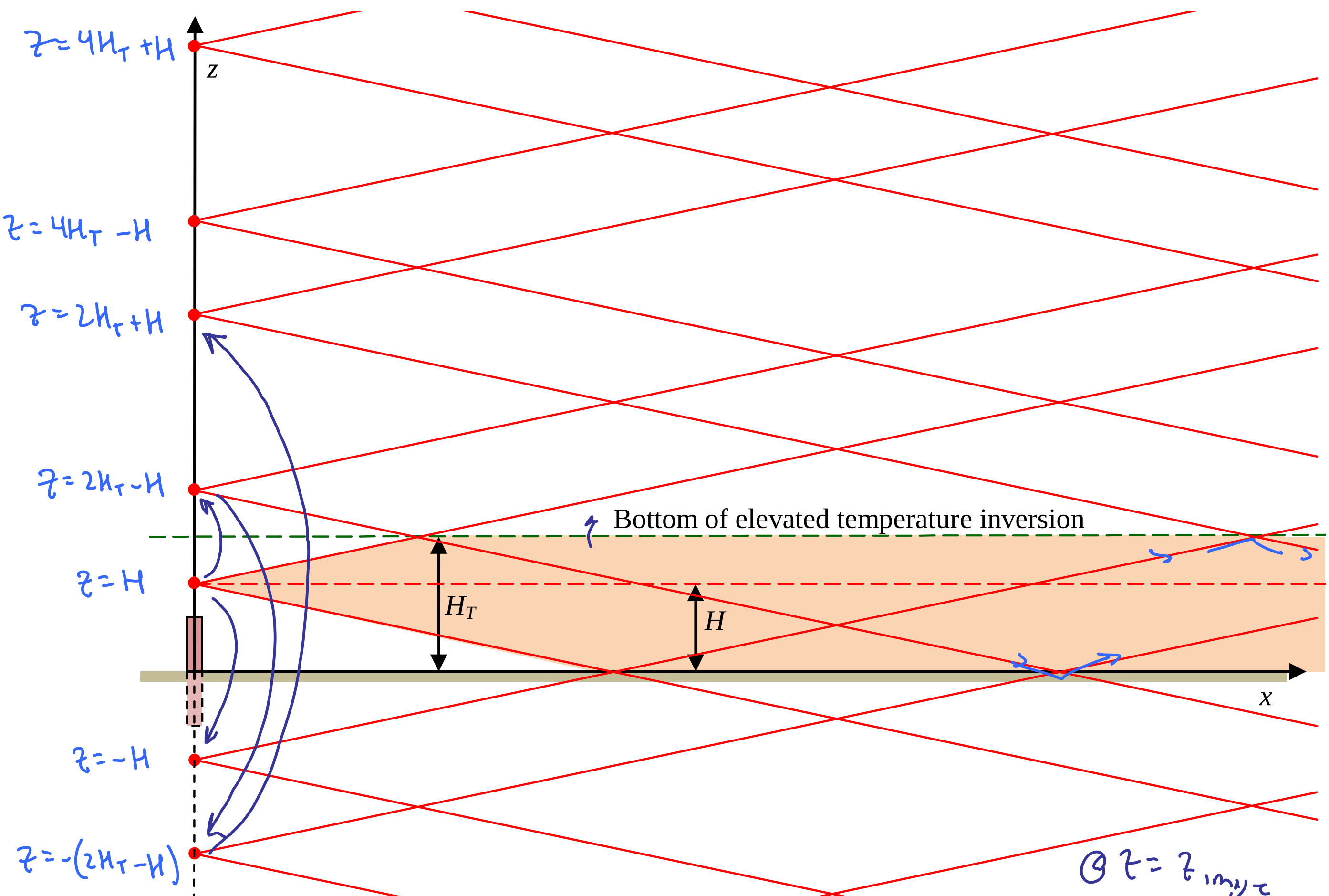
Start with the standard Gaussian plume model with ground reflection:

$$c_j = \frac{\dot{m}_{j,s}}{2\pi U \sigma_y \sigma_z}\left[\exp\left\{-\frac{1}{2}\left[\left(\frac{y}{\sigma_y}\right)^2 + \left(\frac{z-H}{\sigma_z}\right)^2\right]\right\} + \exp\left\{-\frac{1}{2}\left[\left(\frac{y}{\sigma_y}\right)^2 + \left(\frac{z+H}{\sigma_z}\right)^2\right]\right\}\right]$$

Take the y terms outside and re-write (since $e^{a+b} = e^a e^b$):

$$c_j = \frac{\dot{m}_{j,s}}{2\pi U \sigma_y \sigma_z}\exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]\left\{\exp\left[-\frac{1}{2}\left(\frac{z-H}{\sigma_z}\right)^2\right] + \exp\left[-\frac{1}{2}\left(\frac{z+H}{\sigma_z}\right)^2\right]\right\}$$

But, we have *another* reflection, mirror imaged around the bottom of the elevated temperature inversion. Mathematically, we need an *infinite* number of image sources to account for *both* reflections.



@ $z = z_{image}$, replace z with $(z - z_{image})$ in eq.

Sources are at $\boxed{z = H + 2jH_T \quad \& \quad z = -H + 2jH_T}$ where j = any integer (include zero) → in eq. $\sum_{j=-\infty}^{\infty}$

Now add up *all* the sources in an infinite series (including $j = 0$):

$$\boxed{c_j = \frac{\dot{m}_{j,s}}{2\pi U \sigma_y \sigma_z} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] \sum_{j=-\infty}^{\infty} \left\{ \exp\left[-\frac{1}{2}\left(\frac{z-H-2jH_T}{\sigma_z}\right)^2\right] + \exp\left[-\frac{1}{2}\left(\frac{z+H-2jH_T}{\sigma_z}\right)^2\right] \right\}}$$

Gaussian Plume with stack below elevated temperature inversion & ground reflecting

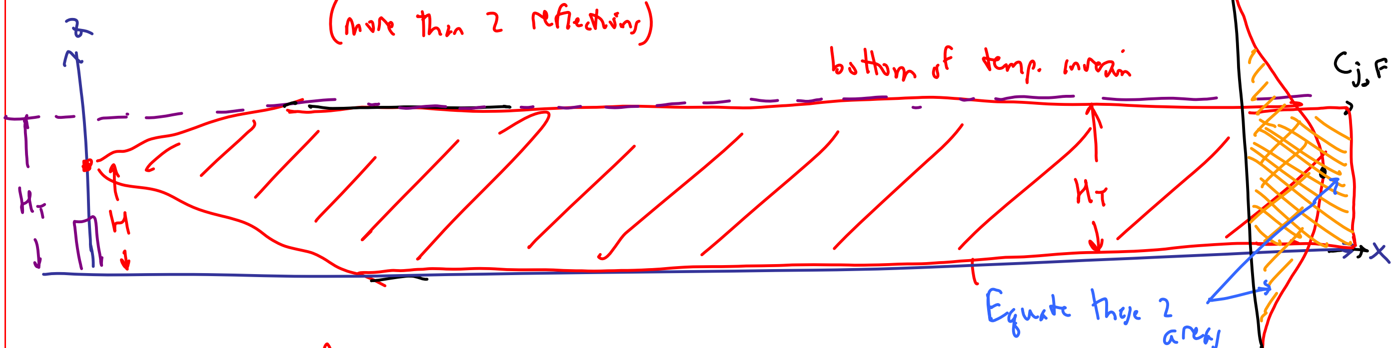
For practical problems → can use a few j's & do a good job

$z - (H + 2jH_T)$

$z - (-H + 2jH_T)$

Approximation for far downstream → Fumigating plume, Ground reflecting

(more than 2 reflections)



Assume the air pollutant is well mixed → c_j = constant in vertical direction between $z = 0$ & H_T

All of the pollutant mass is confined between $z = 0$ & $z = H_T$

Equate areas: @ some x location

$$\int_{z=0}^{\infty} \frac{\dot{m}_{j,s}}{2\pi U \sigma_y \sigma_z} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] \exp\left[-\frac{1}{2}\left(\frac{z-H}{\sigma_z}\right)^2\right] dz = c_{j,F} \cdot H_T$$

far downstream

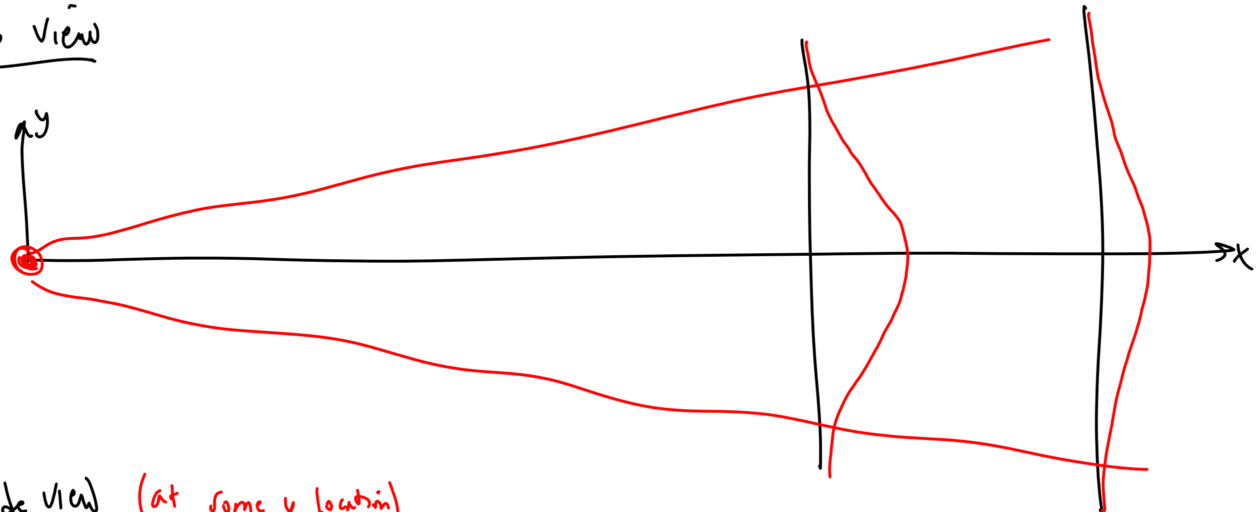
$$= \frac{\dot{m}_{j,s}}{\sqrt{2\pi}\, U \sigma_y} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] \underbrace{\int_{z=-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_z} \exp\left[-\frac{1}{2}\left(\frac{z-H}{\sigma_z}\right)^2\right] dz}_{1} = c_{j,F} H_T$$

$$\boxed{C_{j,F} = \frac{\dot{m}_{j,s}}{\sqrt{2\pi}\, U \sigma_y H_T} \underline{\exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]}}$$

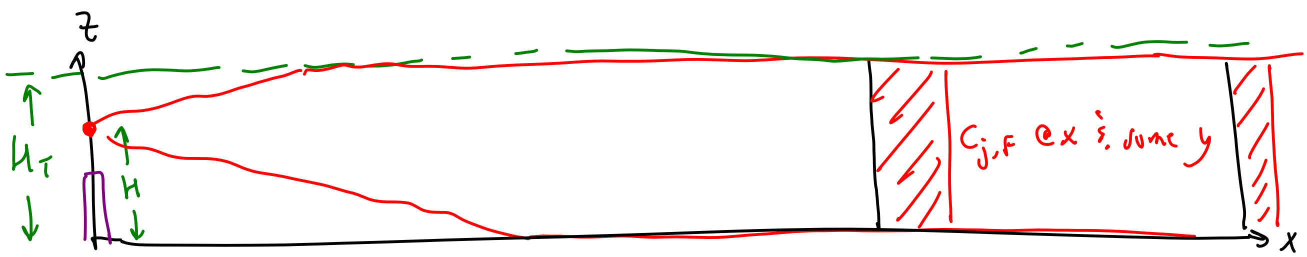
$$\boxed{C_{j,F} = fn\left(x, y\right)}$$

at any x location far downstream

Top view

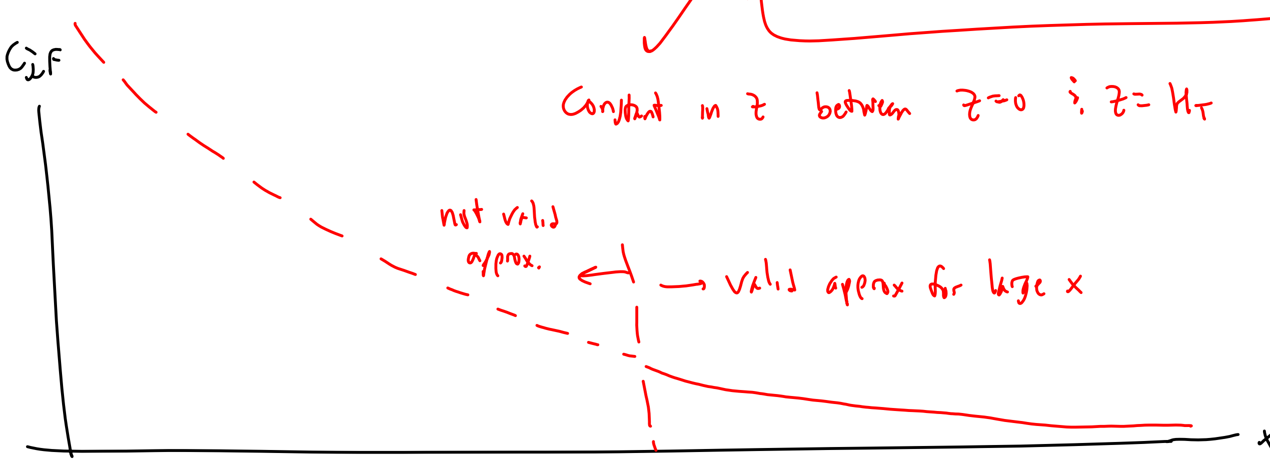


Side view (at some y location)



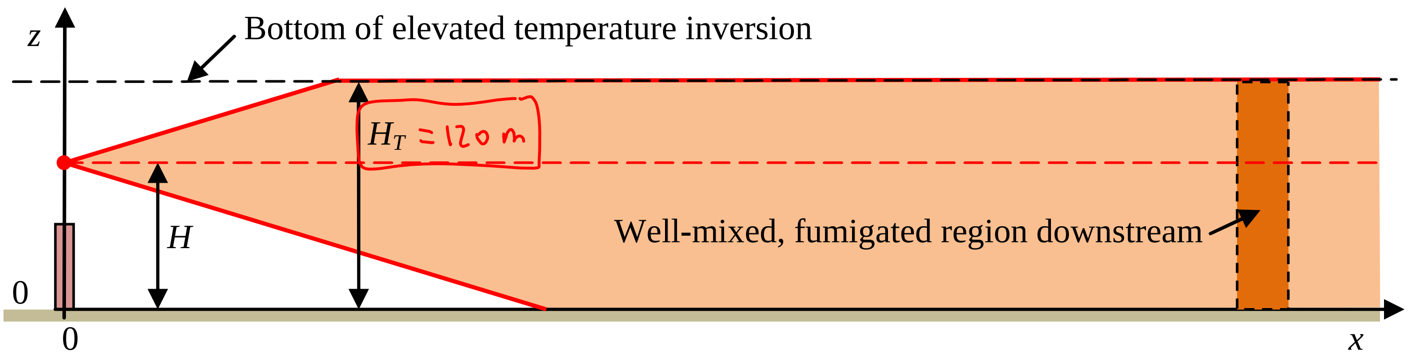
For $y=0$ (centerline), $\exp(0) = 1$ $\rightarrow$ $\boxed{C_{j,F} = \frac{\dot{m}_{j,s}}{\sqrt{2\pi}\, U \sigma_y H_T} \quad @ \; y=0}$

Constant in z between $z=0$ & $z=H_T$



Example: Fumigating Gaussian plume

Given: A buoyant plume emitting air pollution, under the following conditions:



- Stack height = 80 m. Buoyant plume rise = 20 m above stack exit. → $H = 100$ m
- The stack emits the air pollutant at a rate of 110 g/s.
- An elevated temperature inversion is present, extending from 120 m to 140 m.
- The average wind speed is a gentle 1.4 m/s.
- Both above and below the temperature inversion, the atmosphere is very unstable, and is classified as Class A.
- Far downstream, the mass concentration of the air pollutant is well mixed (constant) vertically between the ground and the bottom of the elevated temperature inversion, and people who are downwind of the plume are fumigated, as sketched.
- The ground reflects (does not absorb) the air pollutant.

To do: At the centerline of the plume (y = 0), and at a downwind distance of 2.0 km, estimate the mass concentration of the pollutant experienced by people near the ground.

Solution:

- Use **Table 20.2** to obtain the coefficients for calculation of dispersion coefficients:
  For Class A, we have ***a* = 213, *b* = 0.894**.
- At a given x location, calculate the dispersion coefficient in the y direction:
  $\boxed{\sigma_y = ax^b}$, with x in units of km and σ_y and σ_z in units of m.

  $\sigma_y = 213\,(2.0)^{0.894} = 395.822$ m
- Use the reflecting ground fumigating Gaussian plume equation at y = 0 (centerline) to calculate the well-mixed mass concentration at this particular value of x:

$$\boxed{c_{j,F} = \frac{\dot{m}_{j,s}}{\sqrt{2\pi}U\sigma_y H_T}\exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] \quad \rightarrow \quad \text{at } y=0,\ c_{j,F} = \frac{\dot{m}_{j,s}}{\sqrt{2\pi}U\sigma_y H_T}}$$

$x = 2.0$ km
$\dot{m}_{j,s} = 110$ g/s
$H_T = 120$ m
$U = 1.4$ m/s

Calc $c_{j,F}$ in $\frac{\mu g}{m^3}$ →

$$c_{j,F} = \frac{110 \text{ g/s}}{\sqrt{2\pi}\,(1.4 \text{ m/s})(395.822 \text{ m})(120 \text{ m})}\left(\frac{10^6\ \mu g}{g}\right) = 659.92\ \mu g/m^3$$

$$\boxed{c_{j,F} = 660\ \frac{\mu g}{m^3}}$$

≈ constant from ground to 120 m at this x & at centerline (y=0)