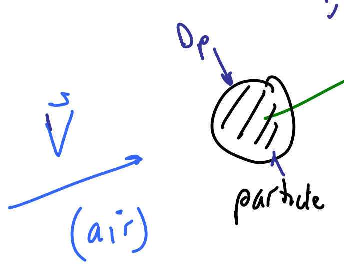
M E 433 Professor John M. Cimbala Lecture 23

Today, we will:
- Discuss drag coefficient and use it in the equations of motion for particle trajectories
- Discuss the **Cunningham correction factor** (small particles; free molecular flow effects)

recall,



$$\boxed{\vec F_{drag} = \frac{1}{2}\rho V^2 C_D A}$$

in direction of $\vec V$ (from fluids class)

drag coeff.

frontal area

sphere, $\boxed{A = \frac{\pi D_p^2}{4}}$

Where this $\vec V$ is the velocity of the air relative to the particle

we want $\vec V_r$ -- -- -- particle " " " air

$$\boxed{\vec V_r = \vec V - \vec U} \qquad \boxed{\vec V_r = -\vec V} \text{ for our analysis}$$

$$\vec F_{drag} = \frac{1}{2}\rho\left[-\vec V_r\left|\vec V_r\right|\right] C_D \frac{\pi D_p^2}{4}$$

"trick" to get both magnitude & direction

$$\therefore \boxed{\vec F_{drag} = -\frac{1}{8}\rho C_D \pi D_p^2 \vec V_r\left|\vec V_r\right|}$$

Acceleration: $m_p \vec a_p = \vec F_{grav} + \vec F_{drag}$

$\rho = \rho_{air}$

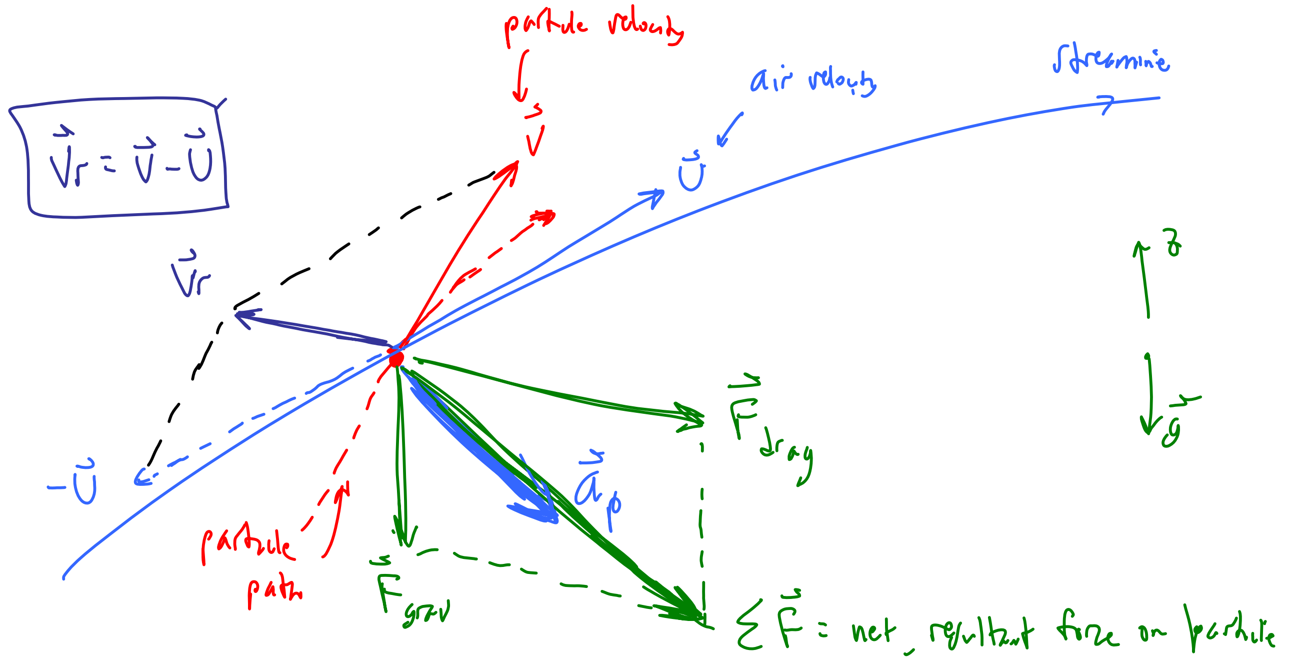
$\rho_p = \rho_{particle}$

$$\boxed{m_p \frac{d\vec V}{dt} = \frac{\pi D_p^3}{6}(\rho_p - \rho)\vec g - \frac{1}{8}\rho C_D \pi D_p^2 \vec V_r\left|\vec V_r\right|}$$

Eq of motion for a particle moving in an air stream (sphere)

$$\left[\vec a_p = \frac{d\vec V}{dt}\right]$$

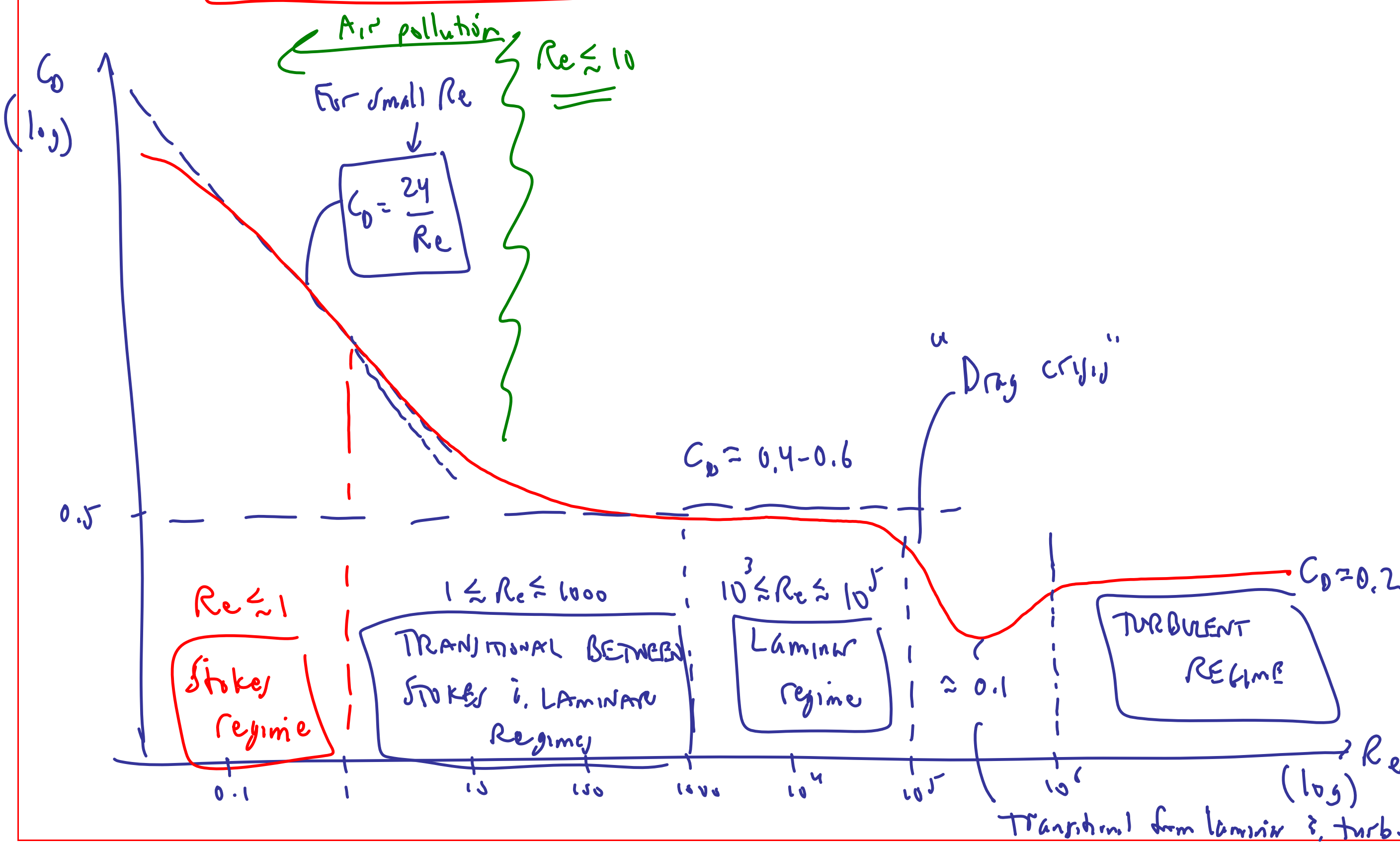
We need eq. for C_D



[$\vec{a}_p$ is in same direction as $\sum \vec{F}$]

Drag coefficient C_D for a sphere $\rightarrow$ $C_D = \text{func}(Re)$

$$\boxed{Re = \text{Reynolds } \# = \frac{\rho |\vec{V}_r| D_p}{\mu} = \frac{|\vec{V}_r| D_p}{\nu}}$$



Eqs for C_D

For $Re \lesssim 0.1$ $\quad \boxed{C_D = \frac{24}{Re}}$ (Stokes regime)

For $0.1 \lesssim Re \lesssim 5$ $\quad \boxed{C_D = \frac{24}{Re}\left(1 + 0.0916\, Re\right)}$ ☆ ← we will use most often

for $5 \lesssim Re \leq 1000$ $\quad \boxed{C_D = \frac{24}{Re}\left(1 + 0.158\, Re^{2/3}\right)}$

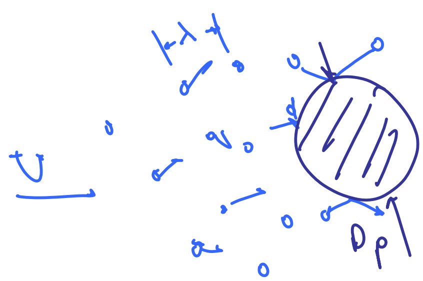
Applies also for $Re \lesssim 0.1$

Modification for very small particles → molecular effects are important

λ = mean free path of air molecules

$$\boxed{\lambda = \frac{\mu}{0.499}\sqrt{\frac{\pi}{8 \rho P}}}$$

@ STP, $\boxed{\lambda = 0.06704\ \mu m}$ *



as $D_p \to \lambda$, free molecular effects become important.

↓

air can no longer be treated as a continuum

Introduce Knudsen # $\quad \boxed{Kn = \frac{\lambda}{D_p}}$ (nondimensional)

Big particles $Kn \ll 1$ – continuum fluid mechanics (generally $Kn \lesssim 0.1$)

Small particles $Kn \gg 1$ – free molecular fluid mechanics (generally, $Kn \gtrsim 10$)

$0.1 \lesssim Kn \lesssim 10$ – transitional regime between continuum & free molecular flow

For free molecular flow, there is "slip" at the wall → no-slip condition is not satisfied

C_D is smaller than it would be because of this

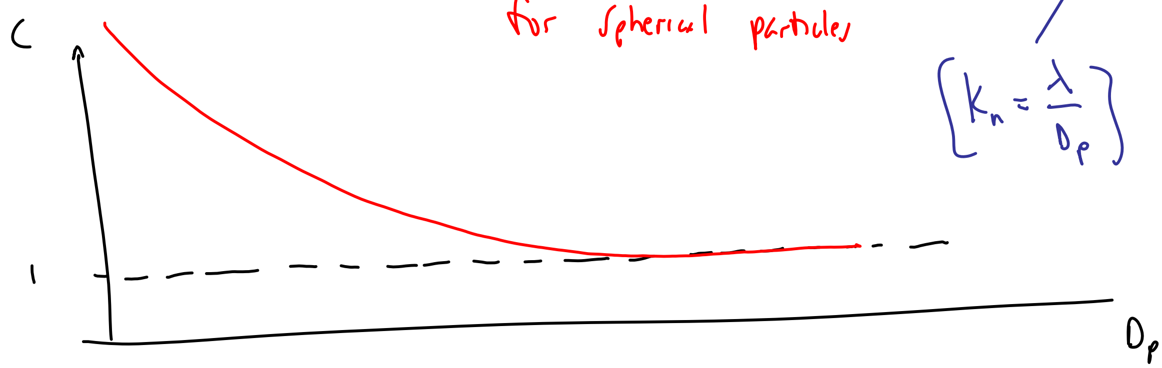
Introduce Cunningham correction factor C (nondimensional)

$C = fnc(K_n)$

empirical eq → ☆ $\boxed{C = 1 + K_n\left[2.514 + 0.80 \exp\left(-0.55/K_n\right)\right]}$

for spherical particles

$\left[K_n = \dfrac{\lambda}{D_p}\right]$



See #s next pg for air @ STP

$\boxed{C \text{ asymptotes to 1 as } D_p \text{ gets larger}}$ ☆

Example: Variation of Cunningham correction factor with particle diameter

Given: Air at STP has mean free path $\lambda = 0.06704$ microns. Knudsen number is defined as $\boxed{\mathrm{Kn} = \lambda / D_p}$. Cunningham correction factor is $\boxed{C = 1 + \mathrm{Kn}\left[2.514 + 0.80\exp\left(-0.55/\mathrm{Kn}\right)\right]}$.

To do:

Calculate C for various values of particle diameter D_p. [Give your answer to 4 significant digits, and be careful with units.]

text D_p, C

Solution:

Table to be filled in during class:

D_p (μm)	Kn	C
0.001	67.04	222.7
0.0025	26.82	89.4
0.006	11.17	37.60
0.01		22.79
0.025		9.489
0.06		4.355
0.1		2.921
0.25		1.702

(Kn decreases) (C decreases)

D_p (μm)	Kn	C
0.6		1.282
1		1.169
2.5		1.067
6		1.028
10		1.017
25		1.007
60		1.003

We should always account for C in air pollution problems

even at $D_p = 10\,\mu m$, C is important

Replace C_D with $\frac{C_D}{C}$

$$\vec{F}_{drag} = -\frac{1}{8}\rho\frac{C_D}{C}\pi D_p^2\,\vec{V}_r\left|\vec{V}_r\right|$$

Cunningham correction factor is a kind of "fudge factor" → makes C_D be smaller due to free molecular effects