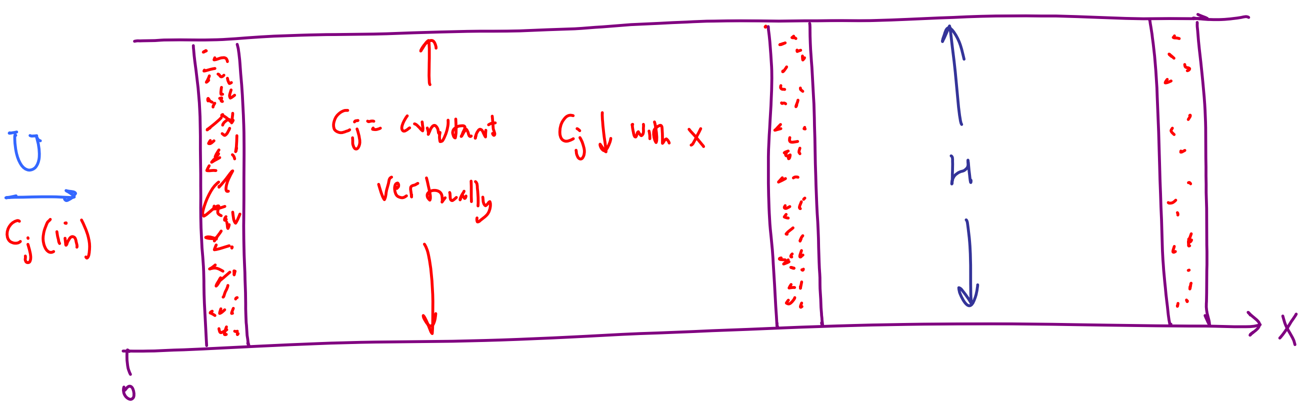
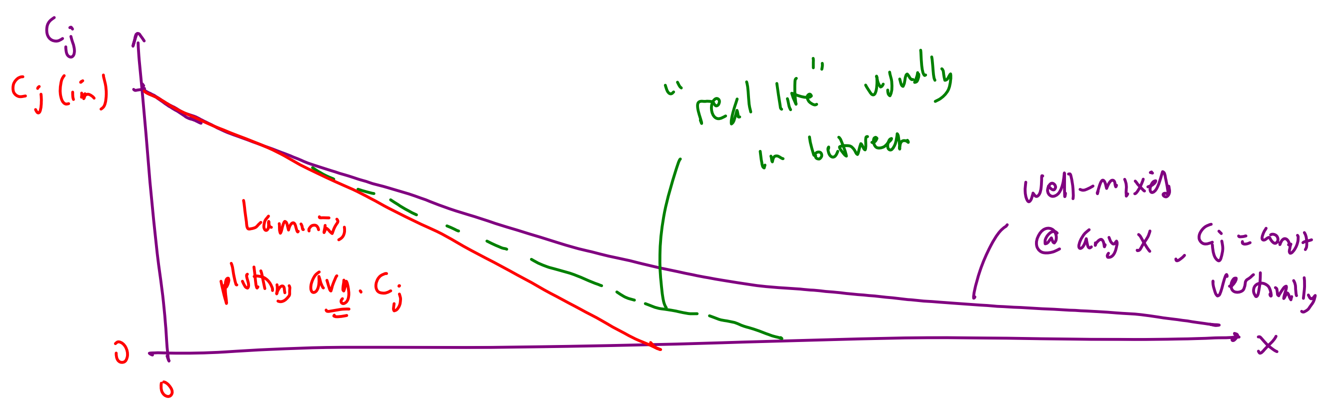
Today, we will:
- Continue discussing gravimetric settling in ducts – well-mixed settling approximation
- Re-visit the Gaussian plume model, but with particle settling included in the analysis
- If time, begin discussing **inertial separation** (particles in curved flows)

Gravimetric Settling in Ducts (continued) — Well-Mixed Settling ("turbulent")





Equations: – consider a monodisperse aerosol (one particle dia at a time)

→ 1st-order ODE in x

Soln:

$$\frac{c_j}{c_j(in)} = \exp\left[-\frac{V_t x}{UH}\right]$$

set x = L = duct length

recall, L_c = critical length

$$\boxed{L_c = \frac{HU}{V_t}}$$

$$\boxed{\frac{c_j}{c_j(in)} = \exp\left[-\frac{L}{L_c}\right]} \quad \star$$

E = removal efficiency

$$\boxed{E = 1 - \frac{c_j}{c_j(in)} = 1 - \exp\left(-\frac{L}{L_c}\right)} \quad \star$$

Example: Removal Efficiency due to Well-Mixed Gravimetric Settling in a Duct

Given: Dusty air enters a horizontal duct of length L = 14.4 m and height H = 6.0 cm at average speed U = 0.20 m/s. Aerosol particles of a certain diameter D_p under consideration have a terminal settling speed of V_t = 0.00025 m/s.

To do: Calculate the removal efficiency E for these particles. Assume well-mixed settling, and assume that all particles that hit the floor of the duct remain there (they stick to the floor). Give your answer as a percentage to two significant digits.

Solution:

$$E = 1 - \frac{c_i}{c_i(in)} = 1 - \exp\left(-\frac{L}{L_c}\right) \qquad L_c = \text{critical length} = \frac{HU}{V_t}$$

$$L_c = \frac{(0.060\text{ m})(0.20\text{ m/s})}{0.00025\text{ m/s}} = \underline{\underline{48\text{ m}}}$$

$$E = 1 - \exp\left(-\frac{14.4\text{ m}}{48\text{ m}}\right) = 0.25918 \rightarrow \boxed{26\% = E}$$

compared to laminar, $\underline{\underline{E = 30\%}}$

Which is it?

@ STP $\rightarrow$ $Re = \dfrac{\rho U H}{\mu} = \dfrac{(1.184\text{ kg/m}^3)(0.20\text{ m/s})(0.060\text{ m})}{1.849\times10^{-5}\text{ kg/m·s}} = \underline{\underline{768.4}}$

Probably laminar since $Re < 2300$ ← value for round pipe

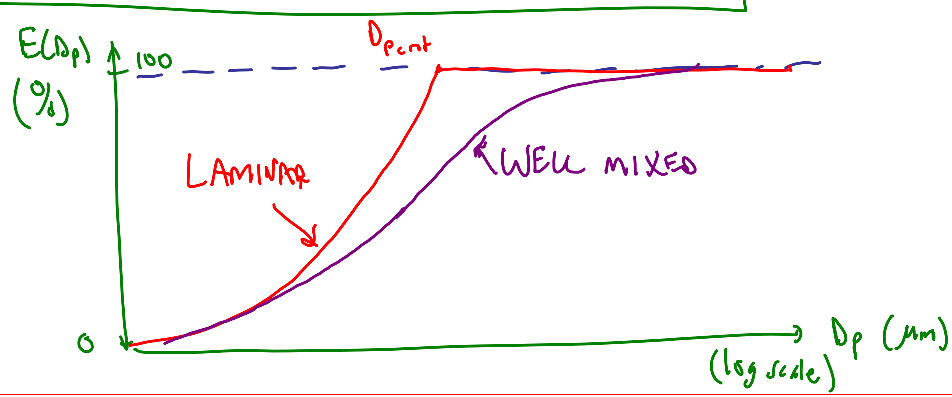
Real life → vibrations, turbulent eddies @ inlet

$\boxed{E \text{ is somewhere between } 26 \text{ \& } 30\ \%}$

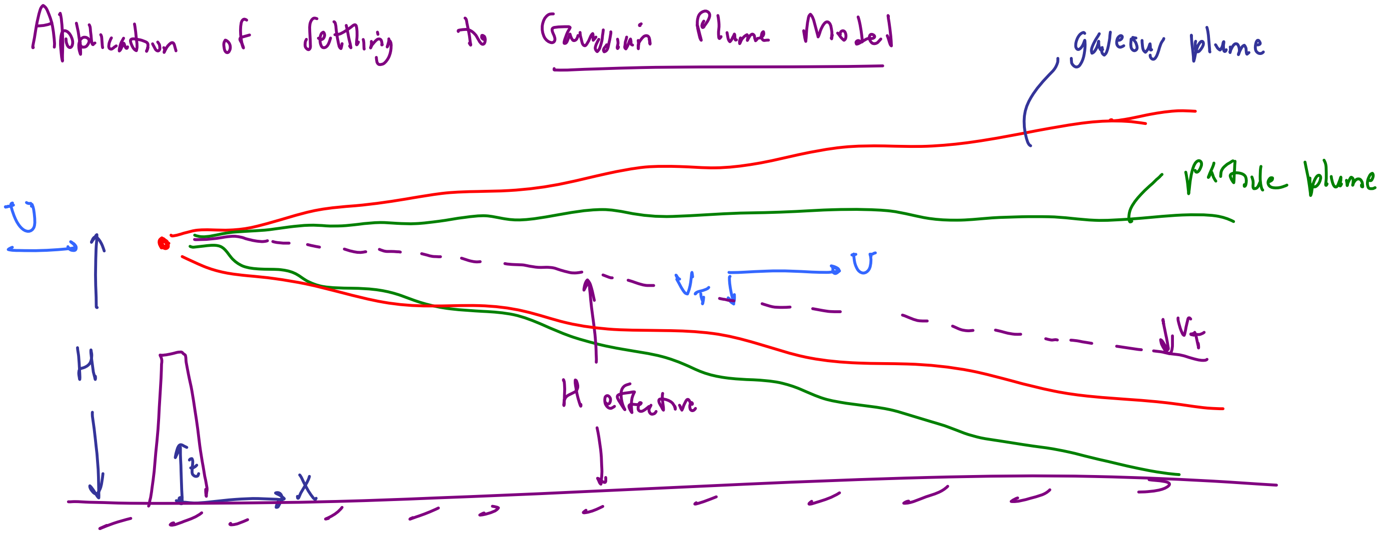
Grade efficiency:

$E(D_p)$

re-do analysis with various D_p's



Application of settling to Gaussian Plume Model



Consider an absorbing ground → particles "stick" when they hit the ground

We had for gaseous plumes

$$\boxed{C_j = \frac{\dot{m}_{j,s}}{2\pi U \sigma_y \sigma_z} \exp\left\{-\frac{1}{2}\left[\left(\frac{y}{\sigma_y}\right)^2 + \left(\frac{z - H}{\sigma_z}\right)^2\right]\right\}}$$

Let $H(x) = H_o - V_t \cdot t$ but @ some x $t = \frac{x}{U}$

H @ $x=0$

Let $\boxed{H(x) = H_o - \frac{V_t x}{U}}$

$$\boxed{C_j = \frac{\dot{m}_{j,s}}{2\pi U \sigma_y \sigma_z} \exp\left\{-\frac{1}{2}\left[\left(\frac{y}{\sigma_y}\right)^2 + \left(\frac{z - \left[H_o - \frac{V_t x}{U}\right]}{\sigma_z}\right)^2\right]\right\}}$$

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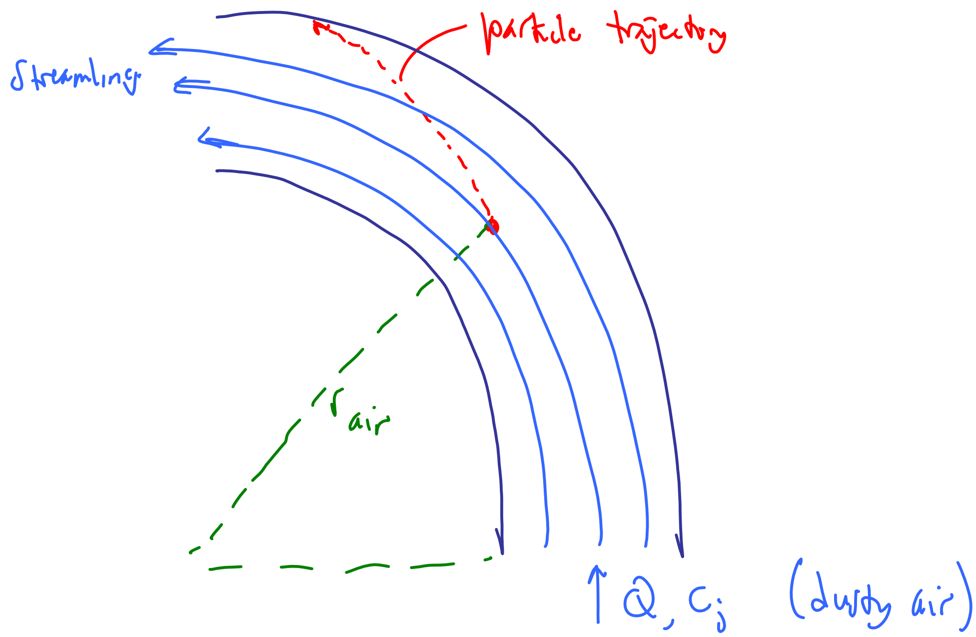
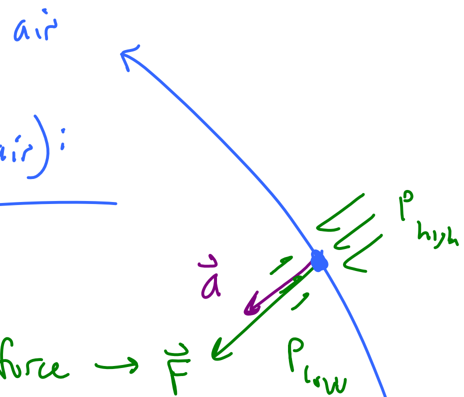
Gaussian plume model for particles (settling)

Inertial Separation (particles in curved ducts)



Air particle ($\rho_p = \rho_{air}$):

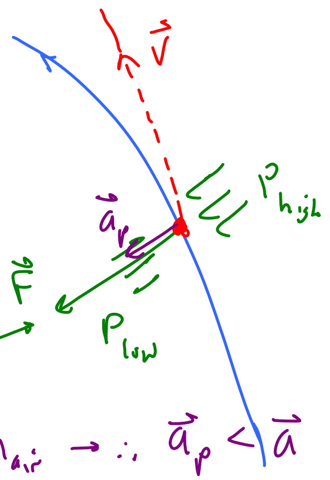
(P increases radially outward)



Centripetal force → $\vec{F}$

Centripetal acceleration $\boxed{\vec{a} = \frac{d\vec{U}}{dt} = \frac{\vec{F}}{m_{air}}} \rightarrow \boxed{a = \frac{U^2}{r}}$ towards the middle

Aerosol Particle ($\rho_p >> \rho_{air}$)



Centripetal force on particle → $\vec{F}$

$\boxed{\vec{a}_p = \frac{\vec{F}}{m_p}}$ $m_p > m_{air} \rightarrow \therefore \vec{a}_p < \vec{a}$ (air)

Particle experiences the same centripetal force, but it is not enough to keep it on the same circular path — it veers outward

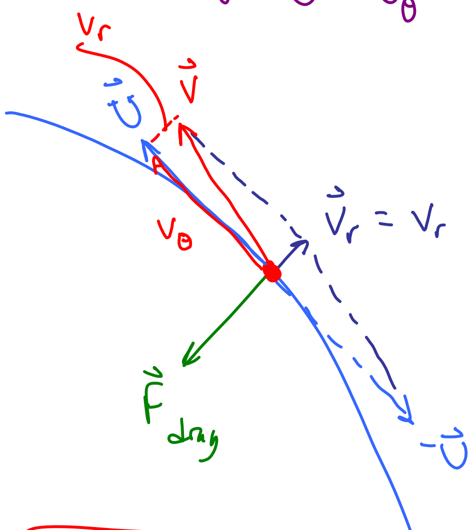
Equations:

Approx: 1) Pressure force >> gravity force (ignore gravity)

2) V_θ → tangential component of particle velocity $\vec{V}$

$V_\theta = U = U_\theta$ (particle moves with the air tangentially)

$\vec{V}_r = \vec{V} - \vec{U}$



Conclusion – $\boxed{\text{The relative velocity is radially outward \& the drag force is therefore radially inward}}$ ☆

Let's balance the outward "centrifugal" force (felt by the particle) & the aerodynamic drag force (felt by the particle):

$$\vec{F}_{centrifugal} = (m_p - m_{air})\frac{U_\theta^2}{r} = \boxed{\frac{\pi D_p^3(\rho_p - \rho)}{6}\frac{U_\theta^2}{r}} \quad \text{in } r\text{-direction}$$

$$\vec{F}_{drag} = -\frac{1}{8}\rho\frac{C_D}{C}\pi D_p^2 \vec{V}_r|\vec{V}_r| \quad \text{(same as previously)}$$

In r-direction, $\boxed{= -\frac{1}{8}\rho\frac{C_D}{C}\pi D_p^2 V_r^2}$ in r-direction

These two must balance for V_r to be constant