Today, we will:

- Continue to discuss **inertial separation** (particles in curved flows)
- Discuss the analogy between gravimetric settling and inertial separation
- Compare laminar vs. well-mixed inertial separation in curved ducts

Review of equations so far for inertial separation in a curved duct:

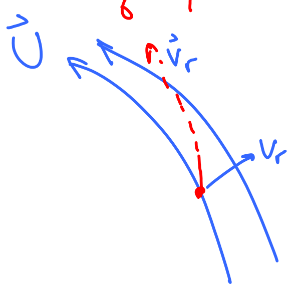
$$\boxed{\vec{F}_{\text{centrifugal}} = \left(m_p - m_{\text{air}}\right)\frac{U_\theta^{\,2}}{r} = \pi\frac{D_p^{\,3}}{6}\left(\rho_p-\rho\right)\frac{U_\theta^{\,2}}{r}}$$ = "centrifugal" force, radially outward

$$\boxed{\vec{F}_{\text{drag}} = -\frac{1}{8}\rho\frac{C_D}{C}\pi D_p^{\,2}\vec{v}_r\left|\vec{v}_r\right| = -\frac{1}{8}\rho\frac{C_D}{C}\pi D_p^{\,2}v_r^{\,2}}$$ = aerodynamic drag force, radially inward

(same aerodynamic drag force that we used previously)

Consider the simplest case in which v_r is *constant*, and the two above forces must balance:

$$\frac{\cancel{\pi} D_p^{\cancel{3}}}{6}\left(\rho_p-\rho\right)\frac{U_\theta^2}{r} = \frac{1}{8}\rho\frac{C_D}{C}\cancel{\pi}\cancel{D_p^2}V_r^2 \Rightarrow \frac{8}{6}D_p\frac{\rho_p-\rho}{\rho}\frac{C}{C_D}\frac{U_\theta^2}{r} = V_r^2$$



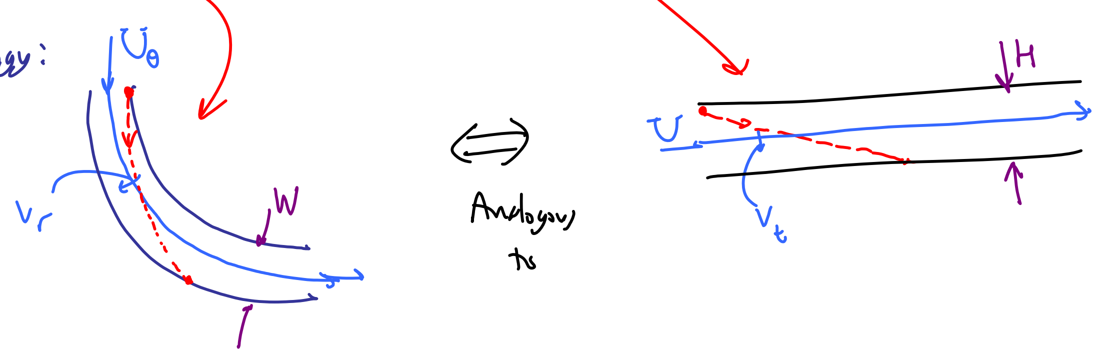
$$\boxed{v_r = \sqrt{\frac{4}{3}\frac{\rho_p-\rho}{\rho}\frac{U_\theta^{\,2}}{r}D_p\frac{C}{C_D}}}$$ = Radial "settling" velocity in a curved duct due to inertial separation

Compare to gravimetric settling in quiescent air (from a previous lecture):

terminal settling velocity $$\boxed{V_t = \sqrt{\frac{4}{3}\frac{\rho_p-\rho}{\rho}gD_p\frac{C}{C_D}}}$$

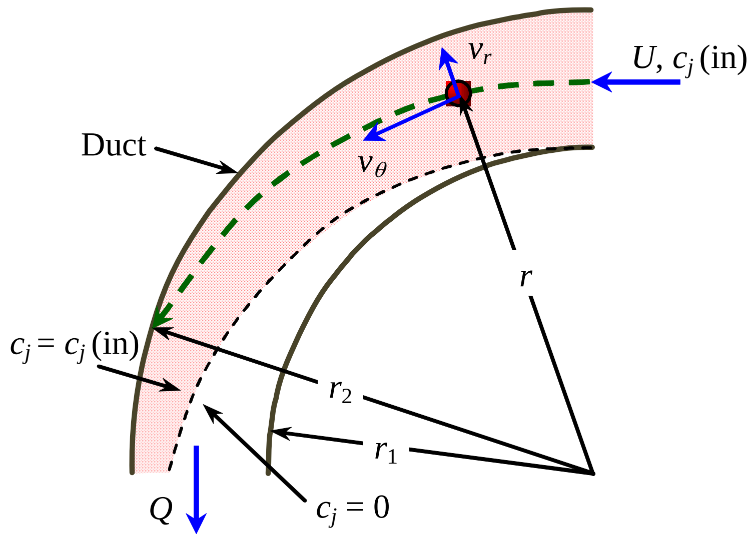
Radial settling is identical to gravimetric settling except replace g by $\frac{U_\theta^2}{r}$

Analogy:



Example: Comparison of Centrifugal and Gravitational Settling

Given: Dusty air enters a curved duct at average speed U. Aerosol particles of a certain diameter D_p have a terminal settling speed of $V_t = 0.00025$ m/s in quiescent air. At the instant of time shown, a particle of diameter D_p is at radius $r = 0.32$ m.



To do: Calculate the air speed U such that the radial velocity v_r of the particle is the *same* as its terminal settling velocity. Give your answer in m/s to three significant digits.

Solution:

• We showed that V_r & V_t have the same equation, except g is replaced by $\frac{U_\theta^2}{r}$

$$\text{For } V_r = V_t \rightarrow \frac{U_\theta^2}{r} = g \rightarrow U_\theta = \sqrt{rg}$$

$$= \sqrt{(0.32\text{ m})(9.807\text{ m/s}^2)}$$

$$= 1.7715\text{ m/s}$$

$$\boxed{U_\theta = 1.77\text{ m/s}}$$

Not very large

$$\text{For } U_\theta = 10\text{ m/s} \rightarrow \frac{U_\theta^2}{r} = \frac{(10\text{ m/s})^2}{0.32\text{ m}} = 312.5\ \frac{\text{m}}{\text{s}^2} \approx 32\ g\text{'s}$$

Laminar vs. well-mixed settling:

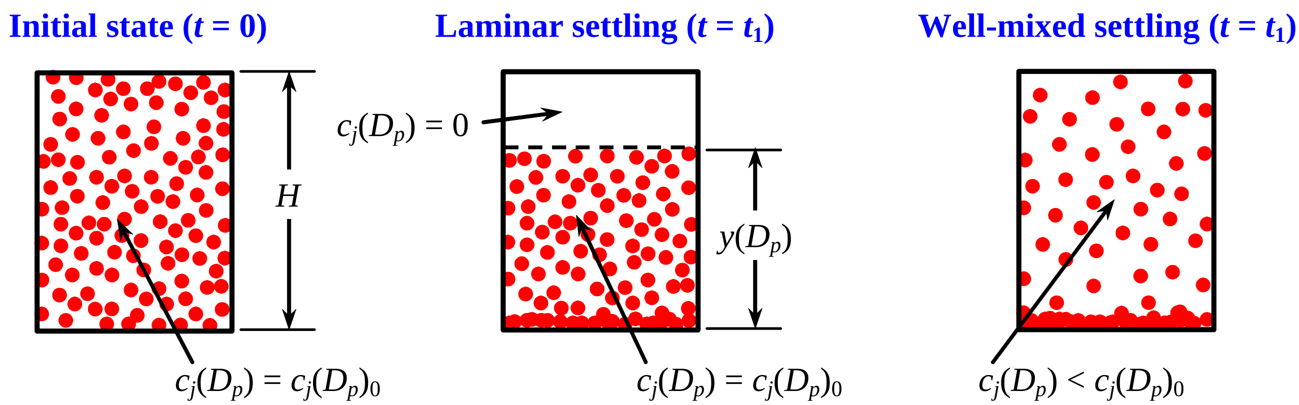
We discussed this twice previously:

1. Gravimetric settling in a room or container
2. Gravimetric settling in a horizontal duct

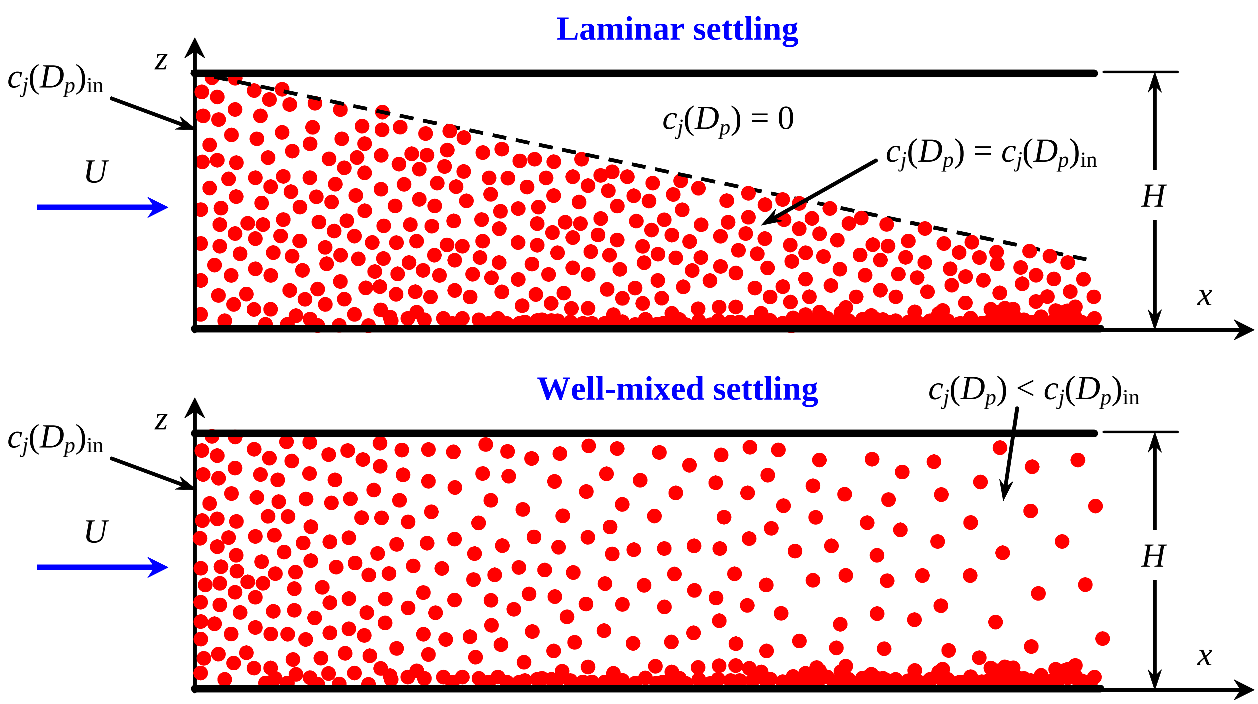
Now we apply the same principles to

3. **Inertial separation in a curved duct**.

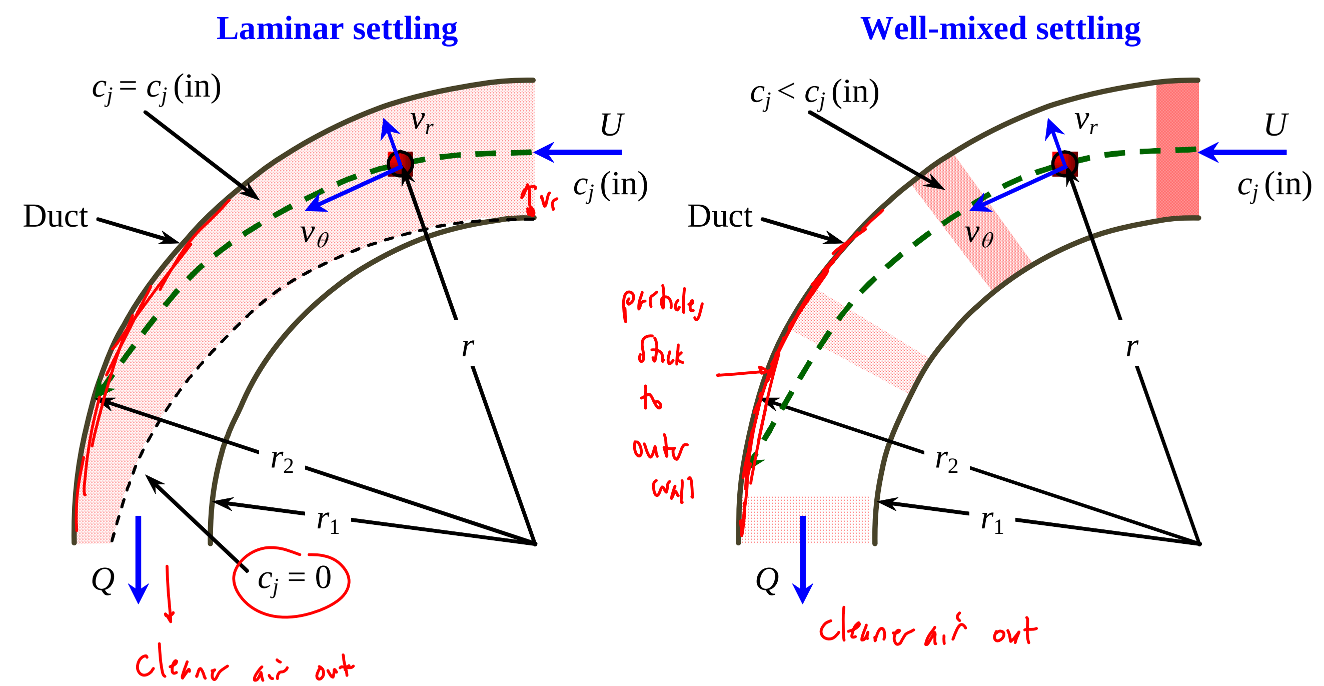
1. Gravimetric settling in a room or container:



2. Gravimetric settling in a horizontal duct:



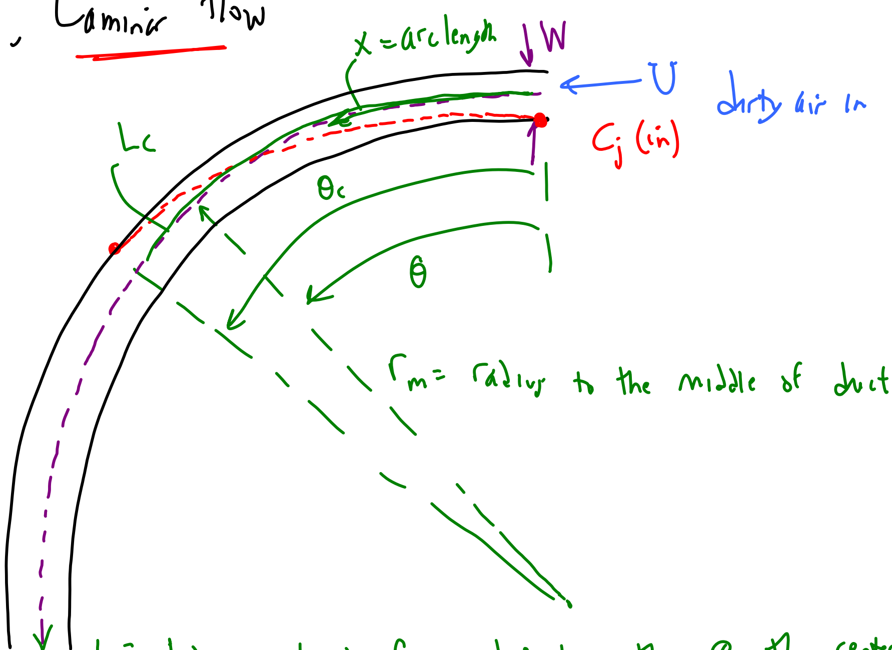
3. Inertial separation in a curved duct:



Simplest Approx , Laminar flow

$\boxed{r_m >> W}$

Then we can use $r \simeq r_m$ $= \text{constant}$



$$\boxed{x = r_m \theta}$$

θ in radians

L = total arc length from inlet to outlet @ the centerline

L_c = critical arc length at which all particles are cleaned up

At $r = r_m \rightarrow$ let's use $r = r_m = \text{constant}$ in our eq.

$$\boxed{V_r = \sqrt{\frac{4}{3} \frac{\rho_p - \rho}{\rho} \frac{U_\theta^2}{r_m} D_p \frac{C}{C_D}}}^{*} \simeq \text{const}$$

Worst-case particle travels distance W radially outward in some time t

$W = v_r t$ & $L_c = U_\theta t$ or Ut

equate t → $\boxed{L_c = \frac{W U_\theta}{v_r}}$ $\boxed{\theta_c = \frac{L_c}{r_m}}$

critical arc length

Removal efficiency:

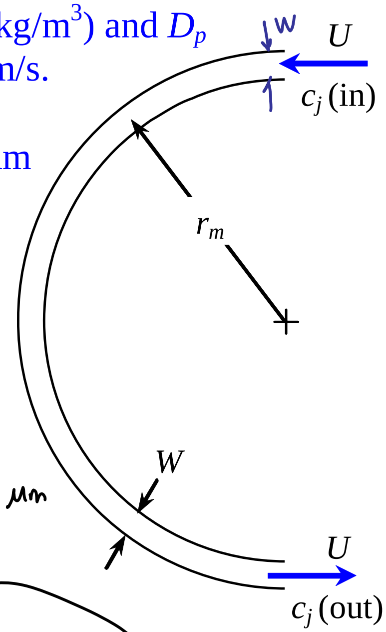
if $x < L_c$, $\frac{C_j}{C_{j(in)}} = 1 - \frac{x}{L_c}$ → $E = 1 - \frac{C_j}{C_{j(in)}}$

$$\boxed{E = \frac{x}{L_c}}$$

If $x > L_c$, $E = 1$ or 100%

Example: Removal efficiency of a curved duct as a function of particle diameter

Given: Monodisperse aerosol particles of unit density (ρ_p = 1000 kg/m^3) and D_p = 15.0 microns enter a 180^o curved duct at average speed U = 10.3 m/s. The air is at STP (ρ = 1.184 kg/m^3, μ = 1.849 × 10^{-5} kg/(m s)). The middle duct radius is r_m = 0.500 m and the duct width is W = 22.5 mm (0.0225 m). We assume laminar settling, and we assume that the particles stick to the outer wall when they hit, and are therefore removed from the flow.



To do: Calculate your best estimate of the predicted removal efficiency E of these particles (% to three significant digits).

Solution: The Cunningham correction factor is C = 1.0168, and Stokes flow is appropriate (C_D = 24/Re).

1.0112 for D_p=15 μm

Equations: $\boxed{v_r = \sqrt{\frac{4}{3}\frac{\rho_p - \rho}{\rho}\frac{U_\theta^2}{r}D_p\frac{C}{C_D}}}$, $\boxed{\mathrm{Re} = \frac{\rho v_r D_p}{\mu}}$, $\boxed{C_D = \frac{24}{\mathrm{Re}}}$,

Laminar settling model: $\boxed{E = 1 - \frac{c_j}{c_j(\text{in})} = \frac{x}{L_c}}$, where $\boxed{L_c = W\frac{U_\theta}{v_r}}$, $\boxed{x = r_m\theta}$.

plug in C_D too

Solve for V_r → $V_r^2 = \frac{4}{3}\frac{\rho_p-\rho}{\cancel{\rho}}\frac{U_\theta^2}{r}D_p\frac{\cancel{\rho}V_r D_p \cdot C}{\mu}$ ⇒ $\boxed{V_r = \frac{1}{18}\frac{(\rho_p-\rho)\,U_\theta^2 D_p^2 C}{\mu r_m}}$

As for D_p=15 μm → $V_r = \frac{1}{18}\frac{(1000-1.184)\,\text{kg/m}^3\,(10.3\,\text{m/s})^2(15\times10^{-6}\,\text{m})^2(1.0112)}{(1.849\times10^{-5}\,\text{kg/m}^3)(0.500\,\text{m})}$

$= 0.1449$ m/s

But $x = (0.50\,\text{m})\pi$ 180°
$= 1.5708$ m

Then $L_c = \frac{WU_\theta}{V_r}$ & $E = \frac{x}{L_c} = \frac{xV_r}{WU_\theta}$

So, $E = \frac{(1.5708\,\text{m})(0.1449\,\text{m/s})}{(0.0225\,\text{m})(10.3\,\text{m/s})} = 0.982 = \boxed{98.2\%}$

Check Re → $\mathrm{Re} = \frac{\rho D_p V_r}{\mu} = \frac{(1.184\,\text{kg/m}^3)(15\times10^{-6}\,\text{m})(0.1449\,\text{m/s})}{1.849\times10^{-5}\,\text{kg/m·s}} = \boxed{0.139 = \mathrm{Re}}$

$\boxed{\text{Ans. } E \approx 98\%}$

A little too big → Re ≲ 0.1 for Stokes flow, but this should be a reasonable estimate