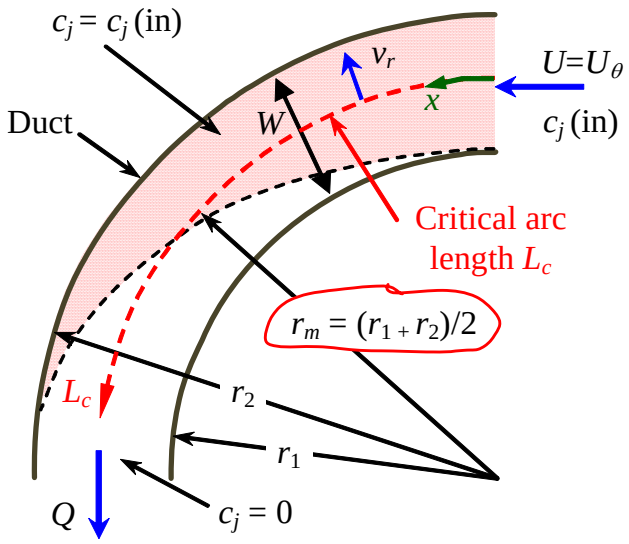
Today, we will:

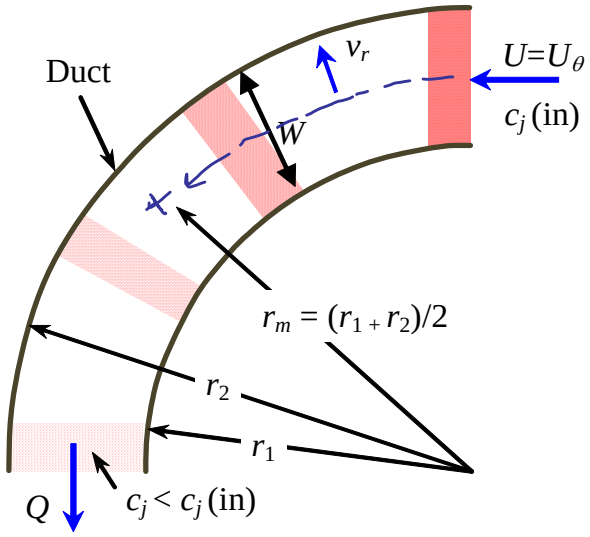
- Continue to discuss inertial separation, and laminar vs. well-mixed settling in ducts
- Discuss an application of inertial separation: **cyclone separators**

Inertial separation in a curved duct (review from previous lecture):

Laminar model



Well-mixed model



The simplest case is when $r_m >> W$, so we treat r and v_r as constants in the equations.

Laminar settling model: $E = 1 - \frac{c_j}{c_j(\text{in})} = \frac{x}{L_c}$, where x is the arc length $x = r_m\theta$, $L_c = W\frac{U_\theta}{v_r}$ is the critical arc length.

Well-mixed settling model: Eqs for E are same as for gravimetric settling in ducts except replace g by U_θ^2/r_m

Well-mixed inertial separation in a curved duct:

$$L_c = W\frac{U_\theta}{v_r}$$

@ some arclength x

$$\frac{c_j}{c_j(\text{in})} = \exp\left(-\frac{x}{L_c}\right)$$

E = removal eff. = $1 - \frac{c_j}{c_j(\text{in})}$

$$\star\quad E = 1 - \exp\left(-\frac{x}{L_c}\right)$$

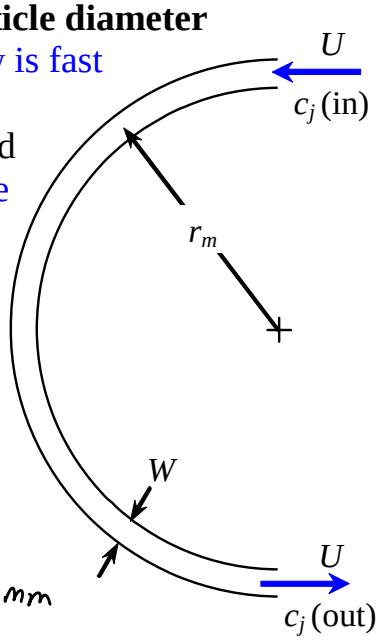
@ end of duct ($x = L$),

$$E = 1 - \exp\left(-\frac{L}{L_c}\right)$$

E will vary with D_p → $E(D_p)$ Grade efficiency

Example: Grade efficiency of a curved duct as a function of particle diameter

Given: A polydisperse aerosol enters a 180° curved duct. The flow is fast enough to be considered turbulent in the duct, so we make the well-mixed settling approximation. For convenience, I have pre-calculated the inertial settling velocity v_r for various particle diameters. Assume that particles stick to the outer wall when the hit the wall, and are therefore removed from the flow. The particles are of unit density (ρ_p = 1000 kg/m^3). The middle duct radius is r_m = 0.500 m, the duct width is W = 22.5 mm (0.0225 m), the air is at STP (ρ = 1.184 kg/m^3, μ = 1.849 × 10^{-5} kg/(m s)), and U = 8.50 m/s.



To do: For each diameter, calculate the removal grade efficiency $E(D_p)$ (in %) for each particle diameter D_p in the table below.

Solution: **Table to be filled in during class:**

$r_m = 0.5\ m$ $W = 22.5\ mm$

D_p (μm)	v_r (m/s)	$E(D_p)$ (%)
1	0.000506747	
1.5	0.001085343	0.89
2	0.001880751	1.5
2.5	0.002892936	2.35
3	0.004121854	3.3
4	0.007229597	
5	0.011203123	
7	0.021741295	16.4

D_p (μm)	v_r (m/s)	$E(D_p)$ (%)
10	0.043982967	
20	0.171476946	75.6
25	0.262745185	88.4
30	0.368588815	
40	0.609606668	99.3
50	0.867242208	99.9
70	1.388132409	

Equations: Well-mixed: $E(D_p) = 1 - \frac{c_j}{c_j(\text{in})} = 1 - \exp\left(-\frac{x}{L_c}\right)$, where $L_c = W\frac{U_\theta}{v_r}$, $x = r_m\theta$.

@ $D_p = 15\ \mu m$ $\quad L_c = \frac{WU_\theta}{v_r} = \frac{WU}{v_r} = \frac{(0.0225\ m)(8.50\ m/s)}{0.097827\ m/s} = 1.95498\ m$

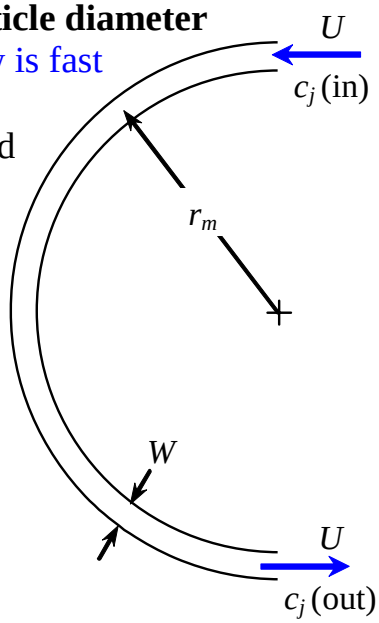
$x = r_m\theta \rightarrow (0.500\ m)(180°)\left(\frac{\pi\ rad}{180°}\right) = 1.57080\ m$ @ $x = L$

$E(D_p) = 1 - \exp\left(-\frac{L}{L_c}\right) = 1 - \exp\left(-\frac{1.5708}{1.95498}\right) = 0.552$

55.2%

@ 15 μm

Example: Grade efficiency of a curved duct as a function of particle diameter



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To do: For each diameter, calculate the removal grade efficiency $E(D_p)$ (in %) for each particle diameter D_p in the table below.

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1	0.000506747	
1.5	0.001085343	
2	0.001880751	
2.5	0.002892936	
3	0.004121854	
4	0.007229597	
5	0.011203123	
7	0.021741295	

D_p (μm)	v_r (m/s)	$E(D_p)$ (%)
10	0.043982967	
20	0.171476946	
25	0.262745185	
30	0.368588815	
40	0.609606668	
50	0.867242208	
70	1.388132409	

Equations: Well-mixed: $E\left(D_p\right) = 1 - \frac{c_j}{c_j(\text{in})} = 1 - \exp\left(-\frac{x}{L_c}\right)$, where $L_c = W\frac{U_\theta}{v_r}$, $x = r_m\theta$.

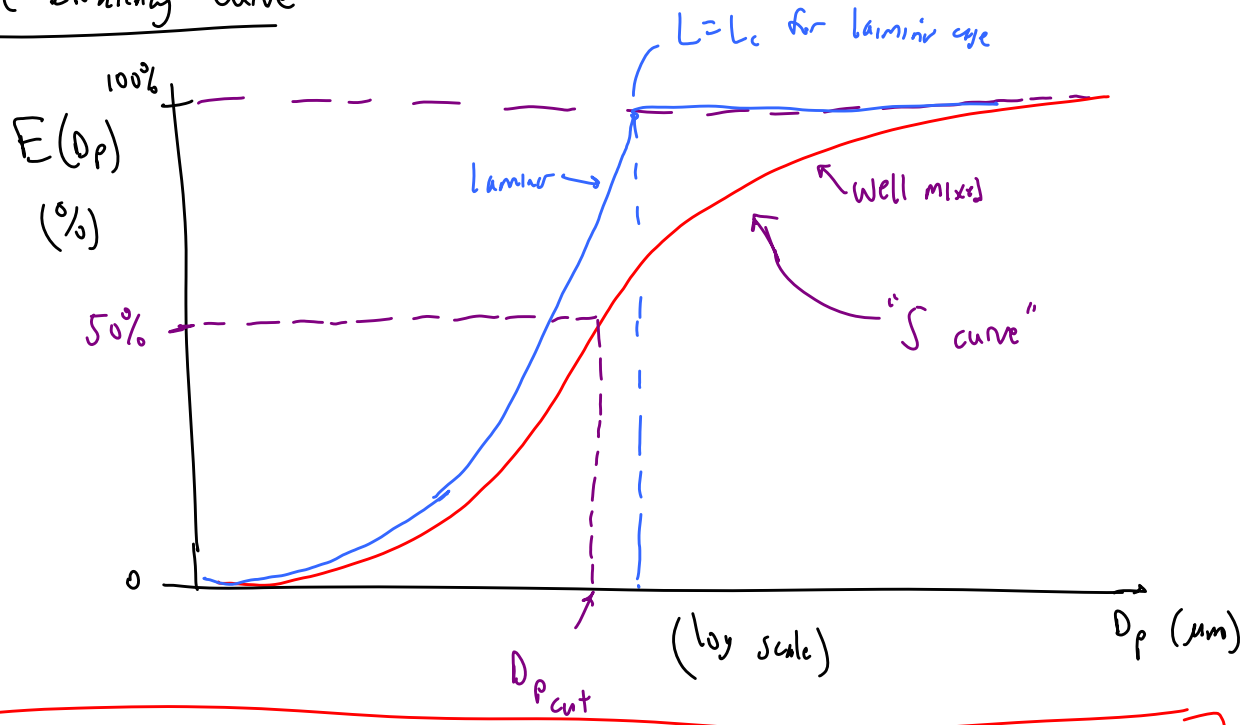
Filled in Table:

D_p (μm)	v_r (m/s)	$E(D_p)$ (%)
1	0.000506747	0.41534198
1.5	0.001085343	0.88746518
2	0.001880751	1.53285045
2.5	0.002892936	2.34805352
3	0.004121854	3.32874447
4	0.007229597	5.76504009
5	0.011203123	8.79083189
7	0.021741295	16.3532896

D_p (μm)	v_r (m/s)	$E(D_p)$ (%)
10	0.043982967	30.3192386
20	0.171476946	75.5464314
25	0.262745185	88.4445008
30	0.368588815	95.1555464
40	0.609606668	99.3308325
50	0.867242208	99.9193602
70	1.388132409	99.9988818

← $D_{p,cut}$

Grade Efficiency Curve



$D_{p_{cut}}$ = "cut diameter" = dia. at which 50% of particles are removed

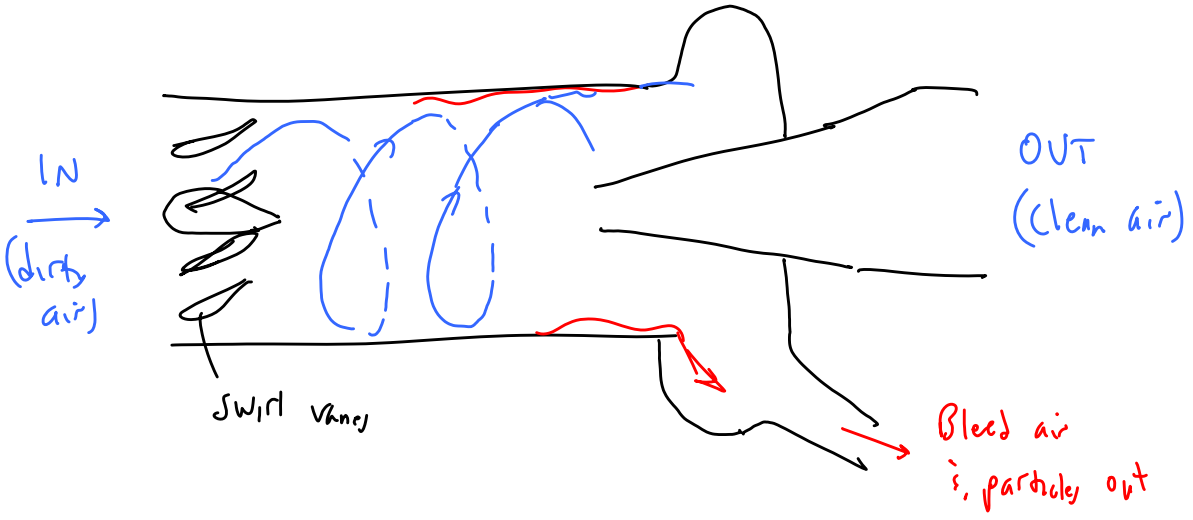
Here $D_{p_{cut}} = 13.9\ \mu m$

Cyclone Separators

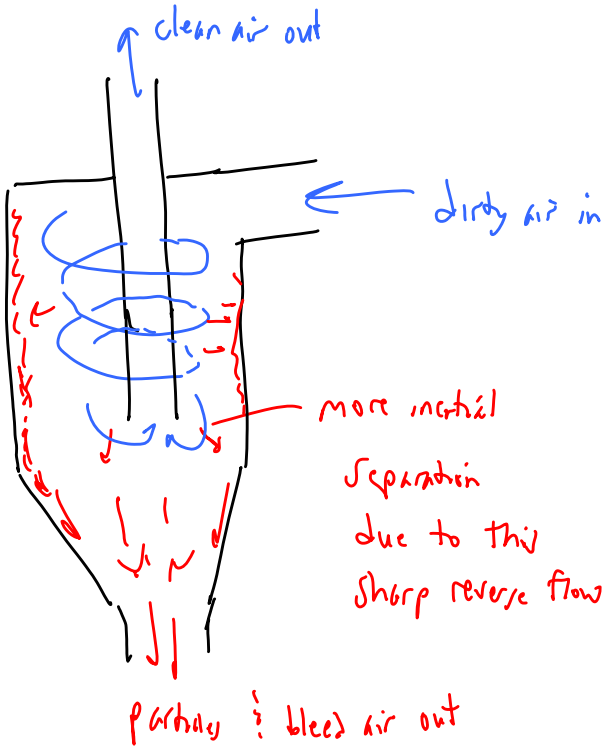
Two types → Straight-through cyclone
reverse-flow cyclone

Very good at removing large particles
Not so good " " small "

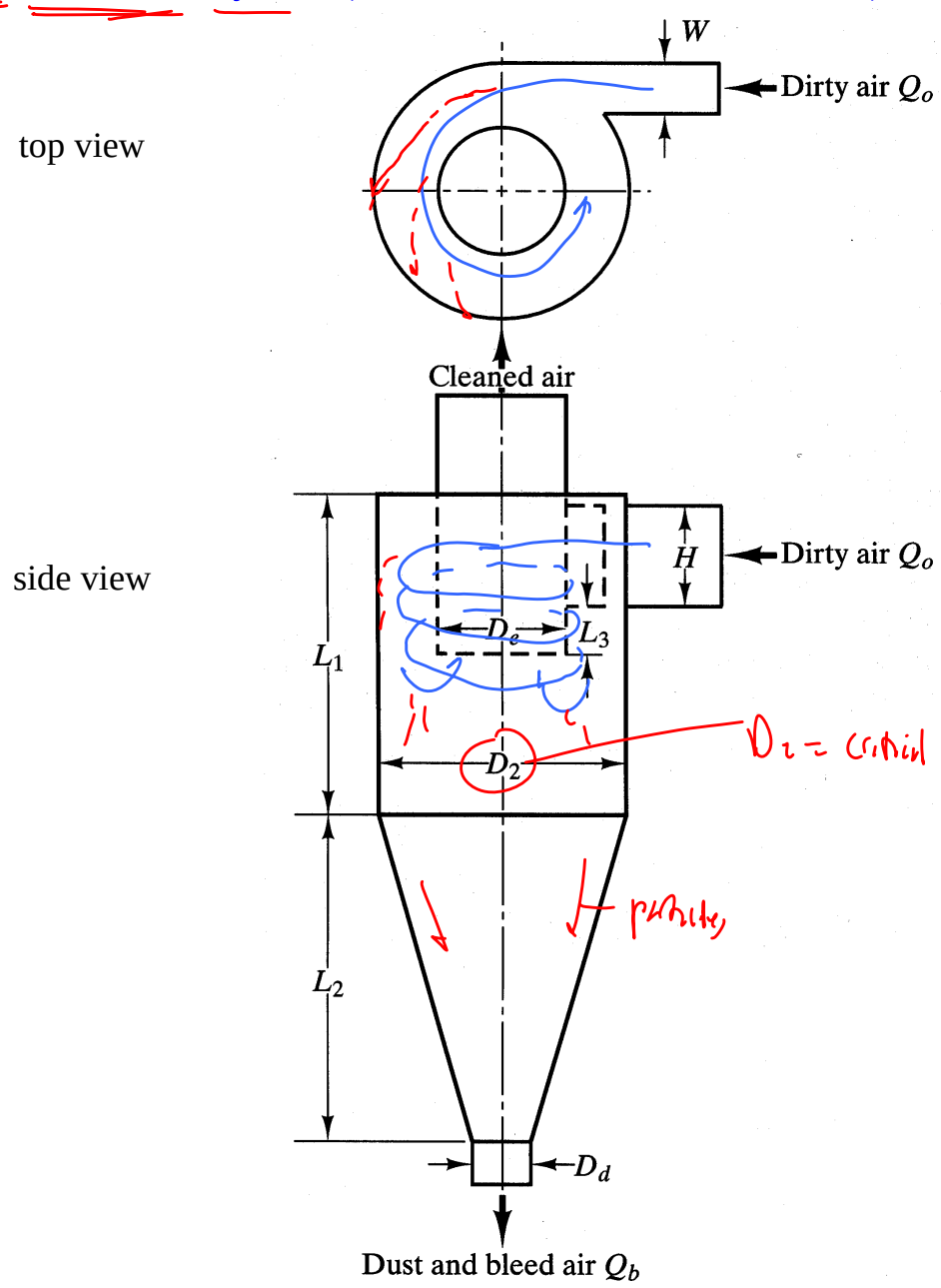
Straight-through cyclone



Reverse flow cyclones



Standard Lapple reverse-flow cyclone (from Heinsohn and Cimbala, 2003):



Standard Dimensions for Lapple Reverse-flow cyclone

$L_1 = L_2 = 2D_2$

$W = D_d = D_2/4$

$L_3 = D_2/8$

$H = D_e = D_2/2$

N_e = # of swirls of the air ≈ 6

} Standards

Empirical eqn for Lapple

$$E(D_p) = \text{grade efficiency} = \frac{1}{1 + \left(\frac{D_p}{D_{p,cut}}\right)^{-2}} \quad \bigstar$$

where $D_{p,cut} = \sqrt{\dfrac{9 \mu H W^2}{2\pi N_e Q (\rho_p - \rho)}}$ (Q = air vol. flow rate)

ρ_w in standard $\rightarrow$ $D_{p,cut} = \sqrt{\dfrac{3 \mu D_2^3}{128 \pi Q (\rho_p - \rho)}}$ $\bigstar$

This one curve works for all Lapple standard cyclones

Pressure drop $\rightarrow$ $\Delta P = 40.96 \varphi \left(\dfrac{Q}{WH}\right)^2$

Power requirement $\rightarrow$ $W_{blower} \propto \Delta P \cdot Q$ * $\leftarrow$ $\propto$ the cost of running the cyclone $\bigstar$