Today, we will:

- Do some examples with the standard **Lapple reverse flow cyclone**
- Discuss how performance of a cyclone varies with size, flow rate, etc.

Example: Lapple Cyclone

Given: A standard reverse flow Lapple cyclone is used to clean up a dusty air flow exhausted by a sanding machine in a wood shop. The main body diameter of the cyclone is D_2 = 45.0 cm (0.450 m).

- particle density $\rho_p = 730$ kg/m^3
- bulk volume flow rate of air $Q = 0.55$ m^3/s
- Air is at STP: $\rho = 1.184$ kg/m^3, $\mu = 1.849 \times 10^{-5}$ kg/(m s).

To do: Calculate the grade efficiency $E(D_p)$ for 10-μm particles. Give your answer as a percentage to 3 significant digits. (%)

Solution: Some equations:

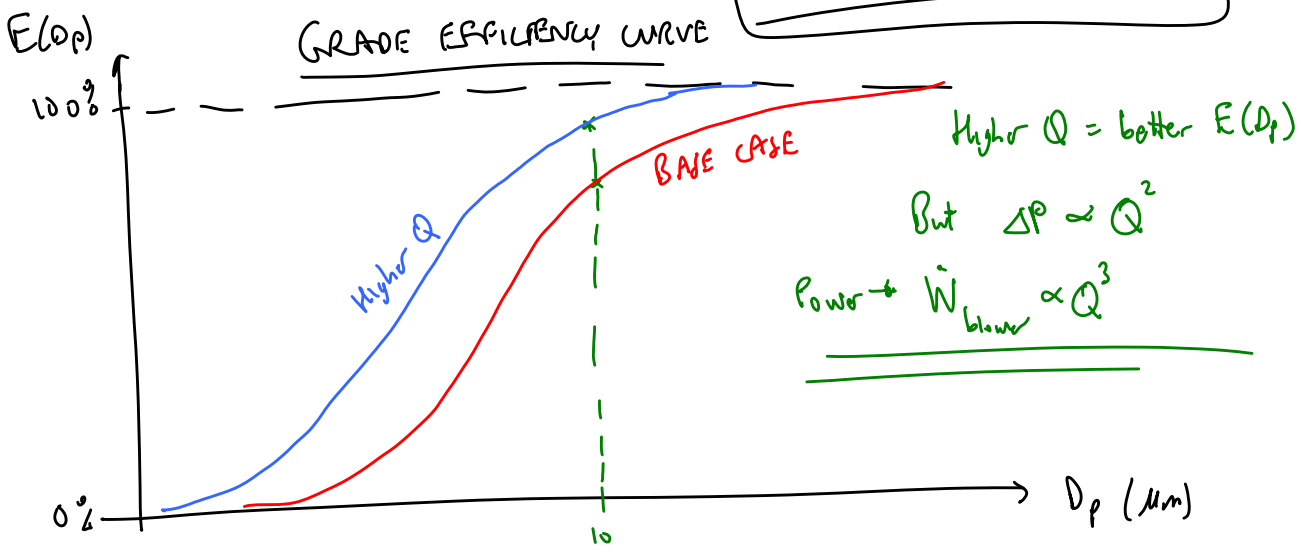
$$D_{p,\text{cut}} = \sqrt{\frac{3\mu D_2^{\,3}}{128\pi Q\left(\rho_p - \rho\right)}}, \qquad E\left(D_p\right) = \frac{1}{1+\left(\frac{D_p}{D_{p,\text{cut}}}\right)^{-2}}. \quad \star$$

$$D_{p,cut} = \sqrt{\frac{3\left(1.849\times10^{-5}\ \text{kg/m·s}\right)\left(0.450\ \text{m}\right)^3}{128\,\pi\,\left(0.55\ \text{m}^3/\text{s}\right)\left(730-1.184\right)\text{kg/m}^3}} = 5.59987\times10^{-6}\ \text{m}$$

$$D_{p,cut} = 5.5999\ \mu\text{m}$$

$$E(D_p) = \frac{1}{1+\left(\frac{10}{5.59}\right)^{-2}} = 0.7613$$

$$76.1\% = E\left(@\ 10\ \mu\text{m}\right)$$

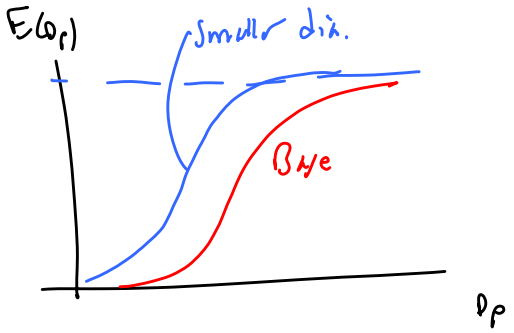


If Q↑, V_θ↑ $\frac{V_\theta^2}{r}$↑ ∴ Better separation

Size comparison:

If $D_2 \downarrow$ $r_m \downarrow$ $\frac{V_\theta^2}{r} \rightarrow \uparrow$

@ same velocity



Design of Lapple Cyclones

*For a given air flow we know Q, ρ_p, ρ, μ → fixed

- Requirement → $D_{p,cut}$ is specified → gives us the performance $E(D_p)$
- Operating cost vary depending on unit size

$$\Delta P = 40.96 \rho \left(\frac{Q}{WH}\right)^2 \rightarrow \text{but } \frac{Q}{WH} = U$$

WH → cross sectional area of inlet

$$= 40.96 \rho U^2$$

- $\dot{W}_{Blower}$ (includes motor & blower) $= \frac{1}{\eta_{blower}} Q \Delta P$

$\rightarrow \Delta P \propto U^2$ $\dot{W}_{blower} \propto U^3$

Example: Design of a Lapple Cyclone

Given: Dusty air from a manufacturing plant needs to be cleaned before being exhausted to the environment. Here is what we know about the dusty air:

- the air is polydisperse, with a wide variety of particle sizes
- particle density $\rho_p = 1500$ kg/m^3
- bulk volume flow rate of air $Q = 0.111$ m^3/s
- the air is at STP: $\rho = 1.184$ kg/m^3, $\mu = 1.849 \times 10^{-5}$ kg/(m s)

To do: Design a standard reverse flow Lapple cyclone to clean the air such that the removal efficiency of 2.5-μm particles is 80%. In particular, calculate dimension D_2, the diameter of the Lapple cyclone canister. Give your answer in meters to 3 significant digits.

Solution: Some equations: $D_{p,\text{cut}} = \sqrt{\dfrac{3\mu D_2^{\,3}}{128\pi Q(\rho_p - \rho)}}$, $E(D_p) = \dfrac{1}{1+\left(\dfrac{D_p}{D_{p,\text{cut}}}\right)^{-2}}$.

$$D_2 = \left[\frac{D_{p,cut}^{\,2}\ 128\,\pi\,Q\,(\rho_p-\rho)}{3\,\mu}\right]^{1/3}$$

$$\left(\frac{D_p}{D_{p,cut}}\right)^{-2} = \frac{1}{E} - 1 = \frac{1-E}{E}$$

$$D_{p,cut} = D_p\left(\frac{1-E}{E}\right)^{1/2}$$

$$= (2.5\ \mu m)\left(\frac{1-0.8}{0.8}\right)^{1/2}$$

$D_{p,cut} = 1.25\ \mu m$ Requirement

#j

$$D_2 = 0.124\ m$$

Suppose $\eta_{\text{blower-motor}} = 0.75\%$

Power requirement → $\dot{W}_{\text{blower}} = \dfrac{1}{\eta_{\text{blower-motor}}} Q\Delta P$

$\Delta P = 40.96\,\rho\left(\dfrac{Q}{WH}\right)^2$

we know that $W = D_2/4$, $H = D_2/2$ (Standard Lapple geometry)

$$\dot{W}_{\text{blower}} = 2621.44\,\rho\,\frac{1}{\eta_{\text{blower}}}\,\frac{Q^3}{D_2^{\,4}}$$ = Power requirement

Here, $\dot{W}_{blower} = 2621.44 \left(1.184 \frac{kg}{m^3}\right) \frac{1}{0.75} \frac{\left(0.111 \frac{m^3}{s}\right)^3}{(0.12352\ m)^4} \left(\frac{N \cdot s^2}{kg \cdot m}\right)\left(\frac{W \cdot s}{N \cdot m}\right)$ (Watts)

$= 24313.7\ W$

$\boxed{\dot{W}_{blower} \approx 24.3\ kW}$

@ 10¢/kWh

run 24/7 $\quad cost = \frac{\$0.10}{kW \cdot hr}(24.3137\ kW)(365.25\ days)\left(\frac{24\ hr}{day}\right)$

$\boxed{= \$21,313 \text{ per year}}$!!

We can save $ by putting several cyclones in parallel

We split Q into m smaller cyclones → $\boxed{Q_{1\ cyclone} = \frac{Q_{overall}}{m}}$

Let's try $m = 4$ $\quad Q = \frac{0.111\ m^3/s}{4} = 0.02775\ m^3/s$ per unit

Keep same requirement → 80% removal @ 2.5 μm

Re-do previous problem @ $Q = 0.02775\ m^3/s$ → $\boxed{D_2 = 0.07782\ m}$

(4 smaller cyclones)

$\dot{W}_{blower} = 2621.44\ \rho \frac{1}{\eta_{blower}} \frac{Q^3}{D_2^4} = 2412.3\ W$

$2.412\ kW$ per unit

Total Power requirement $= 4 \cdot 2.412\ kW = 9.649\ kW$

Cost is now $\$8460$ compared to $\$21,300$ for 1 cyclone (4 in parallel) @ same performance!

- Cyclones in parallel → $Q \downarrow$ in each cyclone
  → $D_2 \downarrow$ in each cyclone
  → ΔP i. $\dot{W} \downarrow$ in each cyclone

get same performance @ lower elec. cost !

Why do parallel cyclones save electricity ?

For same performance → in parallel cyclones have smaller Q i. smaller D_2 for each cyclone

→ This combination leads to an overall drop in ΔP i. in $\dot{W}_{elec}$, as the numbers show.

Example — some vacuum cleaners have many parallel cyclones

- Would cost less electricity than one big cyclone with same performance.