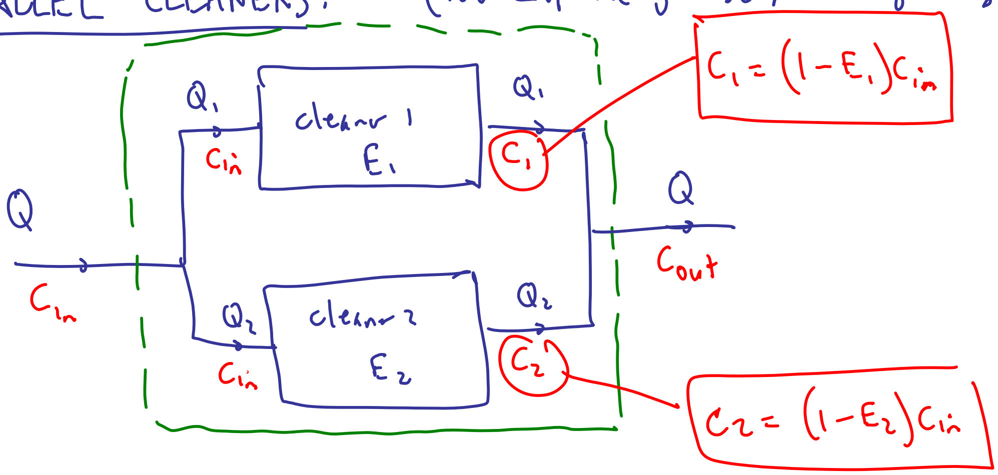
Today, we will:

- Discuss air cleaners in series and parallel

ANALYSIS

☆ **PARALLEL CLEANERS:** (let's drop the j subscript → $C_j = C$)



Volume flow rates: $Q = Q_1 + Q_2$

Let, or define fractional flow rates, f $\quad \boxed{f_1 = \frac{Q_1}{Q}} \quad \boxed{f_2 = \frac{Q_2}{Q}}$

- Cleaner 1 → $C_1 = (1-E_1)C_{in}$

mass flow rate → $C_1 Q_1 = C_{in} Q_1 (1-E_1) = C_{in} f_1 Q (1-E_1)$

cleaner 2 → $C_2 Q_2 = C_{in} Q_2 (1-E_2) = C_{in} f_2 Q (1-E_2)$

overall mass flow rate → $C_{out} Q = C_1 Q_1 + C_2 Q_2$

$$C_{out} \cancel{Q} = C_{in} f_1 \cancel{Q} (1-E_1) + C_{in} f_2 \cancel{Q} (1-E_2)$$

☆ $$\boxed{C_{out} = C_{in} \left[f_1 (1-E_1) + f_2 (1-E_2) \right]}$$

$E_{overall}$ = equivalent overall removal efficiency of these cleaners in parallel

$$\boxed{E_{overall} = 1 - \frac{C_{out}}{C_{in}} = 1 - \left[f_1 (1-E_1) + f_2 (1-E_2) \right]}$$

(for 2 cleaners in parallel)

- Extend to $\underline{\underline{m}}$ parallel cleaners:

$$\boxed{E_{overall} = 1 - \sum_{j=1}^{m} f_j (1-E_j)}$$

☆ E for m cleaners in parallel

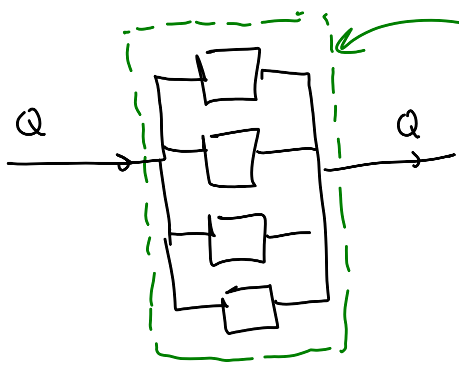
- Simplest case → $\underline{m \text{ identical units}}$ (E_j = same for all units) (f_j = same " ")

$$\boxed{f_j = \frac{Q_j}{Q} = \frac{1}{m}}$$

$\sum_{j=1}^{m} = m$

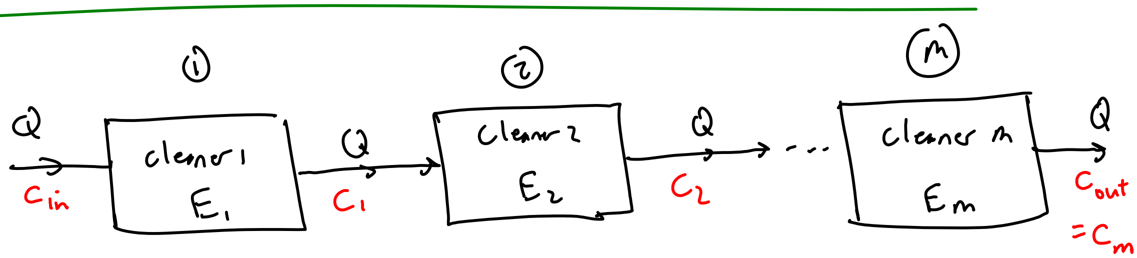
$$\therefore E_{overall} = 1 - m f_j (1-E_j) = 1 - \frac{m}{m}(1-E_j) = E_j$$

$$\boxed{E_{overall} = E_j \text{ for } \underline{m \text{ identical cleaners in parallel}}}$$ ☆



$E_{overall} = E_j$

☆ $\underline{\text{SERIES}}$



Q = constant throughout

$C_1 = (1-E_1)C_{in}$ as previously

$C_2 = (1-E_2)C_1 = (1-E_2)(1-E_1)C_{in}$

$\downarrow$

$C_{out} = C_m = \underline{(1-E_1)(1-E_2)(1-E_3)\cdots(1-E_m)C_{in}}$

$$E_{overall} = 1 - \frac{C_{out}}{C_{in}} = 1 - (1-E_1)(1-E_2)(1-E_3)\cdots(1-E_m)$$

$$\boxed{E_{overall} = 1 - \prod_{j=1}^{m}(1-E_j)}$$

where $\prod$ is like $\sum$ except to product, not sum

Overall removal efficiency for m cleaners in series

Some people like to define Penetration = $\boxed{P_j = 1 - E_j}$

e.g. [$E = 0.9 = 90\%$ → 90% removal, which means 10% penetration, $P_j = 0.10$]

$$\boxed{E_{overall} = 1 - \prod_{j=1}^{m} P_j}$$

(most often used for filters)

Simplest case → m identical cleaners in series

$$\boxed{E_{overall} = 1 - (1-E_j)^m}$$

Comments:
- For gases, use the eq's as is — one E for the species
- For particles, use $E(D_p)$ → Grade efficiency → different $E_{overall}$ depending on D_p

$E_{overall}(D_p)$ = also a grade efficiency

Particles

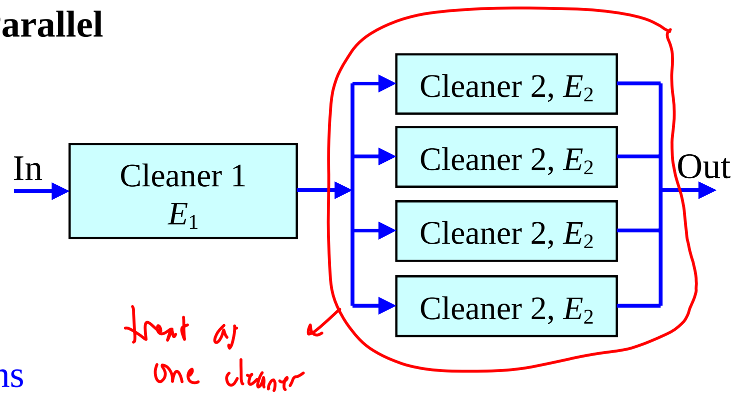
Parallel:
$$\boxed{E(D_p)_{overall} = 1 - \sum_{j=1}^{m} f_j \left(1 - E(D_p)_j\right)}$$

Series:
$$\boxed{E(D_p)_{overall} = 1 - \prod_{j=1}^{m}\left(1 - E(D_p)_j\right)}$$

Example: Lapple Cyclones in Series and Parallel

Given: Dusty air is cleaned by one large Lapple cyclone in series with four smaller Lapple cyclones in parallel. Details:

- particle density ρ_p = 1500 kg/m^3
- bulk flow rate of air Q = 0.111 m^3/s
- air at STP: ρ = 1.184 kg/m^3, $\mu = 1.849 \times 10^{-5}$ kg/(m s)
- $D_{p,\text{cut},1}$ = 10 microns; $D_{p,\text{cut},2}$ = 2.5 microns



To do: Calculate the overall removal efficiency of 2.0-µm particles. Give your answer in percentage to 3 significant digits. Some equations are provided here for convenience.

Parallel:

$$\boxed{E(D_p)_{\text{overall}} = 1 - \sum_{j=1}^{m} f_j \left[1 - E(D_p)_j\right]}, \quad \boxed{f_j = \frac{Q_j}{Q_{\text{total}}}}$$

Series:

$$\boxed{E(D_p)_{\text{overall}} = 1 - \prod_{j=1}^{m} \left[1 - E(D_p)_j\right]}$$

Lapple: $$\boxed{E(D_p) = \frac{1}{1 + \left(\dfrac{D_p}{D_{p,\text{cut}}}\right)^{-2}}}$$

Solution:

Cleaner 1 → $E(D_p)_1 = \dfrac{1}{1 + \left(\dfrac{2.0\ \mu m}{10\ \mu m}\right)^{-2}} = \underline{\underline{0.03846}}$

Cleaner 2 → $E(D_p)_2 = \dfrac{1}{1 + \left(\dfrac{2}{2.5}\right)^{-2}} = \underline{\underline{0.39024}}$

2 cleaners in series:

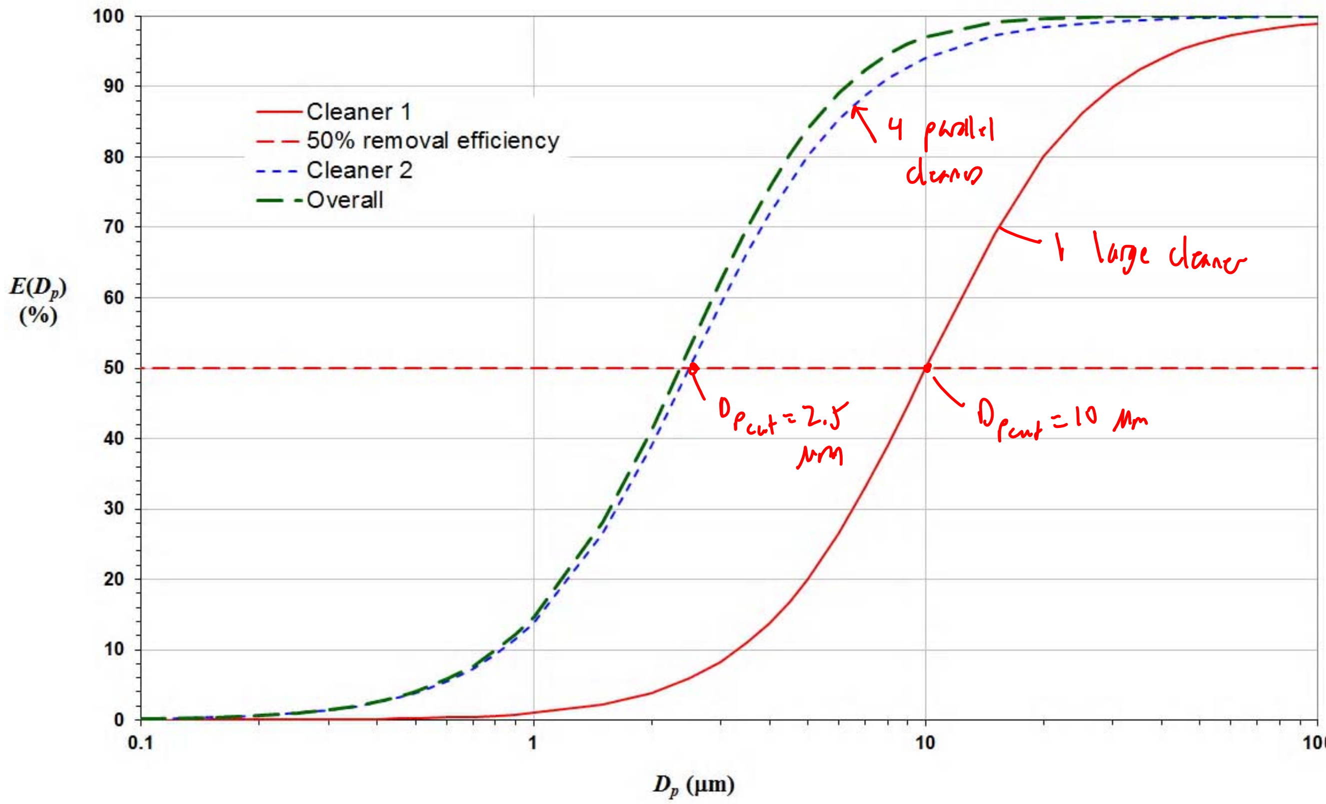
$$E(D_p)_{\text{overall}} = 1 - \left(1 - E(D_p)_1\right)\left(1 - E(D_p)_2\right)$$
$$= 1 - (1 - 0.03846)(1 - 0.39024)$$
$$= 0.4137$$

$$\boxed{E(D_p)_{\text{overall}} = \underline{\underline{41.4\%}}}$$

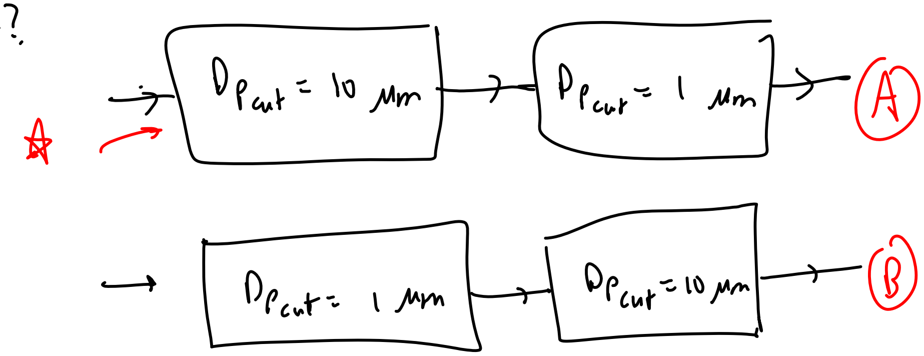
For $D_p = 2\ \mu m$

Repeat for all Diameters to get the overall grade efficiency curve

(I used Excel)



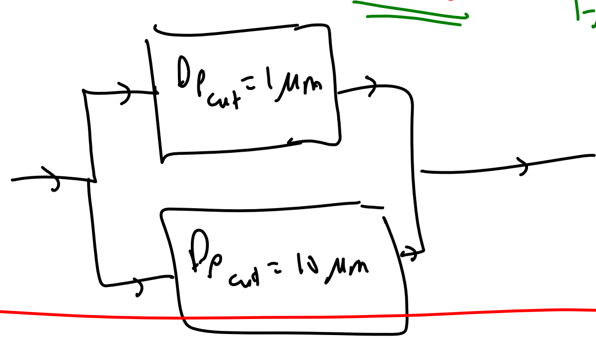
Does the order matter?



Algebra shows no difference — $E_{overall}$ is same for both cases

But, Option A is better practically → [In Option B, the 1-µm cleaner is doing all the work! — But would get clogged up & damaged more easily than it would in Option A.]

In parallel



It is unwise to put unequal cleaners in parallel