

**Today, we will:**

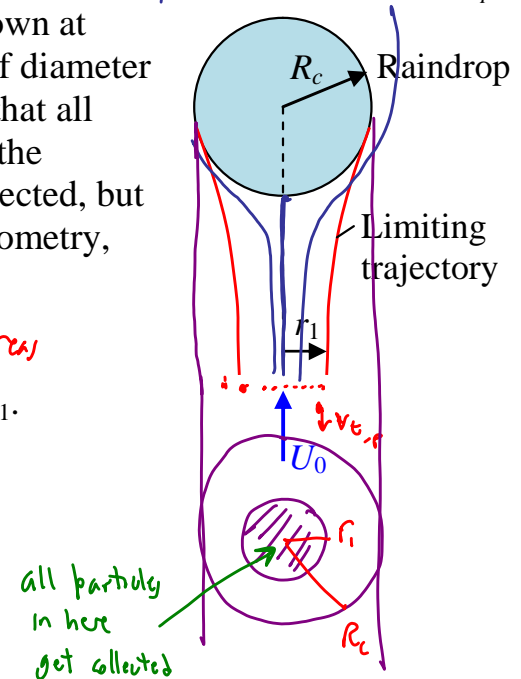
- Continue to discuss particle cleaning by **raindrops** and **spray chambers**
- Discuss the Calvert and Englund model for **single drop grade removal efficiency**
- Discuss **Brownian motion** and its effect on single drop collection grade efficiency

Last time, we developed an expression for the **single drop grade removal efficiency**  $E_d(D_p)$  of a spherical raindrop of diameter  $D_c$  (radius  $R_c$ ) falling down at speed  $U_0$ , and encountering dusty air containing particles of diameter  $D_p$ . We defined  $r_1$  as the **limiting trajectory radius**, such that all particles of diameter  $D_p$  within this radius are collected by the raindrop, and all particles outside of this radius are not collected, but rather miss hitting the raindrop altogether. Based on the geometry, we derived:

$$E_d(D_p) = (r_1/R_c)^2 = \text{ratio of areas}$$

Now the problem reduces to generating an expression for  $r_1$ .

$r_1$  must be  $< R_c$   
otherwise  $E_d > 100\%$

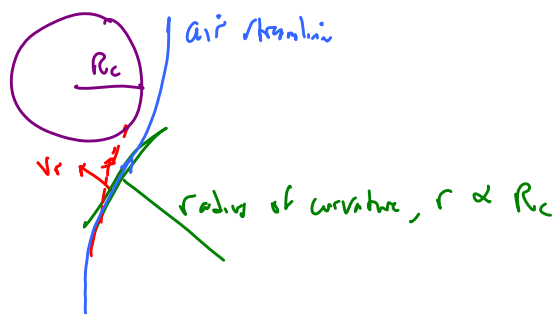


How to calculate  $r_1$ ? (or  $R_c$ )

$$r_1 = f_{nc}(D_p, D_c, \rho_p, \rho_c, \rho_{air}, \mu)$$

$p$  = particle (aerosol, air pollution particles)  
 $c$  = collector = rain drop

Physically



We can use this as our separation accel velocity

$V_r$  = Inertial separation velocity

$$V_r \propto \frac{U_0^2}{R_c}$$

• Use CFD

• OR → do experiments ; make empirical equation (curve fits)

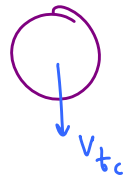
E.g., Calvert i. England (1984) — develop an empirical curve fit (simple eq.)

Notes: 1)  $U_0$  = speed of air relative to the falling drop

$$U_0 = V_{t, \text{drop}} = V_{t, c}$$

2) spherical drops — but can work for any shape

3) Works only when  $D_p < D_c$



$$V_{t,p} < V_{t,c}$$

Define:

$$Stk = \text{Stokes Number} = \frac{(\rho_p - \rho) D_p^2 (U_0 - V_{t,p})}{18 \mu D_c}$$

[some books use  $\Psi$ ]

relative velocity between the drop & particle

C<sub>d</sub>E model  $\rightarrow$   $\frac{r_i}{R_c} = \left( \frac{Stk}{Stk + 0.35} \right) \rightarrow E_d(D_p) = \left( \frac{Stk}{Stk + 0.35} \right)^2$

C<sub>d</sub>E model for single drop collection efficiency of a raindrop

### Example: Single-drop collection grade efficiency for one particle diameter

**Given:** Raindrops of diameter 200 microns are falling through dusty air at STP ( $\rho = 1.184 \text{ kg/m}^3$ ,  $\mu = 1.849 \times 10^{-5} \text{ kg/(m s)}$ ). The uniformly distributed aerosol particles are of unit density ( $\rho_p = 1000 \text{ kg/m}^3$ ), and of diameter 5 microns.

**To do:** Calculate the single-drop collection grade efficiency  $E_d(D_p)$  as a percentage to three significant digits for these particles.

**Solution:**

Calc.  $V_{t_c} = U_0$  for raindrop — gravimetric settling

$$\rho_c = 1000 \frac{\text{kg}}{\text{m}^3} \text{ (water)}$$

→ Do iteration using  $C_0 = C_0(Re)$ ,  $C = \text{Cunningham}$ , etc.

$$\rightarrow V_{t_c} = U_0 = 0.700464 \text{ m/s}$$

$$V_{t_p} \rightarrow \text{repeat for the particle} - V_{t_p} = 0.00076057 \text{ m/s}$$

$$\text{Stokes \#} \quad Stk = \frac{(\rho_p - \rho_{\text{air}}) D_p^2 (U_0 - V_{t_p})}{18 \mu D_c}$$

$$= \frac{(1000 - 1.184) \text{ kg/m}^3 (5 \times 10^{-6} \text{ m})^2 (0.700464 - 0.00076057) \text{ m/s}}{18 (1.849 \times 10^{-5} \frac{\text{kg}}{\text{m.s}}) (200 \times 10^{-6} \text{ m})}$$

$$Stk = 0.26248 \quad [\text{Non-dimensional parameter}]$$

$$E_d(D_p) = \left( \frac{Stk}{Stk + 0.35} \right)^2 = \left( \frac{0.26248}{0.26248 + 0.35} \right)^2 = 0.1837$$

$$E_d(D_p) \approx 18.4\%$$

↓  
for 5  $\mu\text{m}$  particle

$E_d(D_p)$  will vary with  $D_p \rightarrow \text{Grade efficiency}$

**Example: Single-drop collection grade efficiency as a function of particle diameter**

**Given:** A polydisperse aerosol at STP ( $\rho = 1.184 \text{ kg/m}^3$ ,  $\mu = 1.849 \times 10^{-5} \text{ kg/(m s)}$ ) is cleaned by raindrops of diameter 200 microns. The particles are of unit density ( $\rho_p = 1000 \text{ kg/m}^3$ ). The raindrop (the “collector” falls at terminal gravitational settling velocity  $V_{t,c} = 0.700464 \text{ m/s}$ . For convenience, I have pre-calculated the terminal gravitational settling velocity  $V_{t,p}$  for various particle diameters. Assume that particles are absorbed by the raindrop when they hit the raindrop, and are therefore removed from the air.

**To do:** For each diameter, calculate the single-drop collection grade efficiency  $E_d(D_p)$  as a percentage to three significant digits for each particle diameter  $D_p$  in the table below.

**Solution:** Table to be filled in during class:

$D_p$ ( $\mu\text{m}$ )	$V_t$ (m/s)	$E_d(D_p)$ (%)
2	0.000127646	1.15
3	0.000279763	4.52
4	0.000490739	10.5
7	0.001476776	35.4
8.5	0.002168354	46.8
10	0.002992228	56.2
15	0.006692553	75.7
20	0.01185533	84.99

$V_{t,p}$

$D_p$ ( $\mu\text{m}$ )	$V_t$ (m/s)	$E_d(D_p)$ (%)
25	0.018468719	89.9
30	0.026513567	92.7
40	0.046775902	95.7
50	0.072295146	97.1
70	0.13687402	98.3
85	0.194258109	98.7
100	0.25629326	98.9

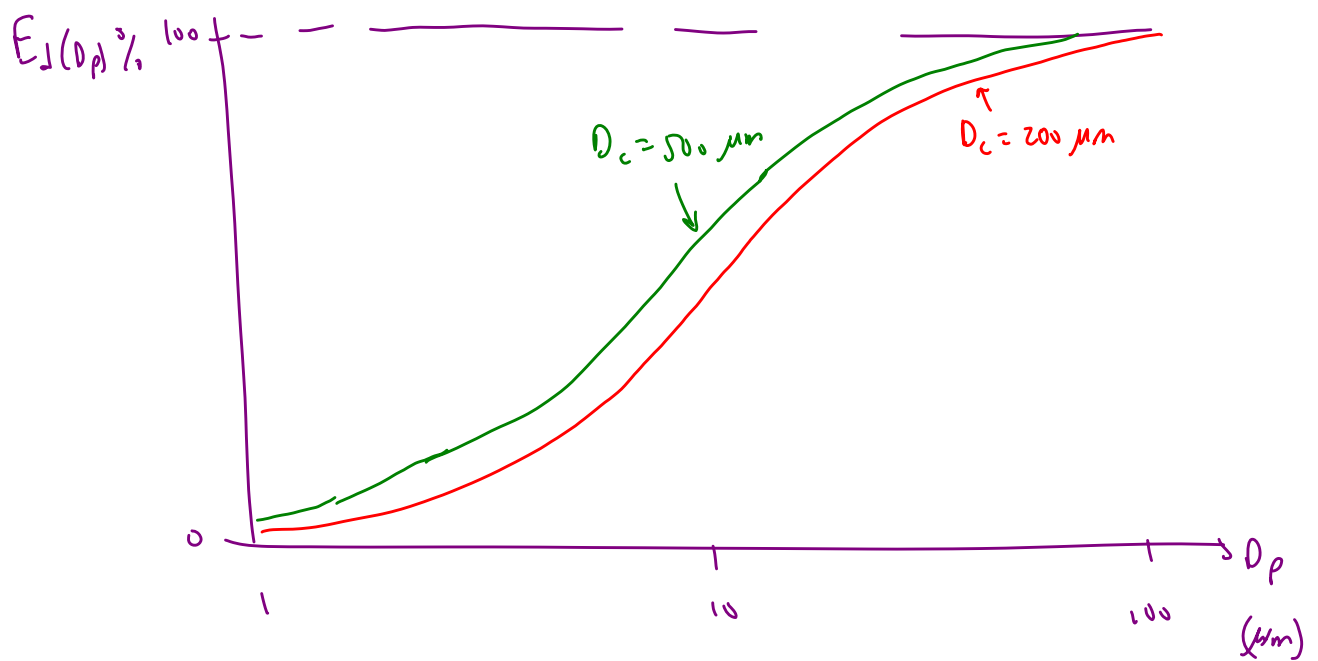
$$Stk = \frac{(\rho_p - \rho) D_p^2 (V_0 - V_{t,p})}{18 \mu D_c}$$

$$E_d(D_p) = \left( \frac{Stk}{Stk + 0.35} \right)^2$$

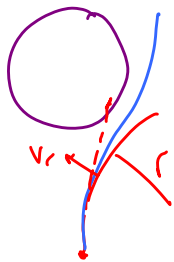
**Filled in Table:**

$D_p$ ( $\mu\text{m}$ )	$V_t$ (m/s)	$E_d(D_p)$ (%)
2	0.000127646	1.149675069
3	0.000279763	4.524218465
4	0.000490739	10.52314578
7	0.001476776	35.38805555
8.5	0.002168354	46.76422259
10	0.002992228	56.15821262
15	0.006692553	75.69004041
20	0.01185533	84.99538738

$D_p$ ( $\mu\text{m}$ )	$V_t$ (m/s)	$E_d(D_p)$ (%)
25	0.018468719	89.89263573
30	0.026513567	92.73095237
40	0.046775902	95.6846146
50	0.072295146	97.09435181
70	0.13687402	98.33191736
85	0.194258109	98.73657951
100	0.25629326	98.95794227



What happens if  $D_c$  is bigger? say  $500 \mu m$  instead of  $200 \mu m$ ?

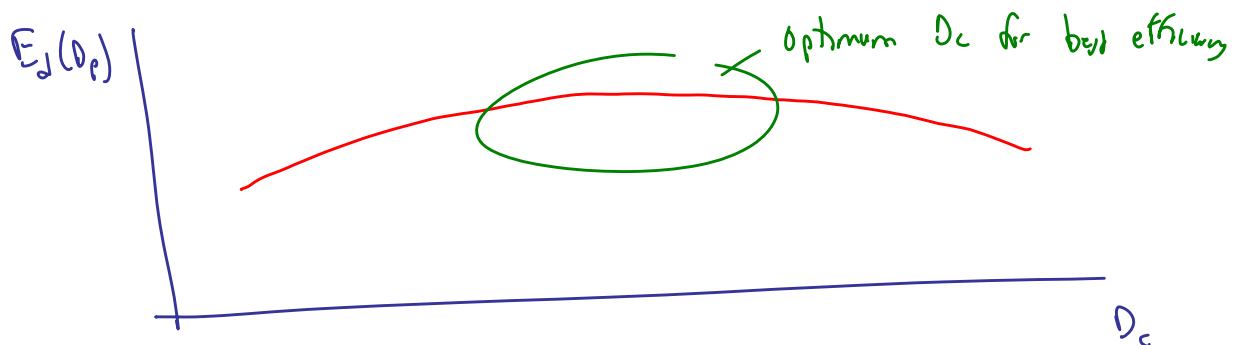


$$V_r \propto \frac{U_0^2}{r} \quad \rightarrow \quad U_0 \uparrow \text{ with } D_c \rightarrow V_r \uparrow$$

$$r \propto R_c \quad r \uparrow \quad \rightarrow \quad V_r \downarrow$$

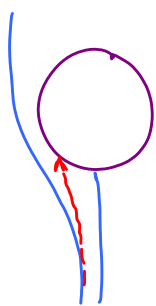
It turns out

that  $E_d(D_p) \uparrow$  as  $D_c \uparrow \rightarrow$  But there is a limit

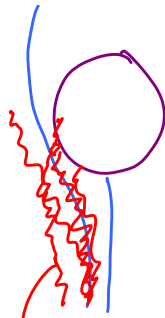


## Diffusion Effects & Brownian Motion

- CiE model breaks down at very small  $D_p$  due to diffusion & Brownian motion
- Around  $0.1 \mu m$  Brownian motion starts to be important

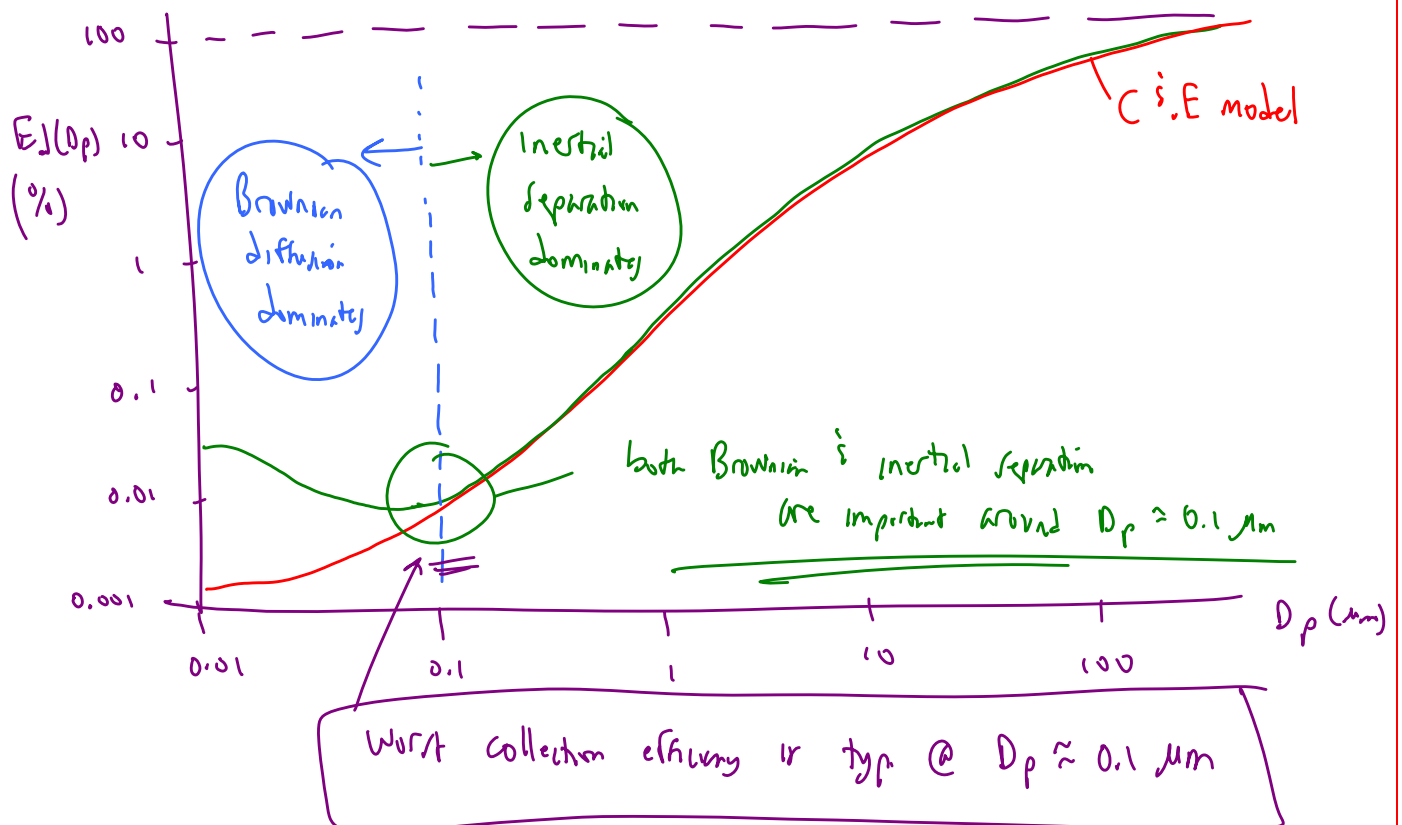


Standard model



trajectories have a random "wobble" due to Brownian motion

$E_d(D_p)$  goes up for very small particles compared to CiE model



IT IS VERY HARD TO CAPTURE  $0.1 \mu m$  PARTICLES WITH INERTIAL SEPARATION \*