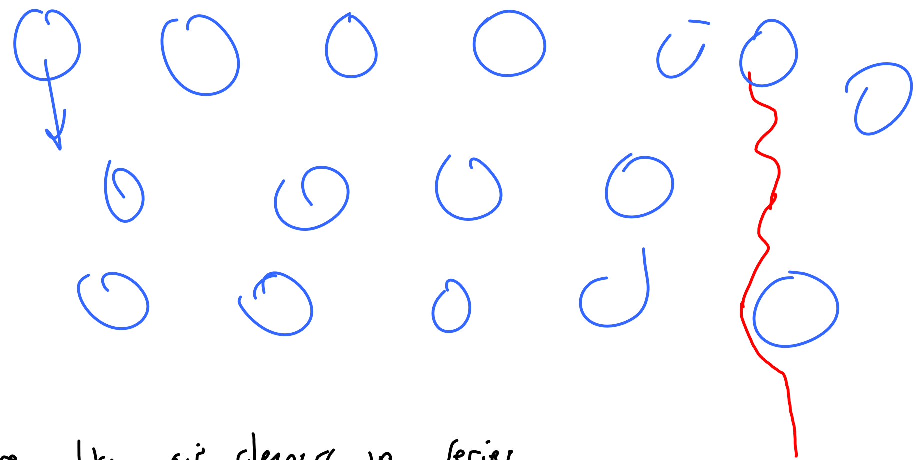
Today, we will:

- Discuss multiple-drop rain drop collection efficiency
- Discuss **spray chambers** (artificial rain chambers to remove particulate matter)

Last time – we discussed single-drop grade collection efficiency $E_d(D_p)$ for one drop of rain

Even if $E_d(D_p)$ is very small, particles encounter many raindrops

$E(D_p)$ may be large



Raindrops are like air cleaners in series

Example: Single-drop collection grade efficiency for one particle diameter

Given: Raindrops of diameter 200 microns are falling through dusty air at STP (ρ = 1.184 kg/m^3, μ = 1.849 × 10^{-5} kg/(m s)). The uniformly distributed aerosol particles are of unit density (ρ_p = 1000 kg/m^3), and of diameter 2 microns. In a previous example problem, we calculated single-drop collection grade efficiency $E_d(D_p)$ = 1.14968% for these particles.

To do: If we model the raindrops as 100 drops lined up in series (above each other), calculate the grade efficiency $E(D_p)$ as a percentage to 3 significant digits for these particles.

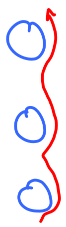
Solution:

Cleaners in Parallel:

$$\cancel{E\left(D_p\right)_{\text{overall}} = 1 - \sum_{j=1}^{m} f_j\left[1 - E\left(D_p\right)_j\right]}, \quad f_j = \frac{Q_j}{Q_{\text{total}}}$$

Cleaners in Series:

$$E\left(D_p\right)_{\text{overall}} = 1 - \prod_{j=1}^{m}\left[1 - E\left(D_p\right)_j\right]$$



$$E(D_p) = 1 - \prod_{j=1}^{m}\left[1 - E_d(D_p)\right]_j$$

$$E(D_p) = 1 - \left(1 - E_d(D_p)\right)^m$$

$$= 1 - \left(1 - 0.0114968\right)^{100}$$

$$= \boxed{68.5\%}$$

It smells good after a rain!

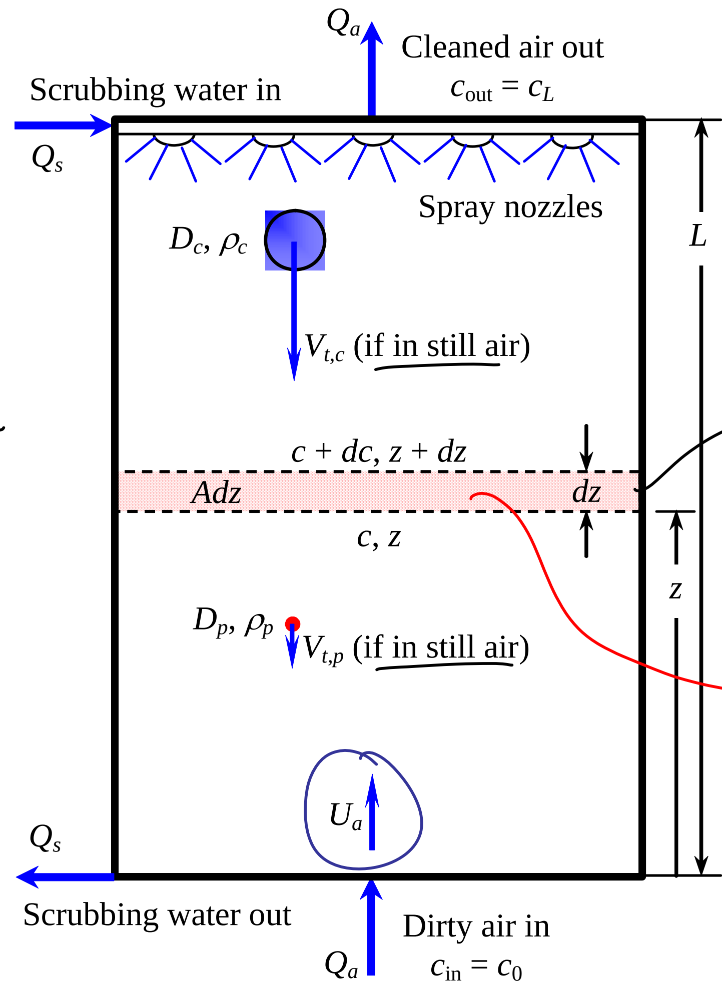
This is the idea behind spray chambers — a type of APCS

(artificial rain)

Counter-Flow Spray Chamber

Schematic diagram:

A = cross-sectional area of chamber



infinitesimal slice through the chamber @ $z = z$ & height = dz

Assume well-mixed in horizontal direction

We adjust U_a such that $V_{t,c} > U_a$ → drops fall at speed

Drops absolute falling velocity of rain drops → $\boxed{V_c = V_{t,c} - U_a}$ (*)

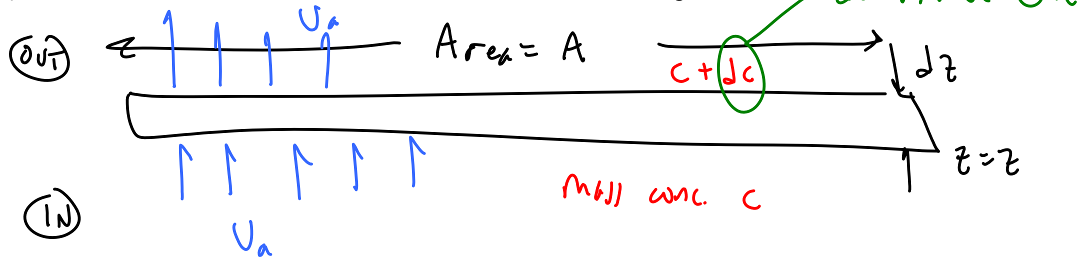
Particles absolute rise velocity of particles — $\boxed{U_a - V_{t,p}}$

[typically $\approx U_a$]

In moving ref. frame moving up @ speed U_a (air speed)

∴ our eqs are still valid → $\boxed{E_d(D_p) = \text{same as for rain drops in still air}}$

Analyze the thin CV shaded in the diagram



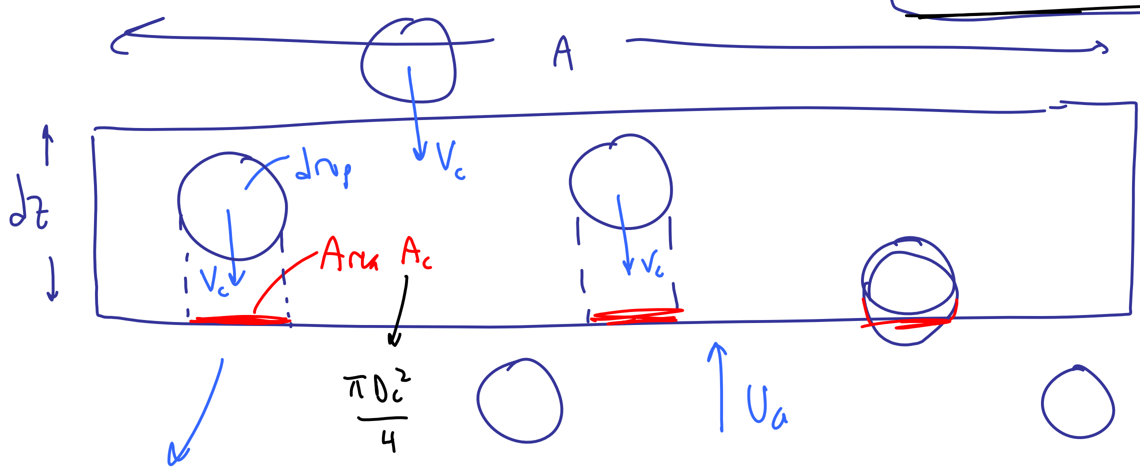
overall grade efficiency $= E(D_p) = 1 - \frac{c_{out}}{c_{in}}$

here, $dE(D_p) = 1 - \frac{c+dc}{c}$ = differential removal eff.

$$c\,dE(D_p) = \cancel{c} - \cancel{c} + dc$$

@ location z

$$dc = -c\,dE(D_p) \rightarrow \boxed{\frac{dc}{c} = -dE(D_p)} \quad (1)$$



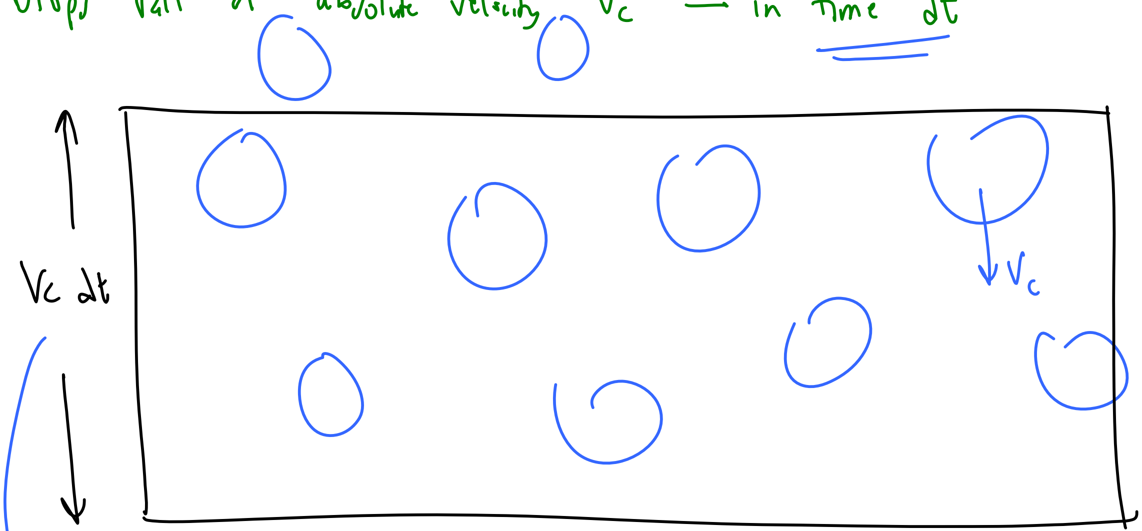
For one drop, $E_d(D_p)$ = single-drop collection efficiency that the air under that drop sees

$$\boxed{dE(D_p) = E_d(D_p) \cdot \frac{A_c}{A} \cdot (\text{\# drops that potentially encounter particles})} \quad (2)$$

overall eff. — single drop eff. — affected area

Let $dt = \frac{dz}{U_a}$ since $dz = U_a\,dt$ — dt = time for the air to rise through the CV (dz)

Drops fall at absolute velocity V_c — in time dt



$$V_c\,dt = \boxed{V_c \frac{dz}{U_a}}$$

Total volume of drops passing through our C.V. that potentially collect particles is

$$\boxed{\underbrace{(V_c + U_a)}_{V_{t,c}}\,\underbrace{dt}_{\frac{dz}{U_a}} \cdot A}$$

The total # drops that encounter particles during time dt for our slice $= \dfrac{(V_c+U_a)\,dz\cdot A}{U_a}\cdot C_{number,c}$ (3)

[number concentration of collector drops]

$$C_{number,c} = \frac{\#\ \text{drops}}{\text{Volume}}$$

Plug (3) into (2) →

$$\boxed{dE(D_p) = E_d(D_p)\cdot\frac{\pi D_c^2}{4A}\,\frac{V_c+U_a}{U_a}\,C_{number,c}\cdot dz\cdot A} \quad (4)$$

how to calculate this?

Example: Number Concentration of Water Drops in a Spray Chamber

Given: A counter-flow spray chamber with the following properties:

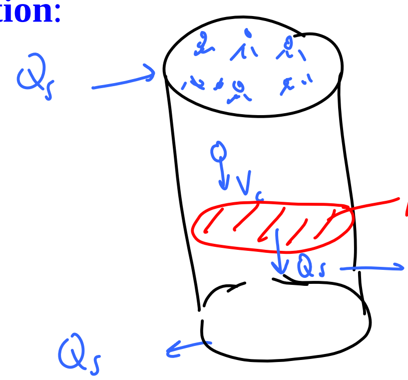
- D_c = 200 microns (collector water drop diameter)
- A = 0.7854 m^2 (cross –sectional area of the spray chamber)
- Q_s = 0.0000521 m^3/s (volume flow rate of the supply water, at the top)
- V_c = 0.38215 m/s (falling speed of water drops in absolute reference frame)

To do: If the drops are randomly and uniformly distributed throughout the spray chamber, calculate the number concentration $c_{\text{number,c}}$ of collector water drops in units of millions of drops per cubic meter (Mdrops/m^3) to three significant digits.

Solution:

let $\forall_c$ = volume of one drop

$$= \frac{\pi D_c^3}{6}$$



A = cross-sectional area

Q_s is same as Q_s supplied since flow is steady

(but Q_s is split up into millions of drops)

$$Q_s = V_c \cdot \forall_c \cdot c_{\text{number},c} \cdot A$$

absolute settling speed

Vol. of one drop $= \frac{\pi D_c^3}{6}$

$\frac{\#\text{ drops}}{\text{Volume}}$ · (cross-sectional area)

$$\boxed{c_{\text{number},c} = \frac{Q_s}{V_c \forall_c A}}$$

We need to generate this eq. by Cons. of mass argument

#'s

$$\boxed{c_{\text{number},c} = \frac{6\,Q_s}{V_c \pi D_c^3 A}} = \frac{6\left(0.0000521\ \text{m}^3/\text{s}\right)}{\left(0.38215\ \frac{\text{m}}{\text{s}}\right)\pi\left(200\times10^{-6}\ \text{m}\right)^3\left(0.7854\ \text{m}^2\right)}$$

$$= 4.144\times10^{7}\ \frac{\text{drops}}{\text{m}^3}$$

or

$$\boxed{c_{\text{number},c} = \frac{41.4\ \text{Mdrops}}{\text{m}^3}}$$

That's a lot of particles!