Today, we will:

- Finish derivation of grade efficiency for **spray chambers**, and do an example problem
- Briefly discuss **wet scrubbers** (another type of APCS to remove PM)
- Begin a discussion about **air filters**, and how they work

Continuing from previous lecture, for a counter-flow spray chamber we had:
Differential grade removal efficiency across our small control volume of volume Adz:

$$\boxed{\frac{dc}{c} = -dE\left(D_p\right)} \qquad (1)$$

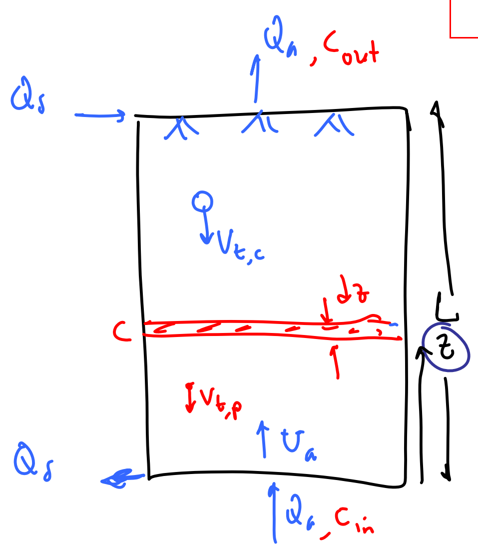
Differential grade removal efficiency across our small control volume of volume Adz:

$$\boxed{dE\left(D_p\right) = E_d\left(D_p\right)\frac{\pi D_c^{\ 2}}{4A}\frac{V_c + U_a}{U_a}c_{\text{number,c}}Adz} \qquad (4)$$

Number concentration of the collector water drops, where Q_s is the water volume flow rate:

$$\boxed{c_{\text{number,c}} = \frac{6Q_s}{V_c \pi D_c^{\ 3} A}} \qquad (5)$$

(From end of previous lecture)



Plug (5) into (4):

$$dE(D_p) = E_d(D_p)\frac{\pi D_c^2}{4A}\frac{V_c + U_a}{U_a}\frac{6Q_s}{V_c \pi D_c^3 \cancel{A}}\cancel{A}\,dz$$

$Q_a = U_a A$ = vol. flow rate of air

$$dE(D_p) = E_d(D_p)\frac{3}{2}\frac{V_c + U_a}{V_c}\frac{Q_s}{Q_a}\frac{dz}{D_c}$$

Plug into (1)

$$\frac{dc}{c} = -\frac{3}{2}E_d(D_p)\frac{V_c + U_a}{V_c}\frac{Q_s}{Q_a}\frac{1}{D_c}dz$$

This $= V_{t,c}$ = settling velocity in still air for the collector drops

We can integrate since we have separated variables

can integrate from $z=0$ to $z=z$ or to $z=L$

$$\int_{c=c_{in}}^{c} \frac{dc}{c} = -\int_{z=0}^{z} \underbrace{\frac{3}{2} E_d(D_p) \frac{V_c+V_a}{V_c} \frac{Q_s}{Q_a} \frac{1}{D_c}}_{\text{const. for a given } D_p} dz$$

$$\ln(c) - \ln(c_{in}) = \ln\left(\frac{c}{c_{in}}\right) = -\frac{3}{2} E_d(D_p) \frac{V_c+V_a}{V_c} \frac{Q_s}{Q_a} \frac{1}{D_c} z$$

take exp of both sides

$$c = c_{in} \exp\left[-\frac{3}{2} E_d(D_p) \frac{V_{t,c}}{V_c} \frac{Q_s}{Q_a} \frac{1}{D_c} z\right]$$

Overall $E(D_p)$ = grade collection efficiency up to height z

$$E(D_p) = 1 - \frac{c}{c_{in}}$$

$$E(D_p) = 1 - \exp\left[\frac{-z}{\frac{2}{3} \frac{Q_a}{Q_s} \frac{V_c}{V_{t,c}} \frac{D_c}{E_d(D_p)}}\right]$$

call this L_c = critical length

$$\boxed{L_c = \frac{2}{3} \frac{Q_a}{Q_s} \frac{V_c}{V_{t,c}} \frac{D_c}{E_d(D_p)}}$$

$$E(D_p) = 1 - \exp\left(-\frac{z}{L_c}\right)$$

For the whole tower, $z = L$

$$\boxed{E(D_p) = 1 - \exp\left(-\frac{L}{L_c}\right)} \quad \star$$

Same form as for elutriators or curved ducts, as discussed previously (well-mixed)

Example: Designing a Spray Chamber

Given: A counter-flow spray chamber is being designed with the following properties:

- D_c = 200 microns (collector water drop diameter)
- A = 0.7854 m^2 (cross –sectional area of the spray chamber)
- Q_s = 0.0000521 m^3/s (volume flow rate of the supply water, at the top)
- $V_{t,c}$ = 0.700464 m/s (falling speed of water drops in still air)
- D_p = 5 microns (diameter of the air pollution particles we are targeting)
- ρ_p = 1000 kg/m^3 (air pollution particles are treated as unit density spheres)
- Q_a = 0.250 m^3/s (volume flow rate of the dirty air, introduced at the bottom)
- Air at STP: ρ = 1.184 kg/m^3, $\mu = 1.849 \times 10^{-5}$ kg/(m s)

To do: Calculate the required height of the spray chamber to remove 90% of these particles. Give answer in meters to three significant digits.

Solution: From a previous problem, for 5-micron particles and 200-micron raindrops, we had $E_d(D_p)$ = 0.183659. ← Single-drop collection efficiency

Equations: $\boxed{E\left(D_p\right) = 1 - \exp\left(-\frac{L}{L_c}\right)}$, where $\boxed{L_c = \frac{2}{3}\frac{Q_a}{Q_s}\frac{V_c}{V_{t,c}}\frac{D_c}{E_d\left(D_p\right)}}$ and $\boxed{V_c = V_{t,c} - U_a}$.

$$U_a = Q_a/A = \frac{0.250 \text{ m}^3/\text{s}}{0.7854 \text{ m}^2} = 0.31831 \text{ m/s}$$

$$V_c = V_{t,c} - U_a = 0.700464 - 0.31831$$

$$V_c = 0.382154 \text{ m/s}$$

$$L_c = \frac{2}{3}\frac{Q_a}{Q_s}\frac{V_c}{V_{t,c}}\frac{D_c}{E_d(D_p)} = \frac{2}{3}\left(\frac{0.250 \text{ m}^3/\text{s}}{0.0000521 \text{ m}^3/\text{s}}\right)\left(\frac{0.382154 \text{ m/s}}{0.700464 \text{ m/s}}\right)\left(\frac{200\times10^{-6} \text{ m}}{0.183659}\right)$$

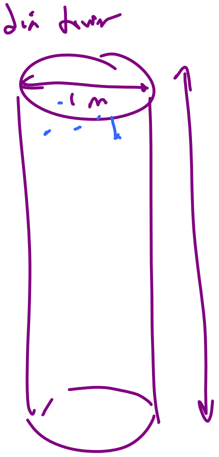
$$L_c = 1.90056 \text{ m}$$

$$1 - E(D_p) = \exp\left(-\frac{L}{L_c}\right)$$

$$\boxed{L = -L_c \ln\left(1 - E(D_p)\right)} \rightarrow = -(1.90056 \text{ m}) \ln(1 - 0.90)$$

$$L = 4.376 \text{ m} \rightarrow \boxed{L \cong 4.38 \text{ m}}$$

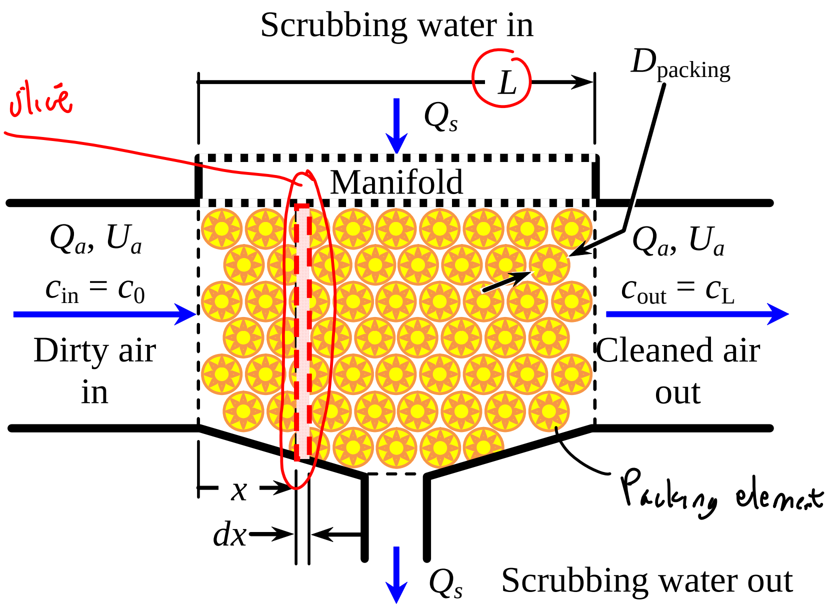
1-m dia tower



The taller the tower, the better the removal efficiency

Wet Scrubbers: → Similar to spray tower, but very compact

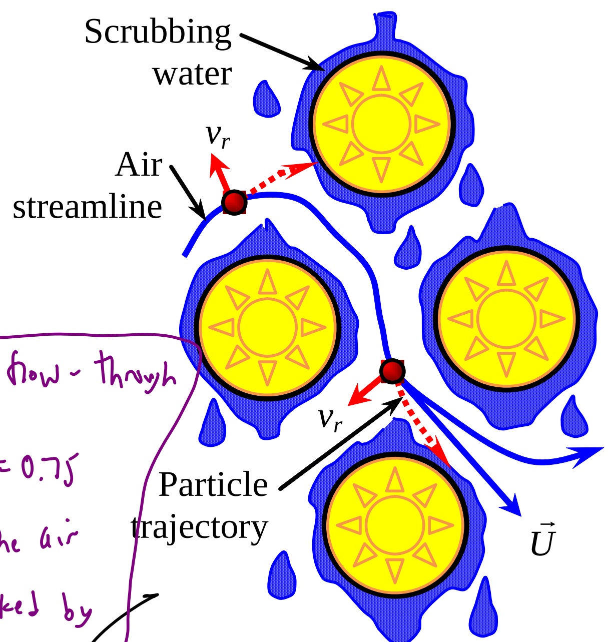
Example: **Transverse Packed-Bed Scrubber**:



Well-mixed assumption vertically in the slice

c decreasing with x

Close-up of packing elements, showing water and inertial separation of small particles:



ε = Porosity = air flow-through factor

e.g. if $\varepsilon = 0.75$

→ 25% of the air duct is blocked by packing elements

D_c = some characteristic collector diameter (depends on the shape, geometry of the packing material)

- Also use principle of inertial separation
- Can be much more compact than a spray chamber ★
- Can draw a CV. slice (vertical slice) since flow is horizontal
- Result:

$$\boxed{E(D_p) = 1 - \exp\left(-\frac{L}{L_c}\right)}$$

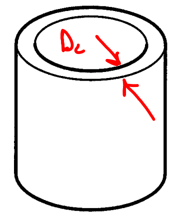
Same eq. as for spray tower

need to calc. L_c

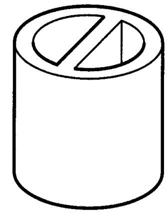
$$L_c = \text{fnc}\left(Q_a, \rho, \mu, \rho_p, D_p, D_c, U_a, \varepsilon, a_p\right)$$

a_p → $\dfrac{\text{packing surface area}}{\forall_{bed}}$

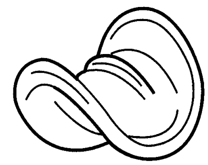
Packing elements come in all kinds of sizes, shapes, and materials:



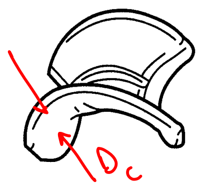
(a) Raschig ring



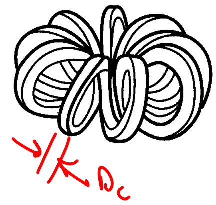
(b) Lessing ring



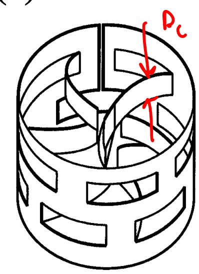
(c) Berl saddle



(d) Intalox saddle



(e) Tellerette ring



(f) Pall ring

D_c is typically associated with the small diameter scale, not necessarily the dia. of the packing element itself.