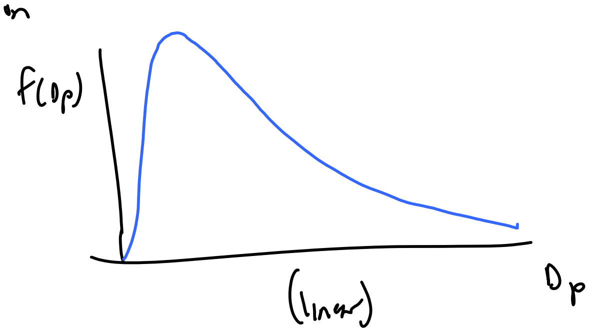
Today, we will:

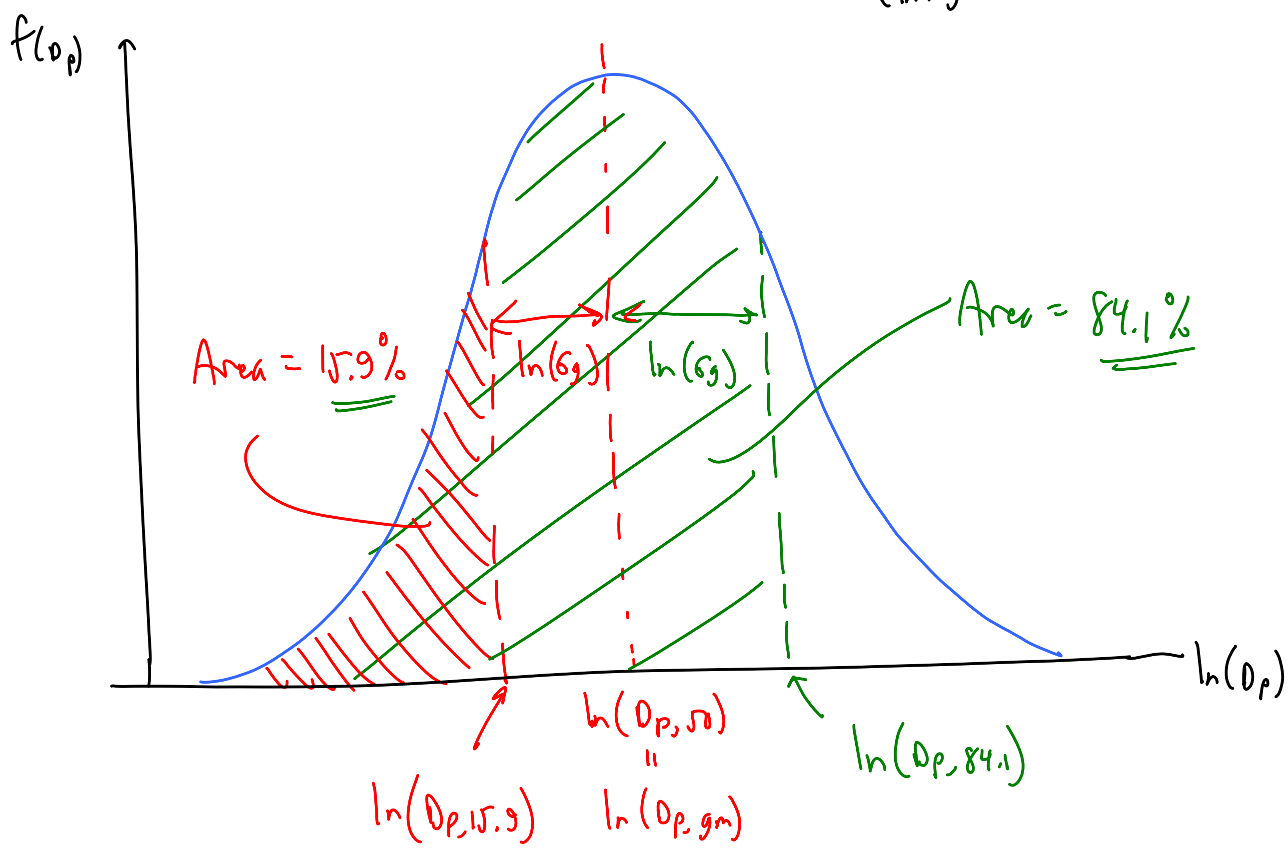
- Continue discussing aerosol particle statistics: **mass distribution**, and **cumulative mass distribution**.
- Finish discussing aerosol particle statistics: Predict how the particle statistics change after passing through an APCS.
- News article presentation by Isaac Moore

σ_g = geometric standard deviation

For a lognormal distribution



On a scale with $\ln(D_p)$



For our example data from the handout → $D_{p,15.9} = 4.7\ \mu m$

$D_{p,50} = 9.0\ \mu m$

$D_{p,84.1} = 17.0\ \mu m$

Calculate $\sigma_g \longrightarrow$ $\boxed{\sigma_g = \frac{D_{p,84.1}}{D_{p,50}}} = \frac{17.0}{9.0} = 1.89\ \mu m$

$\boxed{\sigma_g = \frac{D_{p,50}}{D_{p,15.9}}} = \frac{9.0}{4.7} = 1.91\ \mu m$

$\boxed{\sigma_g = \sqrt{\frac{D_{p,84.1}}{D_{p,15.9}}}} = \sqrt{\frac{17.0}{4.7}} = 1.90\ \mu m$

Same, but not exact due to scatter

From the plot, we see that

$$\ln(D_{p,84.1}) = \ln(D_{p,50}) + \ln(\sigma_g)$$

$\boxed{\ln a + \ln b = \ln(ab)}$

$$\ln(D_{p,84.1}) = \ln(D_{p,50} \cdot \sigma_g)$$

$$e^{(\,)} \rightarrow \quad D_{p,84.1} = D_{p,50} \cdot \sigma_g \longrightarrow \boxed{\sigma_g = \frac{D_{p,84.1}}{D_{p,50}}}$$

MASS DISTRIBUTION

- We know how to convert from number dist. to a mass dist

- mass per particle $= \rho_p \frac{\pi}{6} D_{p,j}^3$

$$\boxed{m_j = n_j \rho_p \frac{\pi}{6} D_{p,j}^3}$$

mass in bin j

particles in bin j

mid-value diameter in bin j

Now we can do the same kind of statistical analysis on mass instead of number

· Define $\boxed{g(D_{p,j}) = \frac{m_j}{m_t}}$ = $\underline{\text{mass fraction}}$

mass in bin j

total mass

= Probability that particles are in bin j based on $\underline{\text{mass}}$ not $\underline{\text{number}}$ (similar to $f(D_{p,j})$)

· Define $\boxed{G(D_p) = \text{cumulative mass distribution}}$ (similar to $F(D_p)$ for number distrib.)

For the data of the previous handout — see next table ↓

Mass Distribution

Additional analysis of the sample particle data (class handout; also see Excel spreadsheet):

j = class (bin number)	$D_{p,\min,j}$ (lower limit)	$D_{p,\max,j}$ (upper limit)	$D_{p,j}$ (middle value)	$m_{j,in} = n_j \rho_p \pi D_{p,j}^3/6$ = mass in class j (mg)	$m_{j,in} / \Delta(D_{p,j})$ (mass in class j divided by class width)	$g(D_{p,j})_{in} = m_j/m_t$ (mass fraction of original aerosol)	$G(D_p)_{in} = (M_j/m_t)_{in}$ = cumulative mass distribution (%)
1	1	4	2.5	8.50848E-07	2.83616E-07	0.000373985	0.037398476
2	4	6	5	1.0472E-05	5.23599E-06	0.004602889	0.497687408
3	6	8	7	2.89147E-05	1.44573E-05	0.012709268	1.768614192
4	8	9	8.5	2.41166E-05	2.41166E-05	0.01060031	2.828645219
5	9	10	9.5	3.00777E-05	3.00777E-05	0.013220447	4.150689966
6	10	14	12	0.000168289	4.20722E-05	0.073970273	11.54771722
7	14	16	15	0.000107796	5.38979E-05	0.047380992	16.28581641
8	16	20	18	0.000241237	6.03092E-05	0.106034	26.88921637
9	20	35	27.5	0.00098003	6.53353E-05	0.430765712	69.96578759
10	35	50	42.5	0.000683305	4.55536E-05	0.300342124	100
11	50	100	75	0	0	0	100

Mass of particles in class j: m_j

Mass of particles in class j divided by class width: $m_j/(\Delta D_{p,j})$

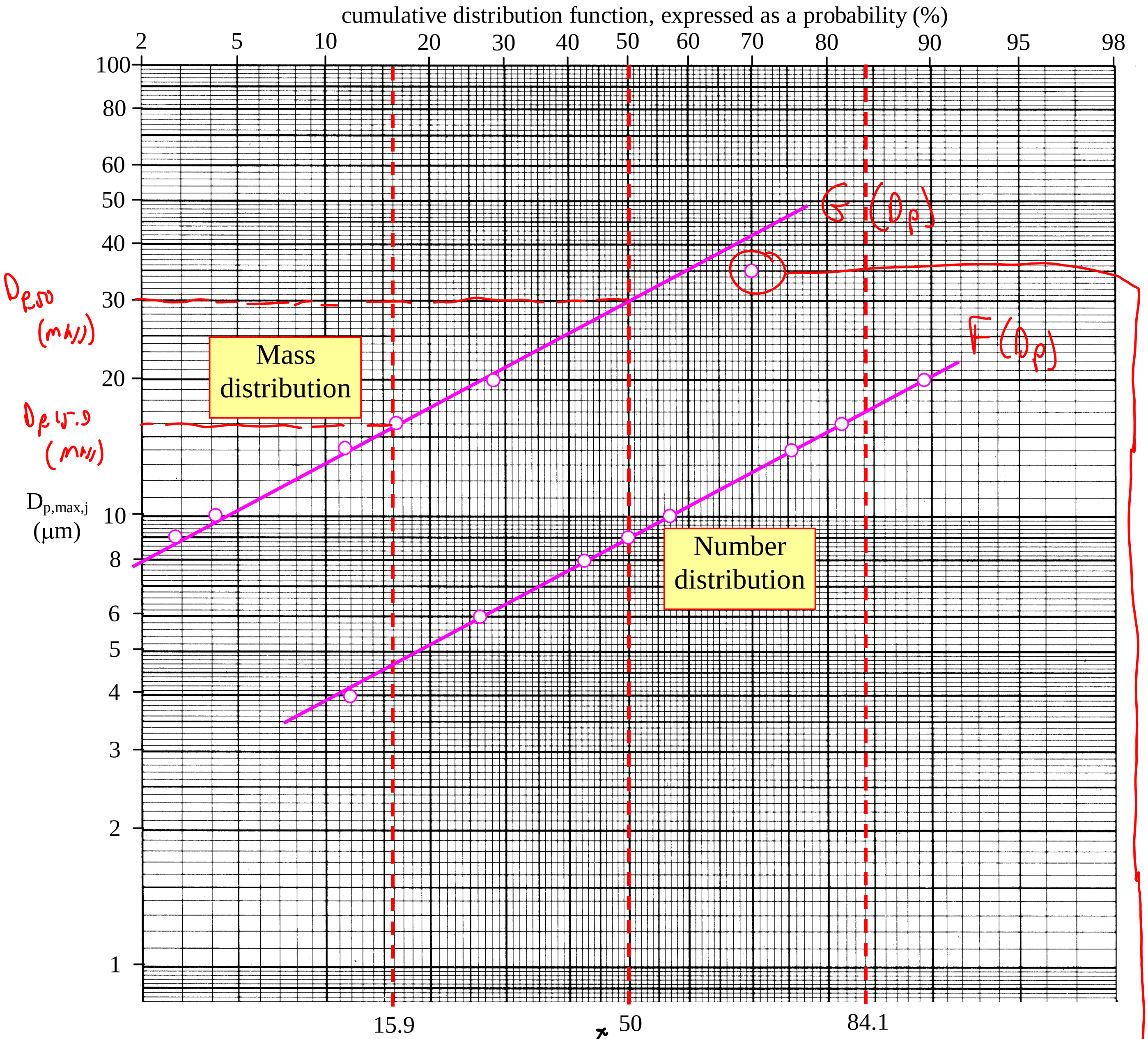
Mass fraction of particles in class j: $g(D_{p,j}) = m_j/m_t$

Cumulative mass distribution function: $G(D_p)$ [sometimes written as M_j/m_t]

WARNING → ☆

When plot this on log-probability paper, remember to use $D_{p,\max,j}$, *not* $D_{p,j}$ since the cumulative distribution function includes the *entire bin* from min to max.

Number distribution and mass distribution for the sample data of the class handout



Comments:

- $D_{p,50}$ (mass) > $D_{p,50}$ (number)
- The two lines are parallel for lognormal distributions
- σ_g is the same for both lines [since slopes are equal]
- Small errors @ large D_p are exaggerated on this kind of plot

Relationship between number & mass distributions

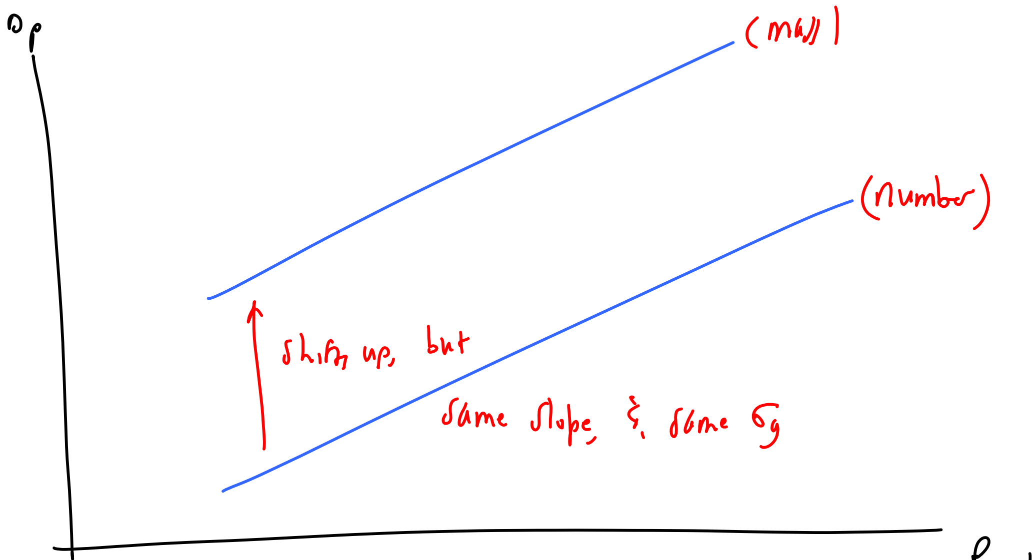
comes from D_p^3 to get mass

$$\boxed{\underbrace{\ln\left(D_{p,gm}(\text{mass})\right)}_{= D_{p,50}(\text{mass})} = \underbrace{\ln\left(D_{p,gm}(\text{number})\right)}_{= D_{p,50}(\text{number})} + 3\left(\ln \sigma_g\right)^2}$$

rearrange:

$$\boxed{\sigma_g = \exp\left\{\sqrt{\frac{\ln\left(D_{p,50}(\text{mass})\right) - \ln\left(D_{p,50}(\text{number})\right)}{3}}\right\}}$$

Log Prob. plot:



Comment about $\ln\left(D_{p,gm}\right)$ $\ln(\sigma_g)$

think of this as $\ln\left(\frac{D_{p,gm}}{1\ \mu m}\right)$

nondimensional so mathematicians can sleep at night