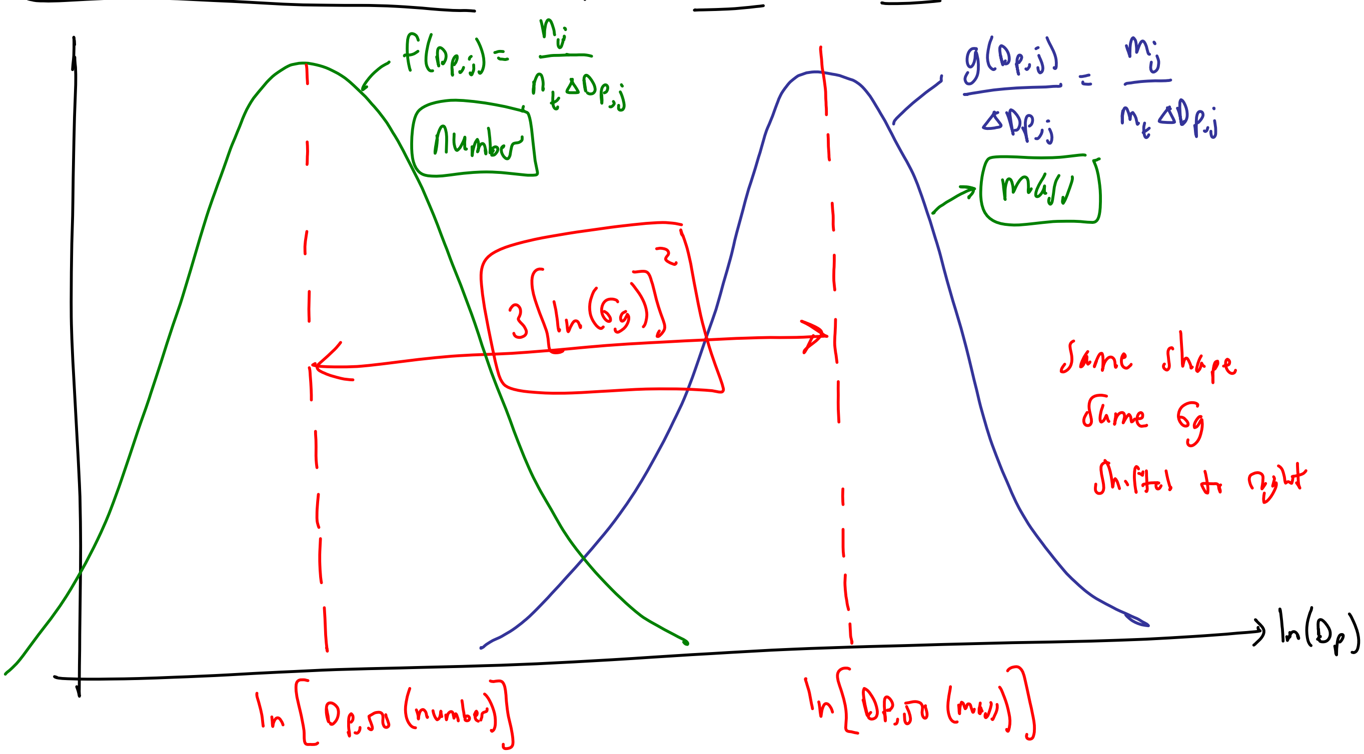
Today, we will:

- Continue discussing aerosol particle statistics: relationship between number and mass distributions
- Predict how the particle statistics change after passing through an APCS.

LOGNORMAL DISTRIBUTIONS – Compare number & mass PDFs



- The PDF shifts to the right (to higher D_p) by an amount $3[\ln(\sigma_g)]^2$

- $$\boxed{\ln\left(D_{p,50}\text{ (mass)}\right) = \ln\left(D_{p,50}\text{ (number)}\right) + 3\left(\ln(\sigma_g)\right)^2}$$

We can convert from mass dist. to number dist. & vice-versa

Example: Converting from mass distribution to number distribution

Given: Mingshou uses a cascade impactor to sample the air quality in Beijing. After carefully weighing all the trays before and after the sample, he plots the cumulative mass distribution on log-probability paper. The data fit fairly nicely into a straight line. From the plot, he determines that $D_{p,15.9}$ (mass) = 2.9 microns, and $D_{p,50}$ (mass) = 5.8 microns.

To do: For the air quality study that Mingshou is performing, he needs to know the median particle diameter based on *number*, not mass. From the given data, estimate $D_{p,50}$ (number). Give your answer in microns to two significant digits.

Solution: Some equations: $\boxed{D_{p,gm}(\text{number}) = D_{p,50}(\text{number})}$, $\boxed{D_{p,gm}(\text{mass}) = D_{p,50}(\text{mass})}$,

$$\boxed{\sigma_g = \frac{D_{p,50}(\text{number})}{D_{p,15.9}(\text{number})} = \frac{D_{p,50}(\text{mass})}{D_{p,15.9}(\text{mass})}}, \quad \boxed{\ln\left(D_{p,50}\ (\text{mass})\right) = \ln\left(D_{p,50}\ (\text{number})\right) + 3\left[\ln\left(\sigma_g\right)\right]^2}$$

Calc. σ_g: $\sigma_g = \dfrac{D_{p,50}}{D_{p,15.9}}$ = use (mass) $\rightarrow$ $\sigma_g = \dfrac{5.8}{2.9} = 2 = \sigma_g$

$$\ln\big(\overset{D_{p,50}}{D_{p,gm}}(\text{number})\big) = \ln\left(D_{p,gm}(\text{mass})\right) - 3\left(\ln \sigma_g\right)^2$$

$$\ln\left(D_{p,50}(\text{number})\right) = \ln(5.8) - 3\left(\ln(2)\right)^2 = 0.3165$$

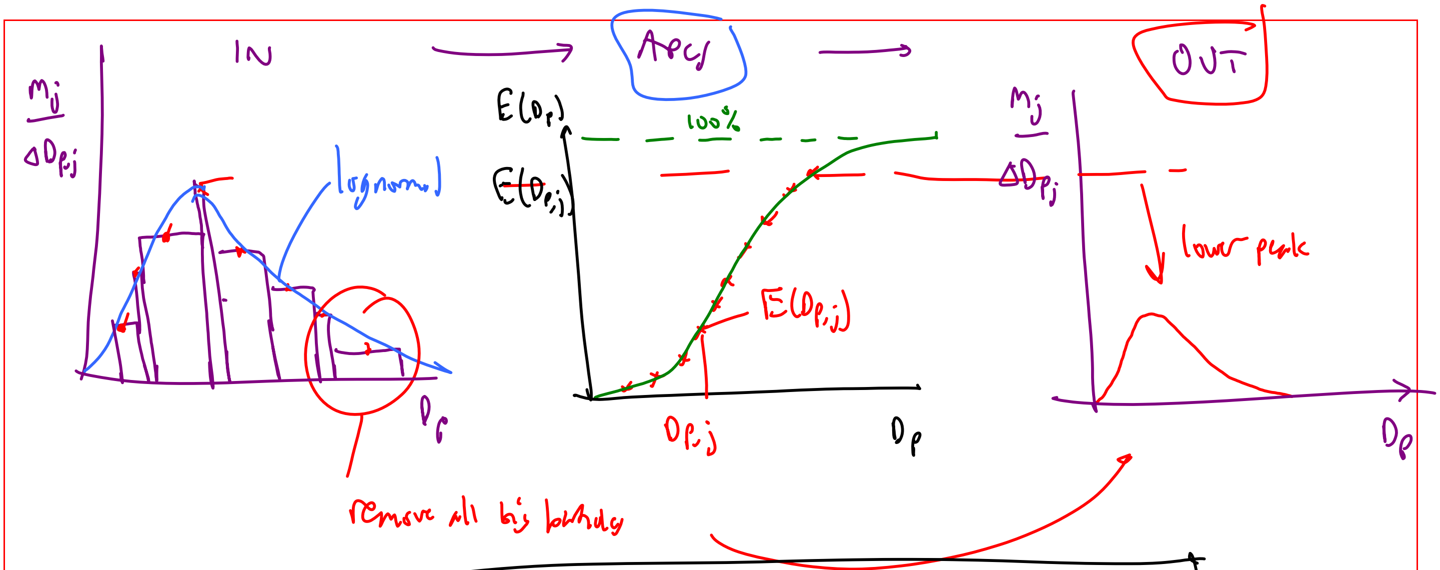
$$D_{p,50}(\text{number}) = e^{0.3165} = 1.37\ \mu m$$

$$\cong \boxed{1.4\ \mu m}$$

Particle Statistics Before and After an APCS

IN: Dirty Air $\longrightarrow$ APCS $E(D_p)$ $\longrightarrow$ cleaner air :OUT

$E(D_p)$ $\rightarrow$ For grouped data, we use $E(D_{p,i})$ @ mid value of each bin



$$\boxed{\underline{E(D_{p,j})} = 1 - \frac{m_{j\,out}}{m_{j\,in}} = 1 - \frac{m_{j\,out}/\forall}{m_{j\,in}/\forall} = 1 - \frac{C_{j\,out}}{C_{j\,in}}} \qquad (1)$$

Overall mass concentration C

$$\boxed{C_{in} = \sum_{j=1}^{J} \underbrace{c(D_{p,j})_{in}}_{C_{j\,in}}} \quad \text{\&} \quad C_{out} = \sum_{j=1}^{J} \underbrace{c(D_{p,j})_{out}}_{C_{j\,out}}$$

Overall collection (or removal) efficiency $E_{overall}$

$$E_{overall} = 1 - \frac{C_{out}}{C_{in}} = 1 - \frac{\sum_{j=1}^{J} C_{j\,out}}{C_{in}}$$

From Eq. (1) $\quad C_{j\,out} = \left[1 - E(D_{p,j})\right] C_{j\,in}$

$$E_{overall} = 1 - \sum_{j=1}^{J} \left[\frac{C_{j\,in}}{C_{in}} - E(D_{p,j})\frac{C_{j\,in}}{C_{in}}\right]$$

$$\left[g(D_{p,j}) = \frac{m_j}{m_t} \right] \qquad = 1 - \sum_{j=1}^{J} \frac{C_{j\,in}}{C_{in}} + \sum_{j=1}^{J} \left[E(D_{p,j}) \frac{C_{j\,in}}{C_{in}} \right]$$

C_{in}

$$\frac{m_j / \forall}{m_t / \forall} = g(D_{p,j})_{in}$$

$$E_{overall} = \cancel{1} - \cancel{1} + \sum_{j=1}^{J} \left[E(D_{p,j})\, g(D_{p,j})_{in} \right]$$

Overall collection Efficiency $\rightarrow$

$$\boxed{E_{overall} = \sum_{j=1}^{J} \left[E(D_{p,j})\, g(D_{p,j})_{in} \right]} \;\star$$

For grouped data (not necessarily lognormal)

Derive an expression for $g(D_{p,j})_{out}$

$$g(D_{p,j})_{out} = \left(\frac{m_j}{m_t} \right)_{out} = \left(\frac{m_j / \forall}{m_t / \forall} \right)_{out} = \frac{C_{j\,out}}{C_{out}}$$

By definition $\quad E(D_{p,j}) = 1 - \frac{C_{j\,out}}{C_{j\,in}} \rightarrow C_{j\,out} = C_{j\,in}\left(1 - E(D_{p,j})\right)$

$$g(D_{p,j})_{out} = \frac{C_{j\,in}}{C_{out}} \left(1 - E(D_{p,j})\right)$$

$$= \frac{C_{j\,in}}{C_{in}} \frac{C_{in}}{C_{out}} \left(1 - E(D_{p,j})\right)$$

$$\left[E_{overall} = 1 - \frac{C_{out}}{C_{in}} \right]$$

$g(D_{p,j})_{in} \qquad \frac{1}{1 - E_{overall}}$

$$\therefore \quad \boxed{g(D_{p,j})_{out} = g(D_{p,j})_{in} \frac{1 - E(D_{p,j})}{1 - E_{overall}}}$$

let's do an example using the same data from the handout

Example

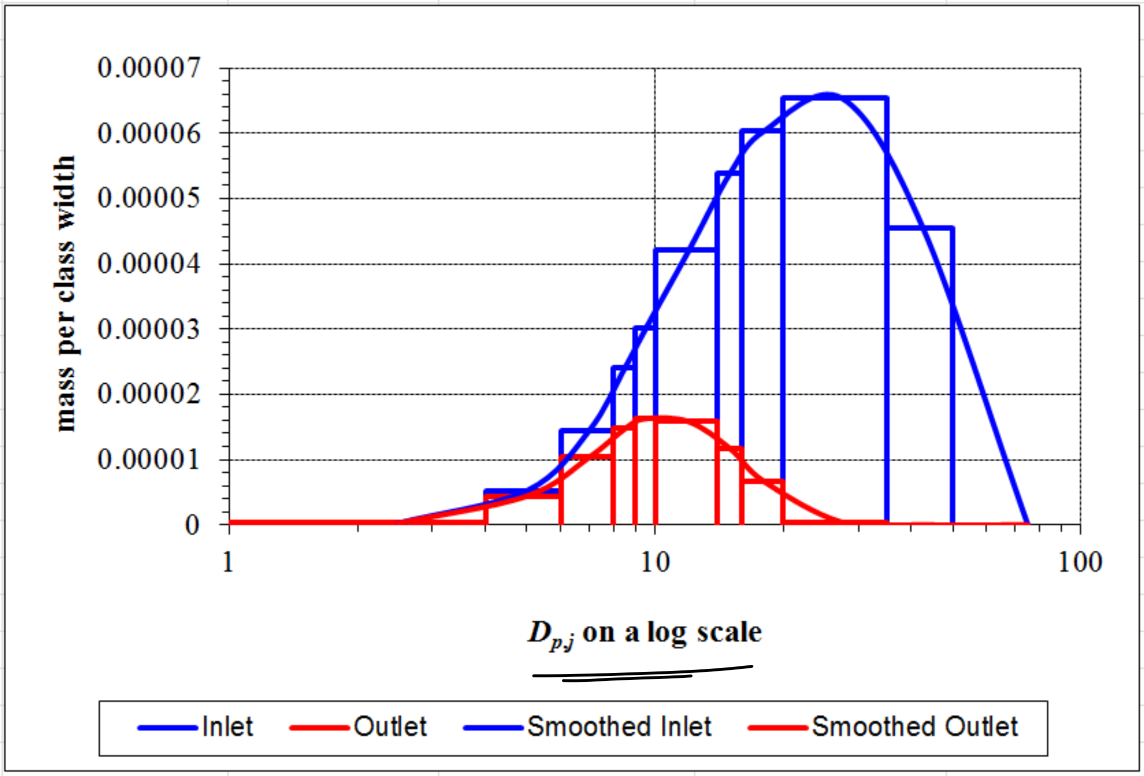
ε

Given: The particle distribution from the class handout passes through an air cleaner with grade efficiency $\eta(D_p) = 1 - \exp(-0.0068 D_p^2)$.

To do: Compare the mass distribution into and out of the air cleaner. Plot the inlet and outlet cumulative mass distributions on log probability paper and compare.

Solution: See the Excel spreadsheet on the course website.

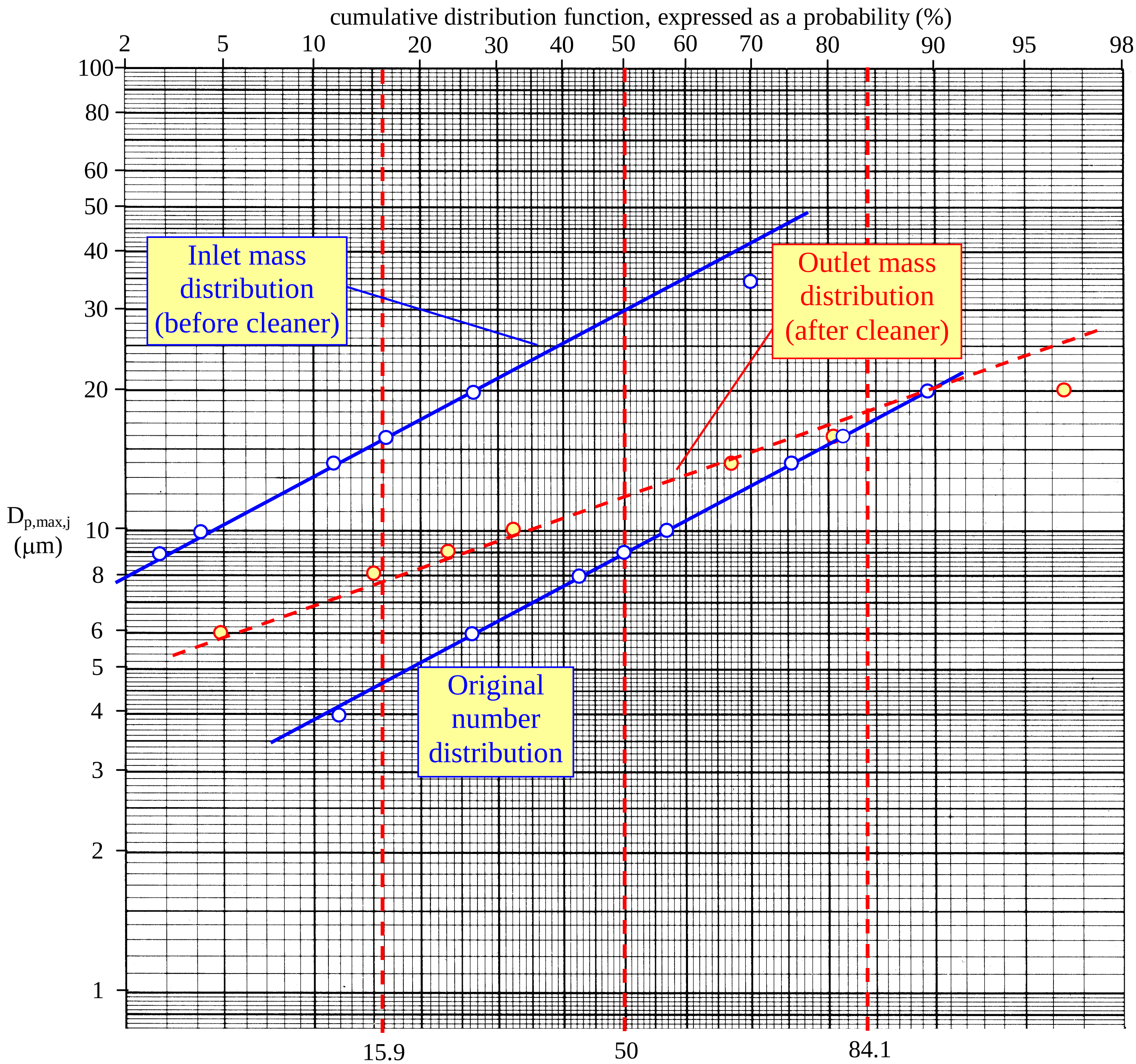
Comparison of mass distribution histogram at inlet and outlet:



★ SEE EXCEL FILE ON WEBSITE

(we utilize the eq's that we just derived)

Log probability plots: sample data of the class handout (before and after air cleaner)



NOTE: HERE, the outflow is also lognormal, but this is not always the case when passing through an APCS

If the inflow to the APCS is lognormal, the outflow is not necessarily lognormal